Ivan SHATSKYI [1], Vasyl PEREPICHKA [2]

SHOCK WAVE IN A PILE IMMERSED INTO VISCOPLASTIC MEDIUM

Abstract

The wave problem of perturbation propagation along an elastic pile interacting with the medium is investigated using the model of viscoplastic friction. An exact solution of the problem is obtained using the Laplace transforms for an arbitrary time of the loading period. The diagrams for velocity and stresses have been constructed.

Keywords

Elastic pile, viscoplastic medium, dry friction, shock wave, velocity, stress.

1 INTRODUCTION

Analysis of energy dissipation due to the frictional interaction of deformable contacting bodies is of great importance in the research of applied problems of the structural dynamics. In this proceeding the nonstationary dynamics of an elastic pile with viscoplastic external resistance under shock loading has been studied. The obtained results generalize the cases of purely dry [1–3] and purely viscous [4] frictions and complement the papers [5, 6].

This problem can be used for modelling the dynamics of drilling equipment to eliminate the sticking of the tool or for research the process of the immersion (pulling) of piles in building.

2 FORMULATION OF THE PROBLEM

We consider the propagation of longitudinal shock wave in immersed into the viscoplastic medium semi-infinite elastic rod with constant cross section induced by sudden loading of the end. The classical theory of rods dynamics has been used. The friction forces on the lateral surface are modeled by Voigt parallel connection of Saint-Venant and Newton elements (the model of interaction through a thin Bingham layer). We study the process of wave attenuation due to action of this external viscoplastic resistance.

The initial-boundary-value problem is the following:

$$\frac{u''}{L} + \frac{\tau_x}{E} = \frac{\ddot{u}}{L}, \qquad x > 0, \quad t > 0 ; \tag{1}$$

$$\tau_x = -\left(\tau_c \, \mathrm{sgn}\, \dot{u} + 2BE\, \frac{\dot{u}}{L} \right), \quad \dot{u} \neq 0 \quad \text{or} \quad \dot{u} = 0, \quad | \tau_x | < \tau_c ; \tag{2}$$

$$u(x, 0) = \dot{u}(x, 0) = 0, \quad x > 0 ; \tag{3}$$

[1] Prof., Ivan Shatskyi, DSc., Department of modelling of dampingsystems, Ivano-Frankivsk Branchof Pidstryhach Institute for Applied Problems of Mechanics and Mathematics, NAS of Ukraine,Mykytunetska str., 3; 76002, Ivano-Frankivsk, Ukraine, phone: (+380) 99 4444 967, e-mail: ipshatsky@gmail.com

[2] Assoc. Prof., Vasyl Perepichka, PhD., Department of modelling of dampingsystems, Ivano-Frankivsk Branchof Pidstryhach Institute for Applied Problems of Mechanics and Mathematics, NAS of Ukraine,Mykytunetska str., 3; 76002, Ivano-Frankivsk, Ukraine, phone: (+380) 97 2141 780, e-mail: : an_w@i.ua

$$\frac{u'(0,t)}{L}=-\frac{\sigma_0}{E}H(t),\quad u(\infty,t)=0,\quad t>0\,.\qquad(4)$$

where:

u – is the axial displacement [m],

τ_x – is the shear stress [Pa],

X,T – are axial coordinate and time [m, s],

$x=X/L,\ t=cT/L$ –are dimensionless coordinate and time,

$L=F/\Pi,\ F,\ \Pi$ – are the characteristic size, area and perimeter of cross-section [m, m², m],

$c=\sqrt{E/\rho}$ – is the wave velocity [m/s],

$E,\ \rho$ –   are Young's modulus and density of the rod material [Pa, kg/m³],

$2B=\beta/\sqrt{E\rho}$,

β – is the dynamic viscosity [kg/ (m² s)],

σ_0 – is the stress at the rod [Pa] and

$H(t)$ – is a Heaviside function.

The primes and the dots denote the partial derivate with respect to the dimensionless coordinate and to the dimensionless time respectively.

3   ANALYTICAL SOLUTION

Divining the sign of the velocity, we linearize the nonlinear problem (1)–(4) and construct the solution using the Laplace transforms over the time coordinate. In particular, analytical expressions for the axial displacement and its derivations are the following [3]:

$$\frac{u(x,t)}{L}=\frac{\sigma_0}{E}\int_x^t\left[(1+\kappa)e^{-Bt}I_0(B\sqrt{\tau^2-x^2})-\kappa\right]d\tau\,H(t-x)\,,$$

$$\frac{\dot{u}(x,t)}{L}=\frac{\sigma_0}{E}\left[(1+\kappa)e^{-Bt}I_0(B\sqrt{t^2-x^2})-\kappa\right]H(t-x)\,,$$

$$\frac{u'(x,t)}{L}=-\frac{\sigma_0}{E}\left[(1+\kappa)\left(e^{-Bx}+Bx\int_x^t e^{-B\tau}\frac{I_1(B\sqrt{\tau^2-x^2})}{\sqrt{\tau^2-x^2}}d\tau\right)-\kappa\right]H(t-x)\,.\qquad(5)$$

Here:

$\kappa=(\tau_c/\sigma_0)/(2B)$ and

$I_0(z)$, $I_1(z)$ – are modified cylindrical Bessel functions of the first kind.

4   ANALYSIS OF THE RESULTS

We gave an example of calculations for the parameters values $\tau_c/\sigma_0=1$, $B=0.6$, then $\kappa\approx0.833$.

Wave pattern of nonstationary perturbation in the pile including the prefrontal zone of rest, the area of motion and the domain of stationary residual stresses has been built (Fig. 1). The regions A and B are separated by the characteristic $x=t$ which describes the wave front and is a line of strong discontinuity of velocity and axial stress. The curve that separates zones B and C is found numerically from the condition $\dot{u}(x,t)=0$ and makes a line of weak discontinuity of acceleration,

shear stress and axial stress gradient. The solution (5) is valid in area B where the velocity is positive. In domain C the displacement and axial stress do not change with time and have values recorded on the line of arrest of the motion.
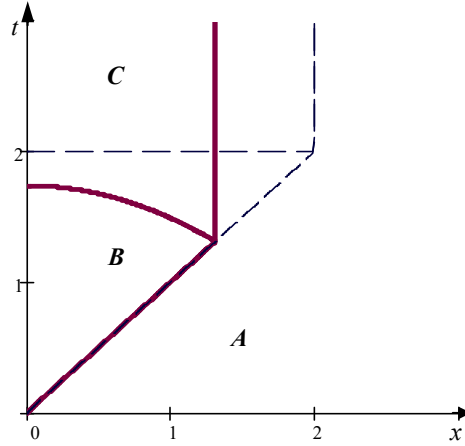

Fig. 1: Wave pattern of nonstationary perturbation in the rod:
A is the prefront zone of rest, B is the area of motion, C is the domain of stationary residual stresses; the dashed lines are the boundaries of corresponding zones for the case of dry friction [1]

The cessation of the motion is first observed at a distance $L_* = (L/B)\ln(1+\kappa^{-1})$ from the end of the pile.

The three-dimensional diagrams for nonstationary fields of velocity and stresses in the pile have been constructed on the Fig. 2–4 too.

Intent analysis of results designate the sphere of correctness of solution (5) in the form of inequality: $\kappa \geq 1/(e-1) \approx 0.577$. Under such condition the line of cessation of the motion does not advance the characteristic of reflected wave and the residual shear stress is smaller than the threshold value. It should be noted that the numerical calculations in the article [5] are executed for $\tau_c/\sigma_0 = 1$, $B=1$, $\kappa = 0.5 < 0.577$, i.e. outside of the validity of the solution in the form (5).
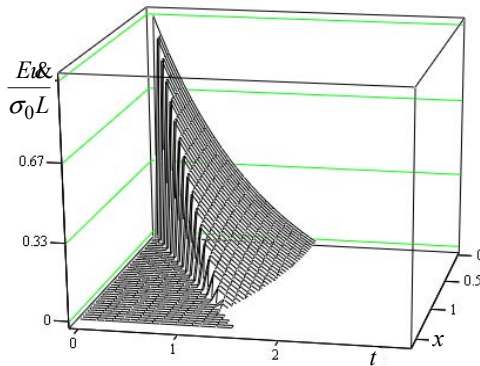

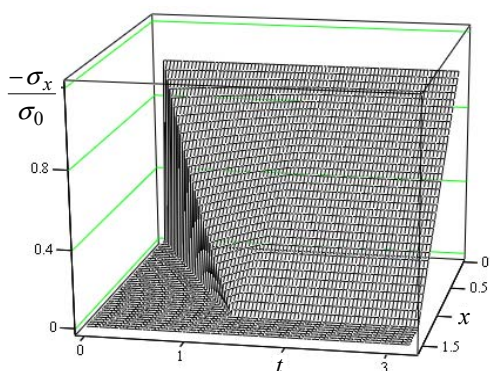Fig. 2: Distributions of the velocity in the rod

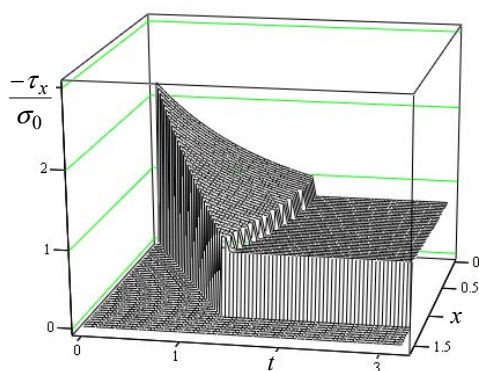Fig. 3: Distributions of the normal stresses
in the rod

Fig. 4: Distributions of the shear contact stress
in the rod

5 CONCLUSIONS

In the presence of viscoplastic external resistance, the motion of rod cross-sections is similar in nature to the motion in the presence of dry friction, and the viscosity effect is less significant.

In contrast to the case of dry friction, where the perturbed part of the pile moves as a perfectly rigid body and the motion stops simultaneously over the entire length from the front end to the wave front, the presence of the viscous component of the resistance is responsible for the motion of the perturbed zone of the rod as a deformable region, and the cessation of motion in the region between the front of the wave and the end of the rod occurs at a rate exceeding the speed of propagation of the perturbation.

LITERATURE

[1] NIKITIN, L. V. *Statics and dynamics of solids with external dry friction*. Moscow: Mosc. Litsei, 1998. 272 p. ISBN 5-7611-0163-7. (in Russian).

[2] ALEKSANDROVA, N. I., Numerical-analytical investigation into impact pipe driving in soil with dry friction. Part 1: Nondeformable external medium, *Journal of Mining Science.* 2012, XLVIII. Nr. 5, pp. 856 -869. ISSN 1062-7391, DOI: 10.1134/S1062739148050103.

[3] ALEKSANDROVA, N. I., Numerical-analytical investigation into impact pipe driving in soil with dry friction. Part 2: Deformable external medium, *Journal of Mining Science.* 2013, XLIX. Nr. 3, pp. 413–425. ISSN 1062-7391, DOI: 10.1134/S106273914903009X.

[4] VEKLICH, N. A. & MALYSHEV B. M. Longitudinal wave's propagation in an elastic rod placed in viscous liquid. *Mechanics of Solids.* 1967. Nr. 5, pp. 174–178. ISSN 0572-3299 (in Russian).

[5] SHATSKII, I. P. & PEREPICHKA V. V. Shock wave propagation in an elastic rod with a viscoplastic externel resistance. *Journal of Applied Mechanics and Technical Physics.* 2013, LIV. Nr. 6. pp. 1016–1020. ISSN 0021-8944, DOI: 10.1134/S0021894413060163.

[6] SHATSKYI, I. P. & PEREPICHKA V. V. Non-stationary twisting of an elastic bar with viscoplastic external resistance. *Proc. Nat. Acad. Sci. of Ukraine.* 2014, Nr. 10, pp. 63–69. ISSN 1025-6415 (in Ukrainan). DOI: 10.15407/dopovidi2014.10.0633.