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DYNAMIC RESPONSE OF RAILWAY BRIDGES SUBJECTED TO PASSING VEHICLES

Abstract

This paper discusses some issues related to dynamic effects in railway bridges focussed on the dynamic behaviour of the small and medium span simply supported railway bridges subjected to a series of moving vehicle. Presented parametric study is focused on the dynamic deflection of the simply supported railway bridge of the span L_b = 38 m, due to the series moving loads representing a conventional train with the IC-coaches, with the impact to the speed up to 160 km/h applied in Slovakia.

Keywords

Dynamic response, railway bride deflections, the modal superposition method

1 INTRODUCTION

The dynamic response of railway bridges subjected to moving trains is influenced by a number of factors such as the speed of load, the bridge span, natural frequencies of the bridge and railways vehicles, the inertia and damping of the two interaction systems (vehicles and the bridge), the distance between the vehicles, and arranging axles of vehicles. At present, the actual question for the bridge loading follows from high speed trains, which may consist of a number of identical cars connected together moving with the speed *c*. In these cases the resonance caused by configuration of the train consisting of a number of vehicles similar types (Fig.1) may occur especially at high speed ranges.

To solve indicated problems need to apply the special dynamic analysis depending on the type of the bridge structure with regard of the static determination of the structure. For statically indeterminate structures, like continuous deck bridges or frame structures, more sophisticated methods of analysis (FEM) must be applied. For the simple bridges the solution is based on the modal superposition method.

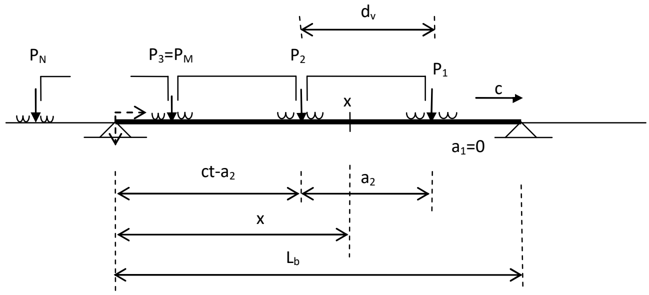

Fig. 1. Loading of the bridge by a series of identical IC-vehicles.

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Railway vehicles travelling along the bridge are modelled as a series of identical moving loads and assuming the vehicle/bridge mass ratio is small m_v<<m_b and loads move along the bridge, the close form of solution can be obtained. The dynamic displacement $w(x,t)$ and acceleration $\ddot{w}(x,t)$ of the bridge are governed at different extents by two sets of frequencies:

- driving frequency of a vehicle $\omega_{(j),dr}$ crosses the bridge. j=1,2...,
- natural frequency of the bridge $\omega_{(j)}$, j=1,2,...

One of the actual problems is the solution the dynamic behaviour of the bridge subjected to a series of identical loads $\sum_n P_n$, $n=1,2,..N$, with identical space intervals d_v (the length of the vehicle) travelling across the bridge with a constant speed c as is shown in Fig.2.

In this paper the dynamic behaviour of the simply supported railway bridge with the span L_b = 38 m, subjected to the successive identical moving loads is solved. The close form solution is applying by means of the modal superposition method [1, 2]. The presented parametric study is focused on the dynamic deflection of the bridge at the mid-span $w(L_2/2,\mathrm{t})$ due to loading of the conventional train with the IC-coaches (coaches are supported by the two bogies, (Fig.2) and with the impact to the moving speed up to 160 km/h.

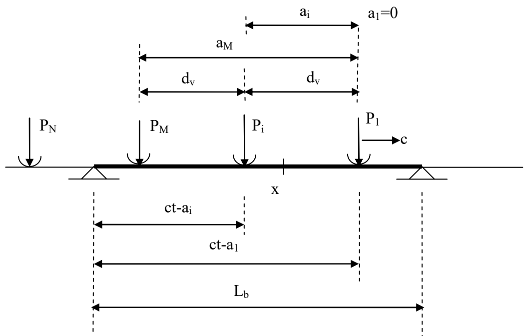

Fig. 2. The single-span bridge subjected to moving loads $\sum_n P_n$.

Using the analytical approach, the key parameters that govern the dynamic displacement response – the vertical beam vibration ${}^{(c)}w_{(P_1,P_2,....P_n)}(x,t)$, for n = 1, 2, 3…M loads moving on the bridge are applied on the railway steel girder of the length L_b = 38 m loaded by the IC-cars length d_v= 24,5 m with the magnitude P_v= 524 kN, Fig. 1.

2 FORMULATION OF THE THEORY FOR THE BRIDGE RESPONSE INDUCED BY MOVING LOAD SERIES

Consider a simply supported beam (without damping) subjected to a series of concentrated constant loads P which move at a uniform speed c in the meaning of Fig. 2. The motion equation for the beam subjected periodically loading of moving load series can be writing as

$$EI\frac{\partial^4 w(x,t)}{\partial x^4}+m_1\frac{\partial^2 w(x,t)}{\partial t^2}=\sum_{n=0}^{M-1}P\delta(x-\mathrm{c}(t-\frac{nd_v}{c})),\ n=1,2,...M, \tag{1}$$

where:

$w(x,\mathrm{t})$ - is the displacement of the beam at point x and time t [m],

EI - is the bending stiffness of the beam [kNm^2],

m_1 - is the mass per unit length of the beam [t],

$P_{v,i} \equiv P_i$ - is the loading force [kN],

$\delta(x)$ - is the Dirac function,

c - is a uniform speed of moving loads [m/s],

L_b - is span length of the bridge [m],

d_v - is identical interval between loading forces [m].

For a simple beam the solution of the vertical deflection $w(x,\mathrm{t})$ in Eq. (1) becomes the harmonic analysis. The particular solution $w(x,\mathrm{t})$ for a simply supported beam can be expressed in term of modal time coordinates $q_{(j)}(t)$ for the beam vibration and the modal shapes $\phi_j(x)$ as

$$w(x,t) = \sum_j q_{(j)}(t)\phi_j(x) = \sum_j q_{(j)}(t)\sin(\frac{j\pi x}{L_b}),\ j=1,2,3..., \tag{2}$$

where:

$q_{(j)}(t)$ - are the generalized coordinates that define the amplitude of vibration with time t,

$\phi_j(x) = \sin(\frac{j\pi x}{L_b})$ - is a fundamental mode shape and for the simply beam is of the sinusoidal type (the first mode is a one-half cycle; the second mode is a full cycle).

Thus, the dynamic deflection may be represented by the summation of modal components. When the first and last moving load on the bridge span be P_1 and P_M at time t, Eq. (1) can be expressed in terms of the generalized coordinates as

$$\frac{d^2 q_{(j)}(t)}{dt^2} + \omega_{(j)}^2 q_{(j)}(t) = \frac{2P}{m_1 L_b}\sum_{n=0}^{M-1}\sin(\frac{j\pi c}{L_b}(t-\frac{nd_v}{c}))H_n,\ n=1,2,..M \tag{3}$$

for M forces moving on the bridge,

where:

$H_n = \theta(t-\frac{nd_v}{c}) - \theta(t-(\frac{nd_v}{c}+\frac{L_b}{c}))$ is the Heaviside function determining whether the load P_n is on the bridge or not.

The modal coordinate $q_{(j)}(t)$ for the j-th mode of vibration of the beam from Eq. (3) can be expressed as [1, 2]:

$$q_{(j)}(t) = \frac{\hat{q}_{(\mathrm{j})\mathrm{st}}}{1-\alpha_{(j)}^2}\sum_{n=0}^{M-1}\left[\sin(\omega_{(j)\mathrm{dr}}(\mathrm{t}-\frac{nd_v}{c})) - \alpha_{(j)}\sin(\omega_{(j)}(\mathrm{t}-\frac{nd_v}{c}))\right]H_n(t), \tag{4}$$

where:

$\hat{q}_{(j)\mathrm{st}} = \frac{2P}{m_1 L_b \omega_{(j)}^2} = \frac{2PL_b^3}{j^4\pi^4 EI}$ - is the modal amplitude - the static deflection caused by the force P with respect to the j-th mode,

$\omega_{(j)} = \frac{\pi j^2}{L_b^2}\sqrt{\frac{EI}{m_1}}$ - is the j-th circular frequency of the beam vibration,

$\alpha_{(j)} = \frac{\omega_{(j)dr}}{\omega_{(j)}} \equiv \frac{c}{c_{cr}}$ is the non-dimensional speed parameter,

$c_{cr} = 2f_{(1)}L_b = \frac{\omega_{(1)}L_b}{\pi}$ is the critical speed,

$\omega_{(j)\mathrm{dr}} = \dfrac{j\pi c}{L_b}$ - is the circular driving frequency of the moving force for the j-th mode of vibration.

2.1 Single-mode analytical solution

The vertical deflection $w(x,\mathrm{t})$ in Eq. (1) for a simply supported beam for a moving load problem can be well simulated by considering the first mode of vibration $\omega_{(1)}$ only. The corresponding modal coordinate from Eq. (4), taking into account M-vehicles on the bridge, is given by superposition of a forced response (a quasi-static response) due to the moving load and a transient response (the dynamic part of the response) can be expressed as follows:

$$ {}^{(c)}q_{(1),(P_1,P_2,\dots\dots P_M)}(t) = \frac{\hat{q}_{(1)st}}{1-\alpha_{(1)}^2}\sum_{n=0}^{M\text{-}1}\left[\sin(\omega_{(1)dr}(t-\frac{nd_v}{c})) - \alpha_{(1)}\sin(\omega_{(j)}(t-\frac{nd_v}{c}))\right]H_n(t)\,, \tag{5} $$

where:

$\alpha_{(1)} = \dfrac{\omega_{dr(1)}}{\omega_{(1)}} = \dfrac{\pi c}{L_b\omega_{(1)}} = \dfrac{c}{c_{cr}}$ is the non-dimensional speed parameter corresponding to the first mode of vibration $\omega_{(1)}$,

The vertical deflection ${}^{(c)}w_{(1),(P_1,P_2,\dots\dots P_N)}(L_b/2,t)$ for the mid-span $x = L_b/2$ is equal the modal coordinate ${}^{(c)}q_{(1),(\mathrm{P}_1,P_2,\dots\dots P_N)}(L_b/2,t)$, because $\sin(\dfrac{\pi x}{L_b)} = \sin(\pi/2) = 1{,}0$.

$$ \begin{aligned} {}^{(c)}w_{(1),(P_1,P_2,\dots P_M)}(L_b/2,t) &= {}^{(c)}q_{(1),(\mathrm{P}_1,P_2,\dots\dots P_N)}(t) = \\ &= \frac{\hat{q}_{(1)st}}{1-\alpha_{(1)}^2}(1{,}0)\sum_{n=0}^{M\text{-}1}\left[\sin(\omega_{(1)dr}(t-\frac{nd_v}{c})) - \alpha_{(1)}\sin(\omega_{(j)}(t-\frac{nd_v}{c}))\right]H_n(t) \end{aligned} \tag{6} $$

2.2 The displacement response at the mid-span of the beam

The important practical significance is just the dynamic deflection ${}^{(c)}w_{(1),(P_1,P_2,\dots\dots P_N)}(L_b/2,t)$ at the mid-span $x = L_b/2$ for loads a moving series (P_1, P_2 ,....P_N). If the number of the load moving out of the span is K and the number of moving forces moving just on the span is M at the time t, the displacement response of the beam can be generalized considering the superposition of the next loading effects [1,6,7]:

$$ \begin{aligned} {}^{(c)}w_{(1),(\mathrm{P}_1,P_2,\dots P_N)}(\mathrm{L}_b/2,t) &= {}^{(c)}q_{(1),(\mathrm{P}_1,P_2,\dots P_N)}(\mathrm{L}_b/2,t)\phi_1\left(L_b/2\right) = \\ &= \frac{\hat{w}_{(1)st}(L_b/2)}{1-{}^{(c)}\alpha_{(1)}^2}(1{,}0)\sum_{n=K}^{M-1}\sin{}^{(c)}\omega_{(1)dr}(t-\frac{nd_v}{c}) - \\ &-\frac{\hat{w}_{(1)st}(L_b/2)}{1-{}^{(c)}\alpha_{(1)}^2}\sum_{n=K}^{M-1}{}^{(c)}\alpha_{(1)}\,\mathrm{e}^{-\omega_d(t-\frac{nd_v}{c})}\sin\omega_{(1)}(t-\frac{nd_v}{c}) - \\ &-\frac{\hat{w}_{(1)st}(L_b/2)}{1-{}^{(c)}\alpha_{(1)}^2}\sum_{n=0}^{K-1}\left({}^{(c)}\alpha_{(1)}\,\mathrm{e}^{-\omega_d(t-\frac{nd_v}{c})}\sin\omega_{(1)}(t-\frac{nd_v}{c}-\frac{L_b}{c})\right) - \\ &-\frac{\hat{w}_{(1)st}(L_b/2)}{1-{}^{(c)}\alpha_{(1)}^2}\sum_{n=0}^{N-1}\left({}^{(c)}\alpha_{(1)}\,\mathrm{e}^{-\omega_d(t-\frac{nd_v}{c}-\frac{L_b}{c})}\sin\omega_{(1)}(t-\frac{nd_v}{c}-\frac{L_b}{c})\right) \end{aligned} \tag{7} $$

for n = 1,2,3,….N = M+K, and for $\sin(\pi/2) = 1{,}0$.

The first term in Eq. (7) represents the component corresponding to each moving load P_n travelling over the beam – it produces *the force vibration response* (the quasi-static part of the response). The second and the thirty term in Eq. (7) produces *the free vibration response* (the dynamic part of the response) considering forces moving just on the beam and forces moving away of the span. The fourth term in Eq. (7) represents the free vibration term after all moving loads left the span. Thus, the beam response ${}^{(c)}w_{(1),(P_1,P_2,.....P_N)}(L_b/2,t)$ can be symbol written as

$$
\begin{aligned}
{}^{(c)}w_{(1),(P_1,P_2,.....P_N)}(L_b/2,t) &= {}^{(c)}w_{(1)\mathrm{st},(P_1,P_2,.....P_M)}(L_b/2,t)+ \\
&+{}^{(c)}w^{(1)}_{(1)\mathrm{dyn},(P_1,P_2,.....P_M)}(L_b/2,t)+{}^{(c)}w^{(2)}_{(1)\mathrm{dyn},(P_1,P_2,.....P_K)}(L_b/2,t)
\end{aligned}
\tag{8}
$$

3. THE DISPLATE SINGLE-SPAN RESPONSE UNDER SEQUENCE OF MOVING LOAD (P1+P2+.....+P10) CROSSING THE BEAM

The steel truss bridge L_b=38 m is subjected by the series of 10 IC-cars ($P_1+P_2+...P_{10}$) in the sense of Fig. 3. The load of equal weight P = 524 kN is spaced at the car interval d_v=24,5 m, the train speed is c_2 =33 m/s=118,8 km/h. As was stated above it is essentially a transient problem with very short acting time.

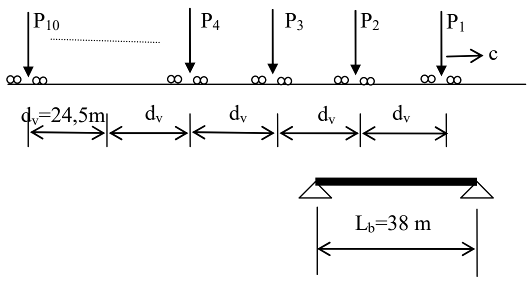

Fig. 3. The beam subjected to the series of loads (P1+P2+.....P10) moving across the beam.

Input parameters: The bending stiffness of the bridge for the two rail loading model is: EI = 7,58.10^7 kNm^2, the beam mass per unit length $m_1 = m_{1(\mathrm{BStr})} + m_{1(\mathrm{Sup})}$ = 3,18 t/m, the first circular frequency of the bridge $\omega_{(1)} = \frac{\pi^2}{L_b^2}\sqrt{\frac{EI}{\bar{m}_1}}$ = 33,34 [s^{-1}], $\hat{w}_{(1)st}(\mathrm{L}_b/2)$ = 0,00786 [m],

${}^{(c_2)}\omega_{(1)dr} = \frac{\pi c_2}{L_b} = \frac{3,14.33}{38}$ = 2,73 m^{-1}, $\frac{\hat{w}_{(1)st}(L_b/2)}{1-\alpha^2_{(1)}}$ = 0,0079[m], $\alpha_{(1)} = \frac{\omega_{(1)dr}}{\omega_{(1)}}$ =0,0819 ,

$t_{b(P)} = \frac{L_b}{c_2}$ =<u>1,1515</u>, $\frac{d_v}{c} = \frac{24,5}{33}$ = 0,7424 s, the damping coefficient ω_d is expressed by means the logarithm decrement ϑ: $\omega_d = f_{(damp)}\vartheta = \frac{\omega_{(1)}}{2\pi}\vartheta$= 0,1327 s^{-1}.

3.1. Components of the beam deflection due to the sequence of load IC-cars ($P_1+P_2+…P_{10}$)

Components of the beam response due to the sequence of load IC-cars ($P_1+P_2+…P_{10}$) are defined by expressions (7).

- **Quasi-static component** ${}^{(c_2=33)}w_{(1)\mathrm{st},(P_1,P_2,\dots P_{10})}(L_b/2,t)$

The quasi-static component of the response includes the effect of forces (P1+P2+.....P10) moving over the beam (without damping) and is defined by the first term in Eq. 7. Because drawing the displacement w downward direction there is apply the sign minus. The result is plotted in Fig. 4.

$$ {}^{(c_2=33)}w_{(1)\mathrm{st},(P_1,P_2,\dots P_{10})}(L_b/2,t) = -\left[\frac{\hat{w}_{(1)st}(L_b/2)}{1-{}^{(c_2=33)}\alpha_{(1)}^2}\sum_{n=0}^{M=9}\sin\left({}^{(c_2=33)}\omega_{(1)dr}(t-\frac{nd_v}{c_2})\right)H_{(\mathrm{n})}\right] \quad (9) $$

Result ${}^{(c_2=33)}w_{(1)\mathrm{st},(P_1,P_2,\dots P_{10})}(L_b/2,t)$

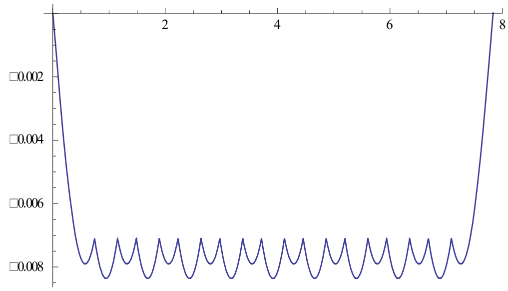

Fig. 4. The quasi-static component of the deflection ${}^{(c_2=33)}w_{(1)\mathrm{st},(P_1,P_2,\dots P_{10})}(L_b/2,t)$: Axes: x= t[s],y= w[m], the load ($P_1,P_2...P_{10}$) =10x528 kN that move over the beam, t_b = 0 to 7,8331s.

- **Dynamic component** ${}^{(c_2=33)}w^{(1)}_{(1)\mathrm{dyn},(P_1,P_2\dots P_{10})}(L_b/2,t)$

The dynamic component of the beam deflection includes the response defined by the second term in Eq. 7. This dynamic response is plotted in Fig. 5.

$$ {}^{(c_2=33)}w^{(1)}_{(1)dyn,(P_1,P_2,\dots P_{10})}(L_b/2,t) = -\left[\frac{\hat{w}_{(1)st}(L_b/2)}{1-{}^{(c_2=33)}\alpha_{(1)}^2}\sum_{n=0}^{M=9}\left(-{}^{(c_2=33)}\alpha_{(1)}\sin(\omega_{(1)}(\mathrm{t}-\frac{nd_v}{c_2}))\right)H_{(\mathrm{n})}\right] \quad (10) $$

Result ${}^{(c_2=33)}w^{(1)}_{(1)\mathrm{dyn},(P_1,P_2\dots P_{10})}(L_b/2,t)$

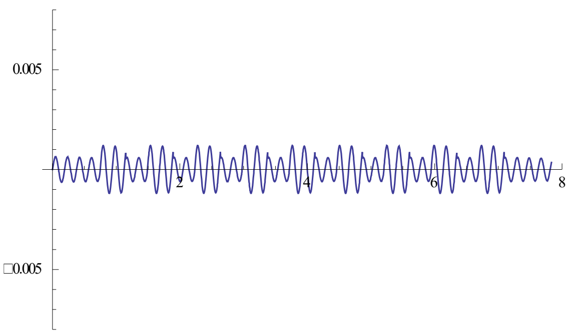

Fig. 5. The dynamic component ${}^{(c_2=33)}w^{(1)}_{(1)\mathrm{dyn},(P_1,P_2\dots P_{10})}(L_b/2,t)$: Axes: x= t[s], y= w [m] for the loads (P1+P2,...+P10)=10x528 kN, that move direct over the beam for t_b = 0 to 7,8331s.

- **The total deflection beam response** ${}^{(c_2=33)}w^{(a)}_{(1),(P_1,P_2\dots P_{10})}(L_b/2,t)$ **for the load (P_1, P_2, ...P_{10}) moving direct over the beam**

The deflection response ${}^{(c_2=33)}w_{(1),(P_1,P_2\dots P_{10})}(L_b/2,t)$, belong to the load ($P_1$, P_2,....P_{10}) =10x528 kN, moving direct over the beam and is superposition quasi-static and the dynamic component.

Because drawing the displacement w downward direction there is apply the sign minus. The result is plotted in, Fig. 6.

$$^{(c_2=33)}w^{(a)}_{(1),(P_1,P_2...P_{10})}(L_b/2,t) = {}^{(c)}w_{(1)\,\mathrm{st},(P_1,P_2,.....P_N)}(L_b/2,t) + {}^{(c)}w^{(1)}_{(1)\,\mathrm{dyn},(P_1,P_2,.....P_N)}(L_b/2,t) \qquad (11)$$

Result $^{(c_2=33)}w^{(a)}_{(1),(P_1,P_2...P_{10})}(L_b/2,t)$

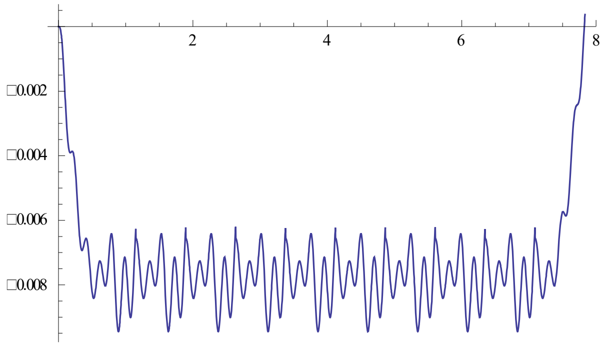

Fig. 6. The deflection response $^{(c_2=33)}w^{(a)}_{(1),(P_1,P_2...P_{10})}(L_b/2,t)$, Axes: x= t[s], y= w [m] for the loads (P1+P2,...+P10)=10x528 kN that move over the beam, $^{(c_2=33)}\Delta^{(\mathrm{a})}_{dyn(P_1,P_2........P_{10})} = \dfrac{0,0095}{0,0085}$ = 1,1176.

- **Dynamic component** $^{(c_2=33)}w^{(2)}_{(1)\,\mathrm{dyn},(P_1,P_2...P_{10})}(L_b/2,t)$ **- the load moving (P1,P2….P10) out of the span**

This dynamic component belong to loads (P1,P2….P10), that have passed the beam, but they still influence the response. The result is plotted in, Fig. 7.

$$^{(c_2=33)}w^{(2)}_{(1)\,\mathrm{dyn},(P_1,P_2...P_{10})}(L_b/2,t) = -\left[\begin{array}{l} \dfrac{\hat{w}_{(1)st}(L_b/2)}{1-{}^{(c_2=33)}\alpha^2_{(1)}} \\ \displaystyle\sum_{n=0}^{K=9}\left(-{}^{(c_2=33)}\alpha_{(1)}\,\mathrm{e}^{-\omega_d\left(t-\frac{nd_v}{c_2}\right)}\sin\omega_{(1)}\left(t-\frac{nd_v}{c_2}-\frac{L_b}{c_2}\right)\right)H_{(n)} \end{array}\right] \qquad (12)$$

Result $^{(c_2=33)}w^{(2)}_{(1)\,\mathrm{dyn},(P_1,P_2...P_{10})}(L_b/2,t)$

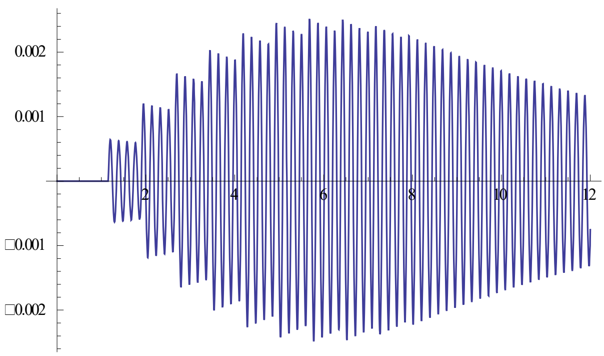

Fig. 7. The dynamic component $^{(c_2=33)}w^{(2)}_{(1)\,\mathrm{dyn},(P_1,P_2...P_{10})}(L_b/2,t)$ **:** Axes: x= t[s], y= w [m], for the loads (P_1, P_2.........P_{10}) = 10x528 kN that have passed the beam, t = 0 to 12 s.

- **Components of the total response – superposition**

The components of the total deflection response ${}^{(c_2=33)}w_{(1),(P_1,P_2...P_{10})}(L_b/2,t)$, belong to the load (P1, P2,….P10) that move direct over the beam and to the loads that have passed the beam. The components are described by Eq. (9), (10) and (12). The components of the total deflection ${}^{(c_2=33)}w_{(1)st,(P_1,P_2,.....P_{10})}(L_b/2,t)$, ${}^{(c_2=33)}w^{(1)}_{(1)\mathrm{dyn},(P_1,P_2,.....P_{10})}(L_b/2,t)$, ${}^{(c_2=33)}w^{(2)}_{(1)\mathrm{dyn},(P_1,P_2,.....P_{10})}(L_b/2,t)$ are plotted in, Fig. 8.

Result

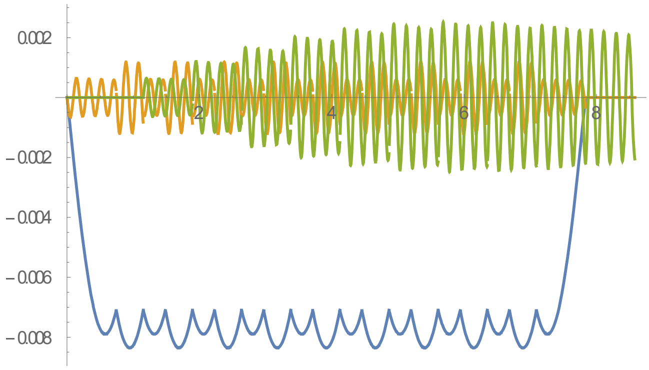

Fig. 8. Components of the total deflection: ${}^{(c_2=33)}w_{(1)st,(P_1,P_2,.....P_{10})}(L_b/2,t)$, ${}^{(c_2=33)}w^{(1)}_{(1)\mathrm{dyn},(P_1,P_2,.....P_{10})}(L_b/2,t)$, ${}^{(c_2=33)}w^{(1)}_{(1)\mathrm{dyn},(P_1,P_2,.....P_{10})}(L_b/2,t)$: Axes: x=t[s], y=w[m].

3.2. The total response due to the series of loads IC-cars (P1+P2+…P10)

The total response ${}^{(c_2=33)}w^{(a+2)}_{(1),(P_1,P_2...P_{10})}(L_b/2,t)$ due to the series of load IC-cars (P1+P2+…P10) is defined superposition quasi-static and the dynamic components described by expressions (9), (10) and (12). The beam response ${}^{(c)}w^{(a+2)}_{(1),(P_1,P_2,.....P_N)}(L_b/2,t)$ can be symbol written as

$$\begin{aligned}{}^{(c_2=33)}w^{(a+2)}_{(1),(P_1,P_2...P_{10})}(L_b/2,t) &= {}^{(c_2)}w_{(1)st,(P_1,P_2,.....P_{10})}(L_b/2,t)+\\ &+{}^{(c_2)}w^{(1)}_{(1)\mathrm{dyn},(P_1,P_2,.....P_{10})}(L_b/2,t)+{}^{(c_2)}w^{(2)}_{(1)\mathrm{dyn},(P_1,P_2,.....P_{10})}(L_b/2,t)\end{aligned} \tag{13}$$

The result is plotted in Fig. 9.

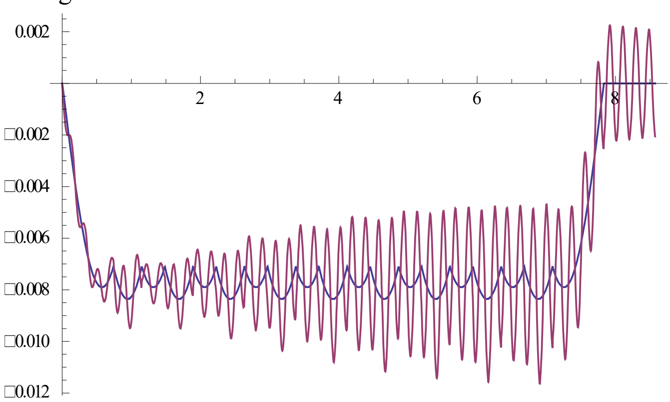

Fig. 9. The total beam displacement response ${}^{(c_2=33)}w^{(a+2)}_{(1),(P_1,P_2...P_{10})}(L_b/2,t)$: Axes: x=t[s], y=w,

for t= 0 to 8.5755 s, ${}^{(c_2=33)}\Delta^{(a+2)}_{dyn(P_1,P_2........P_{10})}=\dfrac{0,012}{0,0085}$ =1,4118.

Comparison the dynamic amplifier factor ${}^{(c_2=33)}\Delta^{(a)}_{dyn(P_1,P_2........P_{10})} = \dfrac{0,0095}{0,0085}$ = 1,1176, for the deflection corresponding to the moving the load directly acting on the beam and ${}^{(c_2=33)}\Delta^{(a+2)}_{dyn(P_1,P_2........P_{10})} = \dfrac{0,012}{0,0085}$ =1,4118 corresponding also the effect of the residual free vibration caused by moving loads that have passed the beam, demonstrates the gain of the DAF.2 at 26,3%.

3.3. Main factors influencing the beam response

When all the moving loads have passed the beam, the force part of the vibration terminates immediately. However, the free vibration part continues to exit until it is eventually damped out. Our parametric studies for different train speeds show that both the phenomena of resonance and cancellation are related to the free vibration component of the beam vibration only. Thus, the sum of the free vibration response components by each moving load can results in the resonance if the total number *N* of the moving loads is large enough.

If the forces act on the bridge at equidistance distances d_v then their repeated action can cause a resonant vibration. The resonant condition follows from the well know condition [Náprstek, Li,Su]:

$$kf_{(d_v)} = f_{(1)} \;\rightarrow\; k\frac{c}{d_v} = \frac{\omega_{(1)}}{2\pi}\ ,\ k=1,2,3\ldots, \tag{14}$$

The resonant condition (14) is calculated from the time necessary for crossing the distance d_v at the speed c which is equal to the k-multiple of the natural beam vibration $f_{(1)}$. From the Eq. (14) results the resonant speed

$$c_{(i,res)} = \frac{\omega_{(1)} d_v}{2\pi k}\quad k=1,2,3... \tag{15}$$

Critical speeds $c_{(i,res)}$ according to Eq. (15) are in Tab. 1.

Tab. 1 Critical speeds for train with IC-cars (d_v=24,5 m) and for L_b = 38 m.

k	$c_{i,res} = \dfrac{\omega_{(1)} d_v}{k2\pi}$ for L_b=38 m, d_v=24,5 m	
1	$c_{1,res}$=130,1 m/s	$\cos(\omega_{(1)} L_b / c_1)$ =-0,9506
2	$c_{2,res}$=65,03 m/s= =234,11 km/h	$\cos(\omega_{(1)} L_b / c_2)$ =0,8072
3	$c_{3,res}$=43,35 m/s	$\cos(\omega_{(1)} L_b / c_3)$ =-0,5843
4	$c_{4,res}$=32,52 m/s	$\cos(\omega_{(1)} L_b / c_4)$ =0,3034

From Eq. (15) results:

- The free-vibration response (Eq. (7) or (13)) results in the resonance response if the total number of moving loads is large enough.
- The resonant-response amplitude decreases when the value of *k* increases.
- The ratio L_b / c of the span L_b to the velocity of the moving loads c expressively influences the response amplitude.

- The difference between phase angles in the dynamic components in Eq. (7) affects the phenomena of resonance and its cancellation.
- The increase of the damping is an effective measure to reduce resonant –amplitude.

4. CONCLUSIONS

An intensive vibration of the bridges similar to the resonance phenomenon for small and medium spans, especially at higher speeds over 160 km/h, can be discovered.

The dynamic response of bridges of small and medium spans, is markedly influenced just composition of the periodic load of moving vehicles, due to the loading interval of vehicles, the rate of the moving loads, and the bridge length. For a bridge the total displacement response is predominated just by the driving frequency ${}^{(c)}\omega_{(1)dr}$, the bridge frequency ${}^{(c)}\omega_{(1)}$, parameters d_v / c and L_b / c. Dynamic deflections of the bridge ${}^{(c_2=33)}w^{(a+2)}_{(1),(P_1,P_2 \dots P_{10})}(L_b/2,t)$ due to the series of load IC-cars ($P_1+P_2+\dots P_N$) is defined superposition quasi-static and the dynamic components described by expressions (9), (10) and (12). For a small and medium span and for low speeds (c<160 km/h) the response do not lead to a significant increase, but at speeds c ≥ 160 km /h the dynamic amplifier factor ${}^{(c_2=33)}\Delta^{(a+2)}_{dyn(P_1,P_2 \dots\dots P_N)}$ increases significantly in particular due to the dynamic components ${}^{(c)}w^{(2)}_{(1),(P_1,P_2,\dots\dots P_N)}(L_b/2,t)$.

The occurrence of resonant speeds $c_{i,res}$ results from parameters of the bridge and the train (distance axles of the train and the train speed). The resonant speeds $c_{i,res}$ are reflected in a cumulative increase in dynamic response. With an increasing number of load forces $n = 1,2, \dots N,$ for *M*-forces moving along the bridge and *K*-forces that have passed the beam, the deflection amplitude ${}^{(c_2=33)}w^{(a+2)}_{(1),(P_1,P_2 \dots P_{10})}(L_b/2,t)$ increase. They are particularly dangerous at high-speed trains and have an adverse effect on the degradation of the superstructure but also fatigue bridges.

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