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INVERSE RESPONSE SURFACE METHOD IN RELIABILITY-BASED DESIGN

Abstract

The paper introduces an inverse response surface method utilized when performing reliability-based design optimization of time-consuming problems. Proposed procedure is based on a coupling of the adaptive response surface method and the artificial neural network-based inverse reliability method. The validity and accuracy of the method is tested using examples with explicit nonlinear limit state functions. Obtained results as well as important aspects of the method are discussed.

Keywords

Response surface, artificial neural network, inverse reliability analysis, reliability-based design, failure probability, reliability index.

1 INTRODUCTION

When structural design and reliability assessment is performed using probabilistic approach the reliability level related to a particular limit state is quantified via reliability indicators such as failure probability or reliability index. Since calculating these indicators for complex systems such as bridges is usually a time-consuming task, the utilization of approximation methods with a view to reducing the computational effort to an acceptable level is an appropriate solution. The general principle of approximation methods (also known as metamodels, surrogate models) is to replace the original limit state function (LSF) with an approximated (simpler) function whose evaluation is not so time-consuming. The failure probability calculation is then performed via the utilization of classical simulation methods but with the approximated function instead of the original one.

A popular approximation method is the response surface method [1], where the LSF is approximated using a suitable function which most often is of the polynomial type [2]. Other methods which have gained popularity among researchers over the last few decades include artificial neural network [3], polynomial chaos [4], support vector machine [5] and the Kriging metamodel [6].

Construction of a response surface requires all variables of stochastic model to be known in advance. However, during the structural design, which is an inverse task, there are design parameters which are subject of reliability-based design optimization procedure and thus not known at the start of the process. For such cases, an adaptive inverse response surface procedure is proposed in the paper.

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2 RESPONSE SURFACE APPROXIMATION

For the approximation of the original LSF, a second order polynomial function is most often employed:

$$\tilde{g}(\mathbf{X}) = a + \sum_{i=1}^{n} b_i X_i + \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} X_i X_j \tag{1}$$

where:

X – are the input basic variables and parameters,

a, b, c – are the unknown regression coefficients of the approximation function which can be obtained by conducting a series of numerical "experiments" with input variables selected according to an "experimental design".

The order of the polynomial selected for fitting to the discrete point outcomes affects the number of unknown parameters of this function, which need to be estimated, and consequently the number of the required evaluations of the original LSF. The number of experiments (original LSF calculations) significantly increases with an increase in the number of variables. In order to obtain the regression coefficients in Eq. (1), $1 + n + n(n + 1)/2$ experiments need to be conducted. Hence, an iterative response surface approach was presented by Bucher and Bourgund [7]. They suggested the polynomial function without the mixed terms $X_i X_j$ in the form of:

$$\tilde{g}(\mathbf{X}) = a + \sum_{i=1}^{n} b_i X_i + \sum_{i=1}^{n} c_i X_i^2 \tag{2}$$

with the same notation as in Eq. (1). Since the number of regression parameters is rather low, i.e. $2n + 1$, only a few numerical experiments are required.

In the next step, the function $\tilde{g}(\mathbf{X})$ is used to obtain an estimate of the "design point", $\mathbf{X}_\mathrm{D}$, for the surface $\tilde{g}(\mathbf{X}) = 0$ based on the assumption of uncorrelated Gaussian variables. Once $\mathbf{X}_\mathrm{D}$ is found, $\tilde{g}(\mathbf{X}_\mathrm{D})$ is evaluated and a new center point, $\mathbf{X}_\mathrm{M}$, for interpolation is chosen on a straight line from the mean vector $\overline{\mathbf{X}}$ to $\mathbf{X}_\mathrm{D}$, i.e.:

$$\mathbf{X}_\mathrm{M} = \overline{\mathbf{X}} + (\mathbf{X}_\mathrm{D} - \overline{\mathbf{X}}) \frac{\tilde{g}(\overline{\mathbf{X}})}{\tilde{g}(\overline{\mathbf{X}}) - \tilde{g}(\mathbf{X}_D)} \tag{3}$$

The same approximation using Eq. (2) is repeated using $\mathbf{X}_\mathrm{M}$ as the new anchor point, hence the total number of LSF evaluations is $4n + 3$. This process of updating the polynomial results usually leads to a sufficiently accurate response surface.

3 ARTIFICIAL NEURAL NETWORK-BASED INVERSE RELIABILITY METHOD

Inverse reliability analysis can be categorized as structural design, i.e. the identification of design parameters that enable the achievement of the desired reliability described by reliability indicators related to particular limit states. The parameters to be identified are deterministic or random design parameters related to the structure itself, the acting load or the surrounding environment. The known (in this case desired) response is the safety level described by reliability indicators. The functional relationship between design parameters and reliability indicators can take the form of an analytical formulation or a stochastic nonlinear finite element method model.

Let's include in addition to the vector of basic random variables $\mathbf{X} = X_1, X_2, \ldots, X_i, \ldots, X_n$ the vector of design deterministic parameters $\mathbf{d} = d_1, d_2, \ldots, d_k, \ldots, d_p$ and the vector of the design parameters of random variables $\mathbf{r} = r_1, r_2, \ldots, r_l, \ldots, r_q$. Note that the design parameters of random variables can be statistical moments of the first and/or second order. In the case of multiple limit states we have several safety margins Z_j and target failure probabilities $p_{\mathrm{f},j}$ or reliability indices β_j, where $j = 1, 2, ..., m$ is number of limit state functions. The inverse problem can generally be stated as:

$$\begin{aligned}&\text{Given}: \quad p_{\mathrm{f},j} \ \text{ or } \ \beta_j\\&\text{Find}: \quad \mathbf{d} \ \text{ and/or } \ \mathbf{r}\\&\text{Subject to}: \quad Z_j = g(\mathbf{X},\mathbf{d},\mathbf{r})_j = 0 \quad \text{for } j = 1, 2, ..., m\end{aligned} \tag{4}$$

A soft computing-based inverse reliability method has been proposed by Lehký and Novák [8]. The method is based on the coupling of a stratified Latin hypercube sampling (LHS) simulation technique and an artificial neural network (ANN). ANN, as a cornerstone of the method, is used as a surrogate model of unknown inverse function describing relation between the design parameters and corresponding reliability indicators:

$$\mathbf{P} = f_{\mathrm{ANN}}^{-1}(\mathbf{I}) \tag{5}$$

where:

$\mathbf{P} = \mathbf{d} \cup \mathbf{r}$ – is the vector of all design parameters (deterministic and random ones),

$\mathbf{I} = \boldsymbol{\beta}$ (or $\mathbf{I} = \mathbf{p}_{\mathrm{f}}$) – is the vector of reliability indicators.

The efficiency of the inverse method is emphasized by utilization of the small-sample LHS simulation method used for the stochastic preparation of the training set utilized in training the ANN. For that purpose, the design parameters **P** (e.g. mean values or standard deviations of selected random variables) are considered as random variables with a scatter reflecting the physical range of design values. Subsequently, the calculation of reliability is performed using appropriate simulation or approximation method and reliability indicators **I** are obtained. Once the ANN has been trained, it represents an approximation consequently utilized in a following way: To provide the best possible set of design parameters corresponding to prescribed reliability. See [8] for more complex explanation of the method.

4 INVERSE RESPONSE SURFACE METHOD

As described in Section 2, the response surface is an alternative to the real LSF. However, in contrast to the forward approach, when designing the structure, the function values that are used to construct the response surface are not available until the desired design variables are determined. Therefore, an inverse response surface method (IRSM) is proposed. It is based on a coupling of the adaptive RSM of Bucher and Bourgund [7] and the ANN-based inverse reliability method of Lehký and Novák [8]. The method is inspired by the procedure of Li [9], which combined the RSM with the Newton-Raphson iterative algorithm to solve inverse reliability problem [10]. The method proposed in this paper utilizes ANN and LHS methods which makes it more robust, efficient and therefore feasible for solving time-consuming problems such as structural design.

An iterative scheme to upgrade the response surface and, at the same time, to accomplish the inverse reliability analysis is proposed as follows:

1. In the first step of the IRSM, with the initial values for the design parameters, the initial response surface is constructed using foregoing RSM (polynomial RSM was used in this paper). Based on this approximate response surface the ANN-based inverse reliability analysis is carried out and a new estimate of design parameters is obtained as well as the design point.
2. In the second step, the new anchor point is calculated from the design point using Eq. (3). It serves together with the previously obtained design parameters for the response surface update. Based on this updated response surface the ANN-based inverse reliability analysis is carried out again to seek the new design parameters and the design point.
3. This process is repeated until the convergence is achieved at the design parameter with acceptable tolerance.

5 NUMERICAL EXAMPLES

5.1 Example 1

An explicit nonlinear LSF function [11] has been selected to show the procedure and demonstrate the validity and accuracy of the proposed method:

$$g(\mathbf{X}) = \exp[0.4(X_1 + 2) + 6.2] - \exp(0.3X_2 + d) - 200 \tag{6}$$

where:

X_1 and X_2 – are standard normal variables,

d – is an unknown deterministic parameter – subject of identification, see stochastic model in Tab.1.

The target reliability index is considered as $\beta = 2.688$ which corresponds to the target value of $d = 5$ (calculated using 10 million Monte Carlo simulations of the original LSF).

In order to construct the response surface and to perform ANN-based inverse reliability analysis the design parameter d has been treated as a uniformly distributed random variable with the initial range <4;8>, see Tab.2. A two-degree polynomial response surface without the mixed terms according to Eq. (2) has been used to substitute the original LSF. In order to calculate unknown coefficients of the response surface, 30 evaluations of the original LSF has been carried out with random samples of input parameters generated by LHS method.

Tab.1: Stochastic model

Variable	Distribution	Mean	Standard deviation
X_1	Normal	0	1
X_2	Normal	0	1
d	Deterministic	?	–

Tab.2: Randomization of the design parameter

Variable	Distribution	Mean	Standard deviation	Min	Max
d	Rectangular	6	1.155	4	8

Based on the constructed response surface the ANN-based inverse reliability analysis has been carried out. Utilized ANN consisted of two nonlinear neurons in a hidden layer and a linear output neuron corresponding to the design parameter d. There was one input to the network corresponding to reliability index. In order to create the training set, the reliability calculations using 1 million Monte Carlo simulations have been performed with 30 random samples of design parameter. After ANN training, the ANN is ready to provide the best design parameter related to the initial response surface. This is performed by means of a network simulation using target reliability index as an input.

With an updated design parameter, an updated response surface has been constructed for the next iteration. Here, the stochastic model has been changed with respect to the updated design parameter and the new anchor point calculated according to Eq. (3), i.e. random sampling was performed in a region closer to the design point. Standard deviation of the design parameter has been reduced to half of the original value in order to speed up the process and improve its convergence.

Tab.3 shows the values of design parameter and reliability index during iteration process. Here, two iterations have been enough for reaching acceptable accuracy. Let’s note that reliability index was calculated by 10 million Monte Carlo simulations of response surface.

Tab.3: Results of iterative process in Example 1

Design parameter, reliability index	Identification		Target value
	iteration 1	iteration 2	
d	4.832	4.999	5
β	2.979	2.669	2.688

5.2 Example 2

In order to give the graphic interpretation of the proposed iteration scheme, the LSF in the previous example has been rewritten as:

$$g(\mathbf{X}) = \exp[0.4(X_1 + 2) + 6.2] - \exp(d) - 200 \tag{7}$$

Stochastic model (excluding variable X_2) as well as target reliability index remain the same, expected design parameter is $d = 5.163$. Number of LHS simulations as well as ANN structure stay unchanged too. The values of design parameter and reliability index during iteration process can be seen in Tab.4. In this case, the results reached the good accuracy after three iterations.

Tab.4: Results of iterative process in Example 2

Design parameter, reliability index	Identification			Target value
	iteration 1	iteration 2	iteration 3	
d	5.468	5.203	5.156	5.163
β	2.655	2.668	2.687	2.688

Figure 1 shows how response surface approach the real LSF (g_{orig}) graphically. In the figure, there are three response surfaces, each depicted for both initial ("ini") value as well as updated ("upd") value of the design parameter in each iteration (1 to 3). Figure also shows design points of all response surfaces (white bullets), i.e. the points that has the highest contribution to the probability integration.

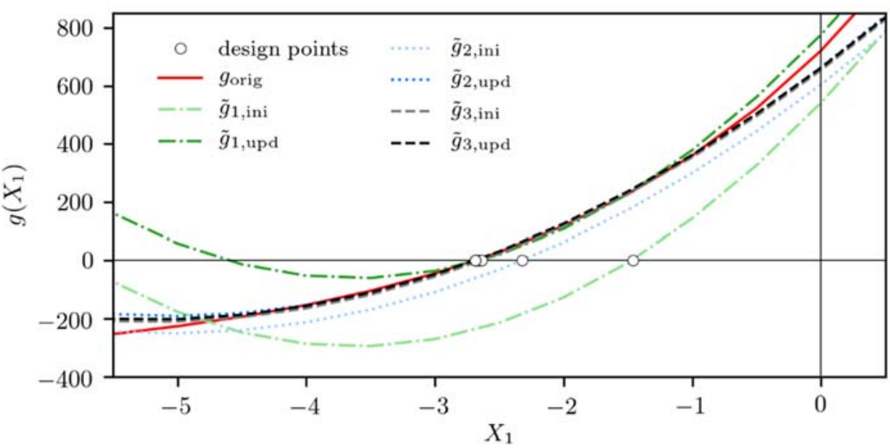

Fig.1: Evolution of response surfaces in iterative process

6 CONCLUSIONS

From results, we can conclude that, even after couple of iterations, the iterative procedure significantly improves the quality of utilized response surface when performing structural reliability-based design. Generally, the initial response surface approximation and consequent inverse reliability analysis cannot result in sufficiently accurate design parameter. A number of iterations and successful convergence of the process is dependent on the shape of the original limit state function and the selection of an initial range of design parameter. Let's also note that for highly nonlinear problems two-degree polynomial function may not be sufficient and use of a more complex surrogate model may be necessary.

In general, for complex systems, the RSM is the only way to approach both forward analysis and inverse analysis since there is no other method which can give the solution with an acceptable level of computational effort. When utilizing RSM for inverse problems, the iterative procedure should be performed to ensure accuracy of reliability-based design and reliability assessment.

The future work will be focused on utilization of the method in multiple design parameter and multiple reliability constraint problems. Following the development of the ANN-RSM method, the use of a more robust ANN surrogate model will also be tested.

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