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Ivan SHATSKYI[1], Mykola MAKOVIICHUK[2], Lubomyr ROPYAK[3]

ANALYTICAL ESTIMATIONS OF STRENGTH OF TWO LAYER COATING LOADED ALONG A LINE

Abstract

The engineering calculation procedure of the stress-strain state of the two-layer coating under the load localized along a line is proposed. The plane coupled problem of the plate equilibrium has been formulated. The analytical solutions of the problem have been obtained and compared with previous results for the uncoupled problem. The analytical estimations of strength of the composite coating have been given.

Keywords

Two-layer coating, Winkler foundation, local load, stress distribution, strength.

1 INTRODUCTION

Workability of machines, mechanisms and equipment subjected to contact loads and to interaction with aggressive or abrasive media is mainly controlled by the state and structure of the surface layers of their critical components. Conventional procedure of surface hardening of machine parts implies deposition of functionally gradient coatings whose integral parts have different functions. Such heterogeneous structures include, in particular, two-layered coatings of the ceramics-aluminum system, which cover the surface of steel [1] or aluminium [2] parts. The external (oxide) layer exhibits a high hardness and ensures the protection from mechanical wear, while the subsurface (aluminum) layer provides the necessary bond with the base metal (steel) and protects the latter from hydrogen saturation in aggressive media. Similar problems rise also in civil engineering within interaction of building walls and base plates on soil bodies.

The available literature in this domain [3–7] provides insufficient insight into the effect of structure and mechanical properties of the two-layer coating components on the limit load values. Especially, this is vital for localized spatial loads. Study [6] describes the approach to analytical calculation of local stresses in a two-layer coating treated as a plate lying on the Winkler bed and provides the assessment of the limit load transferred from the tool applied during the finishing mechanical treatment of the coated part. The procedure for design optimization of a two-layer coating for particular local loading conditions is considered in the article [7].

The aim of the present study is elaboration of the technique [6, 7] in the case of taking into consideration interconnection of the membrane and bending fields in coating.

[1] Prof., Ivan Shatskyi, DSc., Department of modelling of damping systems, Ivano-Frankivsk Branch of Pidstryhach Institute for Applied Problems of Mechanics and Mathematics, NAS of Ukraine, Mykytunetska str., 3; 76002, Ivano-Frankivsk, Ukraine, phone: (+380) 99 4444 967, e-mail: ipshatsky@gmail.com

[2] Assoc. Prof., Mykola Makoviichuk, PhD., Department of modelling of damping systems, Ivano-Frankivsk Branch of Pidstryhach Institute for Applied Problems of Mechanics and Mathematics, NAS of Ukraine, Mykytunetska str., 3; 76002, Ivano-Frankivsk, Ukraine, phone: (+380) 50 7832 439, e-mail: makoviy@ua.fm

[3] Assoc. Prof., Lubomyr Ropyak, PhD, Department of Computer Engineering Manufacturing, Ivano-Frankivsk National Technical University of Oil and Gas, Karpatska Str., 15, Ivano-Frankivsk, 76019, Ukraine, phone: (+380) 50 5408 007, e-mail: l_ropjak@ukr.net

2 PROBLEM FORMULATION AND STRENGTH CRITERION

Let's consider a flat rigid foundation with deposited two-layer coating consisting of a quite stiff surface layer of thickness h_c (external coating layer) and of a quite compliant subsurface layer of thickness h_o (substrate), which is illustrated by Fig. 1.

The external (upper) layer is modeled by a plate subjected to tensile (compressive) and bending loads, while the lower (substrate) layer is treated as an interlayer whose behavior is controlled by the Winkler hypotheses on the proportionality of stresses and elastic displacements. For the sake of the problem simplification, the base is assumed to be perfectly rigid, while the mechanical contact between the coating components is treated as a perfect one. The composite is loaded by distributed force which is uniformly distributed along the straight line (normal to the plane in Fig. 1) and is represented as two components X and Y (N/m). Another assumption is the plane strain state of the composite under study ($\varepsilon_z = 0$). The stress distribution in the coupled two-layer composite and the strength values are to be derived.

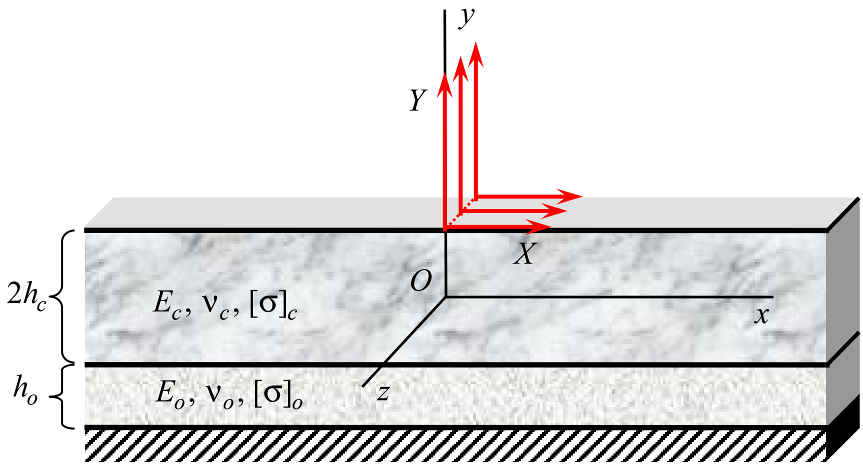


Fig. 1: Scheme of a two-layer coating and load

Based on the above assumptions, the equilibrium equation for the coating on an elastic bed takes the following form [8, 9]:

$$B\frac{d^2u_x}{dx^2} - k_x\left(u_x + \underline{h_c\frac{du_y}{dx}}\right) = -X\,\delta(x)$$

$$D\frac{d^4u_y}{dx^4} + k_y u_y \underline{- h_c k_x \frac{d}{dx}\left(u_x + h_c\frac{du_y}{dx}\right)} = Y\,\delta(x) + \underline{h_c X\frac{d}{dx}\delta(x)}, \quad x \in \mathbf{R}, \qquad (1)$$

where:

u_x, u_y – are components of the elastic displacement of the plate midsection [m],

$\delta(x)$ –is the Dirac function,

$B = 2E_c h_c/(1-\nu_c^2)$ – are stiffness values for tension [Pa·m],

$D = 2E_c h_c^3/(3(1-\nu_c^2))$ – are stiffness values for bending, [Pa·m^3],

$k_x = G_o/h_o = E_o/(2(1+\nu_o)h_o)$, $k_y = E_o/h_o$ – are the Winkler bed coefficients, [Pa/m],

E_c, E_o – are elastic moduli of coating and substrate, respectively [Pa],

ν_c, ν_o – are Poisson's ratios.

At infinity the loads and bending moments turn to zero:

$$B\frac{du_x}{dx}(\pm\infty)=0, \quad D\frac{d^2u_y}{dx^2}(\pm\infty)=0, \quad D\frac{d^3u_y}{dx^3}(\pm\infty)=0. \tag{2}$$

Obtained displacements correspond to stress in the coating, in particular in the external coating midsection ($y=-h_c/2$) have the following form:

$$\sigma_x = E_c\frac{d}{dx}\left(u_x + \underline{h_c\frac{du_y}{dx}}\right). \tag{3}$$

Using the Winkler hypotheses, the stresses in the substrate are assessed as follows:

$$\tau_{xy} = k_x\left(u_x + \underline{h_c\frac{du_y}{dx}}\right), \quad \sigma_y = k_y u_y. \tag{4}$$

For the strength assessment of each layer, the Mises criterion is used, which yields the following equations for the plane-strained coating

$$\sigma_{eq,c}(x) \equiv \sqrt{(1-\nu_c+\nu_c^2)(\sigma_x^2+\sigma_y^2)-(1+2\nu_c-2\nu_c^2)\sigma_x\sigma_y+3\tau_{xy}^2} \le [\sigma]_c, \tag{5}$$

and for the Winkler bed-type substrate

$$\sigma_{eq,o}(x) \equiv \sqrt{\sigma_y^2+3\tau_{xy}^2} \le [\sigma]_o, \tag{6}$$

where

$[\sigma]_c$, $[\sigma]_o$ – are maximal permissible stresses for the coating and substrate materials [Pa].

The underlined members in equations (1) and formulae (3), (4) characterize interconnection of membrane and bending fields and distinguish this statement from the previously considered uncoupled problem [6, 7].

3 STRESSED STATE ANALYSIS

Using the dimensionless values $u_1 = u_x/h_c$, $u_2 = u_y/h_c$, $\xi = x/h_c$ the problem (1), (2) was presented in the form:

$$\mathrm{L}_{11}u_1 + \mathrm{L}_{12}u_2 = B_1, \quad \mathrm{L}_{21}u_1 + \mathrm{L}_{22}u_2 = B_2, \quad \xi \in \mathbf{R}. \tag{7}$$

$$\frac{du_1}{d\xi}(\pm\infty)=0, \quad \frac{d^2u_2}{d\xi^2}(\pm\infty)=0, \quad \frac{d^3u_2}{d\xi^3}(\pm\infty)=0. \tag{8}$$

The elements of symmetrical matrix of differential operators and the loads vector are the following:

$$\mathrm{L}_{11} = \frac{d^2}{d\xi^2} - \varepsilon, \quad \mathrm{L}_{12} = \mathrm{L}_{21} = -\underline{\varepsilon\frac{d}{d\xi}}, \quad \mathrm{L}_{22} = \frac{1}{3}\frac{d^4}{d\xi^4} - \underline{\varepsilon\frac{d^2}{d\xi^2}} + 2(1+\nu_o)\varepsilon,$$

$$B_1 = -\frac{X}{B}\delta(\xi), \quad B_2 = \frac{Y}{B}\delta(\xi) + \underline{\frac{X}{B}\delta'(\xi)}.$$

Here

$$\varepsilon = \frac{k_x h_c^2}{B} = \frac{E_o h_c}{E_c h_o} \frac{1-\nu_c^2}{4(1+\nu_o)} .$$

The analytical solution of boundary problem (7), (8) has been obtained using the Fourier transformation. The result in image space parametrised by ω is the following:

$$u_k^F(\omega) = \Delta_k(\omega) / \Delta(\omega) , \quad k = 1, 2 ; \tag{9}$$

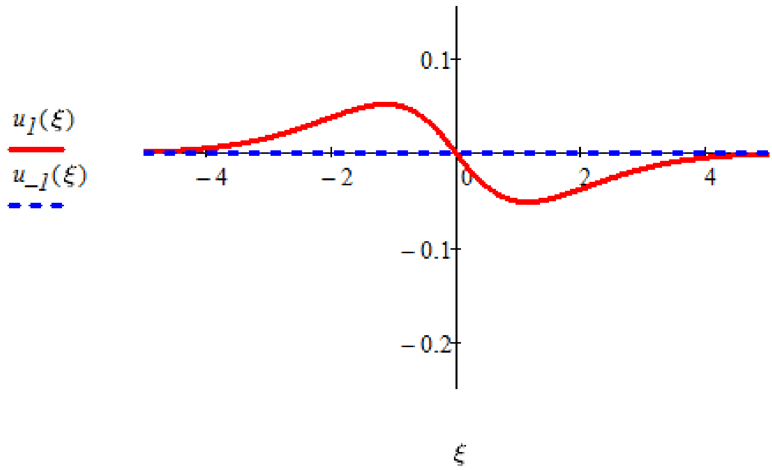
$$\Delta_1(\omega) = -\frac{X}{B}\left(\frac{1}{3}\omega^4 + 2(1+\nu_o)\varepsilon\right) - \underline{\frac{Y}{B}\varepsilon i\omega} , \quad \Delta_2(\omega) = \underline{\frac{X}{B} i\omega^3} + \frac{Y}{B}(\omega^2 + \varepsilon) ,$$

$$\Delta(\omega) = (\omega^2 + \varepsilon)\left(\frac{1}{3}\omega^4 + 2(1+\nu_o)\varepsilon\right) + \underline{\varepsilon\omega^4} .$$

The problem of Fourier inversion of the expression (9) is reduced to finding zeros of the denominator (the roots of the bicubic equation $\Delta(\omega) = 0$).

As we see, rejection of underlined member which is responsible for connectivity of problem significantly simplifies the searching of roots.

We considered the example of the calculation with parameters: $X = 0$, $Y/B = -1$, $\varepsilon = 1$, $\nu_o = 0.25$, $\nu_c = 0.3$. The results of simulation of dimensionless values of the displacement and von Mises stresses are represented on Fig. 2 and Fig. 3.



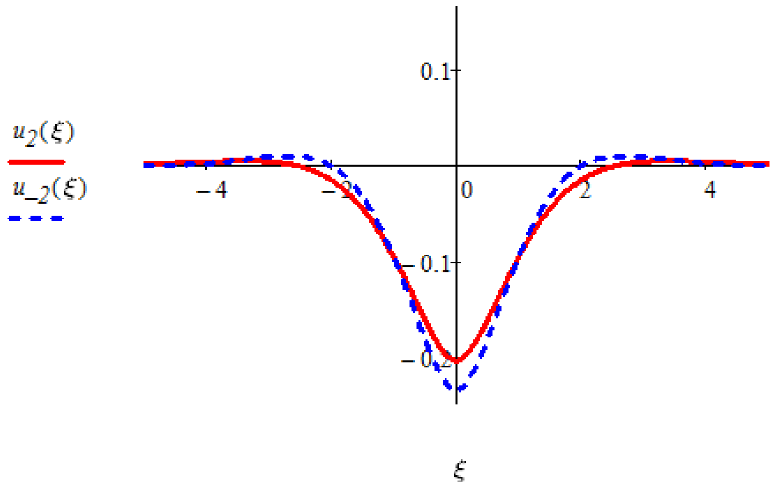
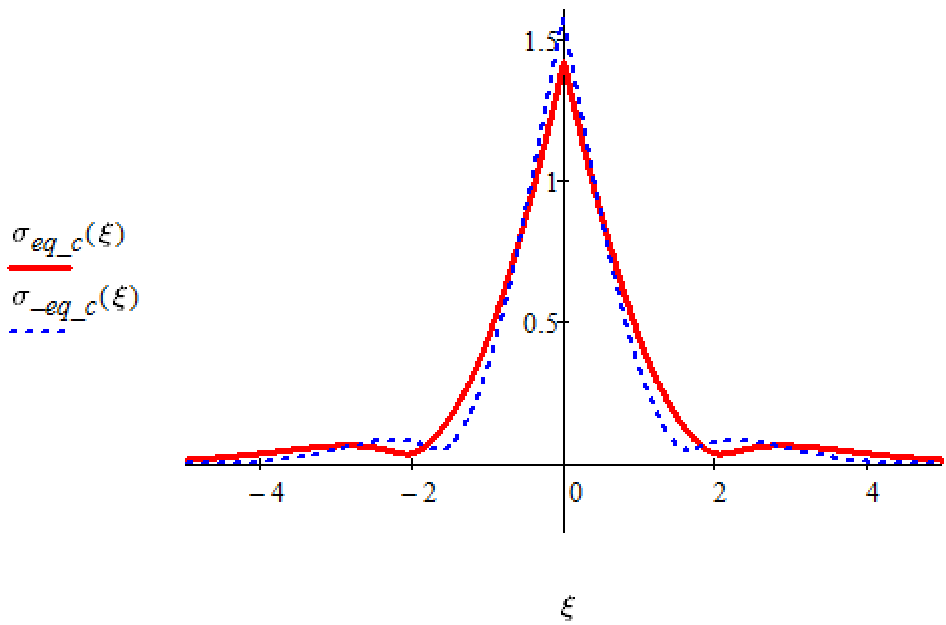


Fig. 2: The horizontal and vertical displacements of upper layer coating: dashed lines correspond to uncoupled problem [6, 7], solid lines correspond to consider coupled problem



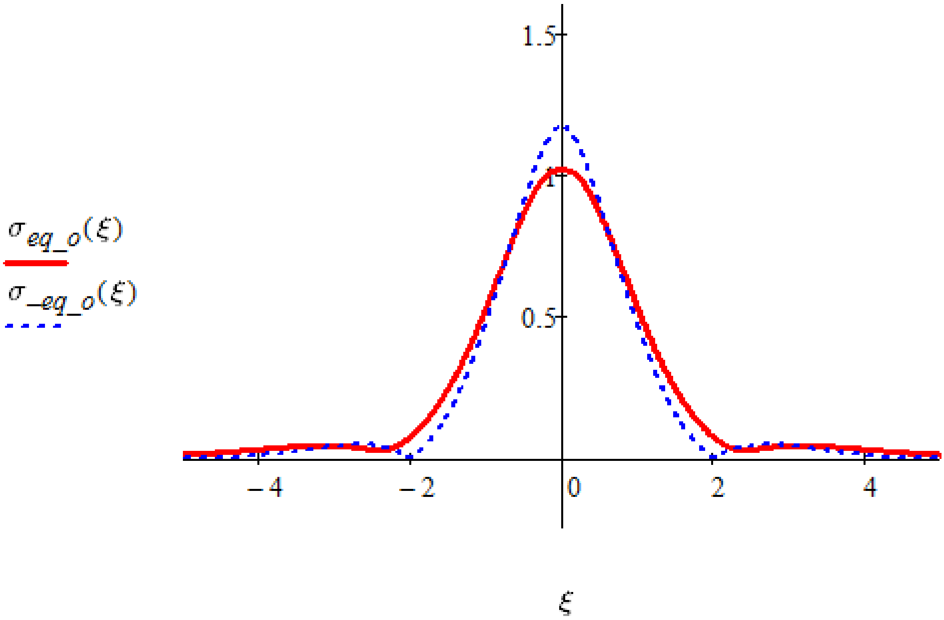


Fig. 3: The equivalent stresses in the coating and in the substrate:
dashed lines correspond to uncoupled problem [6, 7],
solid lines correspond to consider coupled problem

6 CONCLUSIONS

The proposed calculation procedure makes it possible to provide analytical prediction of the stressed state and limiting equilibrium of two-layer coating for various ratios between geometrical and mechanical characteristics of the coating components.

The principal issue of the proposed calculation scheme is application of the strength criteria to both components of the piecewise-uniform body. The conclusion about the real strength of the composition should be done by comparing the value of relationship $\sigma_{eq,c}/[\sigma]_c$ and $\sigma_{eq,o}/[\sigma]_o$.

The comparison of presented model that takes into account the connectivity of membrane and bending fields and previously developed model [6, 7] shows that at small ε the simplified theory is worth using.

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