

Maksym GRZYWIŃSKI¹**OPTIMIZATION OF DOUBLE-LAYER BRACED BARREL VAULTS****Abstract**

The paper deals with discussion of discrete optimization problem in civil engineering structural space design. Minimization of mass should satisfy the limit state capacity and serviceability conditions. The cross-sectional areas of truss bars are taken as design variables. Optimization constraints concern stresses, displacements and stability, as well as technological and computational requirements.

Keywords

Optimization, space truss, braced barrel vaults, double-layer, formex algebra.

1 INTRODUCTION

Barrel vault is one of the oldest type of space structure used since antiquity. This type of structures has lightweight and is cost effective structures which are used to cover large areas such as exhibition halls, stadium, markets, and concert halls. The earlier types of braced barrel vaults were constructed as single-layer structures. Nowadays, double-layer systems are utilized for covering large spans [1 - 6]. Double layer barrel vaults are generally indeterminate from static point of view. In such systems, due to the rigidity, the risk of instability can almost be eliminated. The use of this type of barrel vaults enhances the stiffness of the vault structure and provides structural systems of great potential, capable of having spans in excess of 100 m.

In the last decade, structural optimization has become one of the most interesting branches of structural engineering and many meta-heuristic algorithms have been developed and applied for optimization of truss structures.

2 BRACED BARREL VAULTS

The braced barrel vault is composed of member elements arranged on a cylindrical surface. The basic curve is a circular segment; however, occasionally a parabola, ellipse, or funicular line may also be used. Figure 1 shows the typical arrangement of a braced barrel vault. Its structural behavior depends mainly on the type and location of supports, which can be expressed as L/R , where L is the distance between the supports in longitudinal direction and R is the radius of curvature of the transverse curve.

If the distance between the supports is long and usually edge beams are used in the longitudinal direction (Fig. 1a), the primary response will be beam action. For $1.67 < L/R < 5$, the barrel vaults are called long shells, which can be visualized as beams with curvilinear cross-sections. The beam theory with the assumption of linear stress distribution may be applied to barrel vaults that are of symmetrical cross-section and under uniform loading if $L/R > 3$. This class of barrel vault will have longitudinal compressive stresses near the crown of the vault, longitudinal tensile stresses towards the free edges, and shear stresses towards the supports.

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As the distance between transverse supports becomes closer, or as the dimension of the longitudinal span becomes smaller than the dimension of the shell width such that $0.25 < L/R < 1.67$, then the primary response will be arch action in the transverse direction (Fig. 1b). The barrel vaults are called short shells. Their structural behavior is rather complex and dependent on their geometrical proportions. The force distribution in the longitudinal direction is no longer linear, but in a curvilinear manner, trusses or arches are usually used as the transverse supports.

When a braced barrel vault is supported continuously along its longitudinal edges on foundation blocks, or the ratio of L/R becomes very small, i.e., < 0.25 (Fig. 1c), the forces are carried directly in the transverse direction to the edge supports. Its behavior may be visualized as the response of parallel arches. Displacement in the radial direction is resisted by circumferential bending stiffness. Such type of barrel vault can be applied to buildings such as airplane hangars or gymnasias where the wall and roof are combined together.

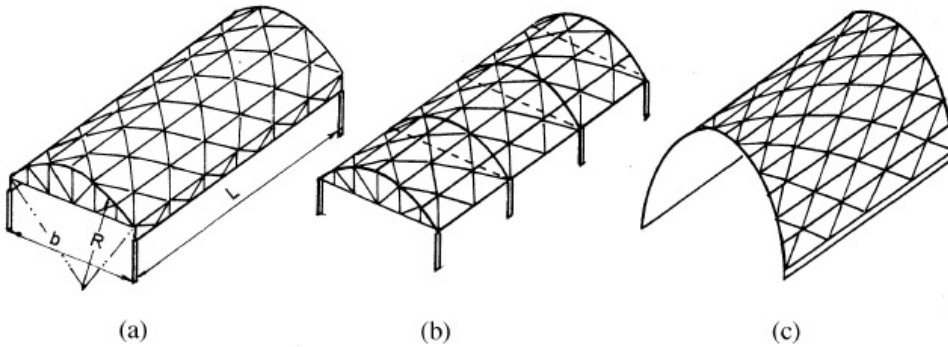


Fig. 1: Braced barrel vaults [4]

3 FORMEX ALGEBRA

Formex algebra is a mathematical system that consists of a set of abstract objects, known as formices and number of rules in accordance with which these objects may be manipulated. It provides a convenient basis for solution of problem in data generation and computer graphics. The early ideas on which formex algebra is based were developed by Prof. H. Nooshin during the years 1972-73. After publication of this book [7], formex algebra started getting worldwide acceptance. Nooshin presents a survey of the current work in formex configuration processing where reference is given to structural applications in relation to double-layer grids, barrel vaults, domes.

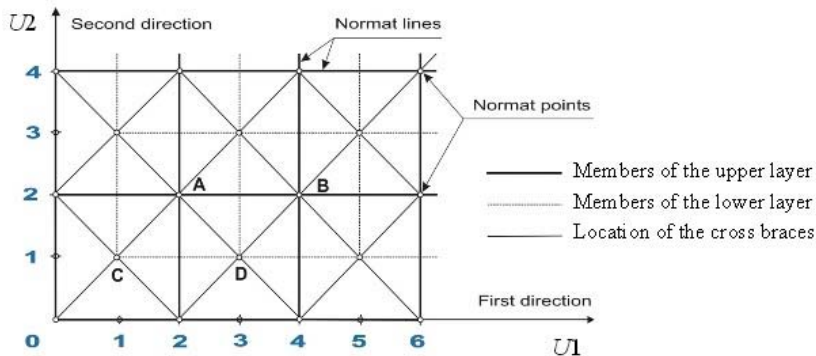


Fig. 2: Arrangement of component parts of space truss [7]

In the formex algebra, a structural configuration is defined according to its position towards three basic directions U1, U2 and U3, which may be considered as suitable counterparts to axes X, Y and Z in the three-dimensional Cartesian system. Lines parallel to these basic directions are called “normat lines” and points of their intersections are called “normat points”. Distances between these “normat lines” can be individually assumed by the programmer. In the example presented in Fig. 2, these distances are assumed as a unity. In the formex algebra, for instance, position on Node C placed in the lower layer is defined by three successive figures separated by colons and put inside square brackets like as follows: [1, 1, 0]; a member of the upper layer located between Nodes A and B, comparing Fig. 2, is defined as: [2, 2, 1; 4, 2, 1], where the third coordinate indicates position of the lower or upper layer measured along the third direction U3. Member CD placed in the lower layer is defined as: [1, 1, 0; 3, 1, 0], while the cross brace AD is described as: [2, 2, 1; 3, 1, 0].

In Fig. 3 and 4 shown double-layer barrel vault with programming language Formian.

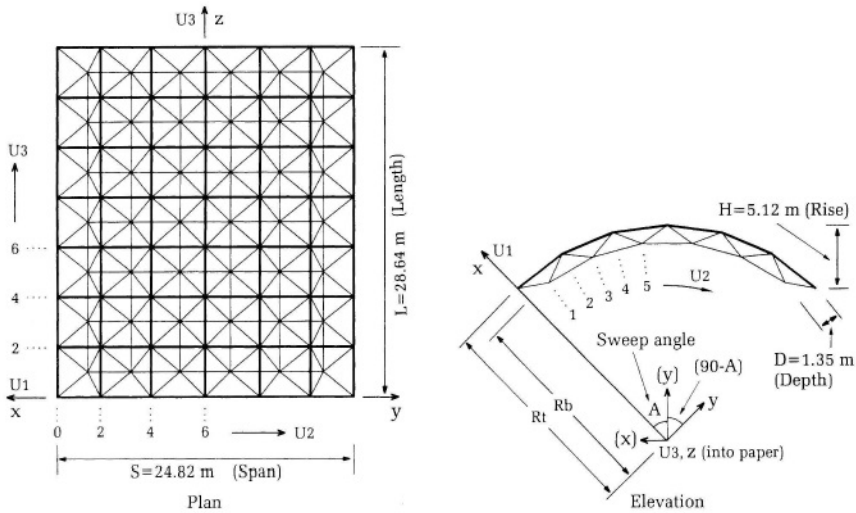


Fig. 3: Plan and elevation of the double-layer barrel vault [7]

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(*) Double-layer Barrel Vault (*)
M=6;      (*) top modules along U2 (*)
N=7;      (*) top modules along U3 (*)
S=24.82;  (*) span (*)
H=5.12;   (*) rise (*)
D=1.35;   (*) depth (*)
L=28.64;  (*) length (*)
v=0;      (*) view adjuster (*)
A=2*atan|(2*H/S); (*) sweep angle (*)
Rt=S/(2*sin|A); (*) top radius (*)
Rb=Rt-D;  (*) bottom radius (*)
TOP=rinit(M,N+1,2,2)|[Rt,0,0; Rt,2,0]#
    rinit(M+1,N,2,2)|[Rt,0,0; Rt,0,2];
BOT=rinit(M-1,N,2,2)|[Rb,1,1; Rb,3,1]#
    rinit(M,N-1,2,2)|[Rb,1,1; Rb,1,3];
WEB=rinit(M,N,2,2)|lamit(1,1)|[Rt,0,0; Rb,1,1];
B=TOP#BOT#WEB;
B1=bc(1,A/M,L/(2*N))|B;
BV=verad(0,0,90-A)|B1;
use &,vm(2),vt(2),
    vh(v,2.75*Rt,-Rt,0,0,Rt,0,1,Rt);
clear; draw BV;

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Fig. 4: A generic scheme for the double-layer barrel vault of Fig. 3

4 OPTIMAL DESIGN OF STRUCTURE

Minimizing the structural weight W requires the selection of the optimum values of number cross-section D_i while satisfying the design constraints. The discrete optimal design problem of truss structure may be expressed as

$$\text{find:} \quad X = [x_1, x_2, \dots, x_{ng}] \quad (1)$$

$$\text{to minimize:} \quad W(X) = \sum_{i=1}^{nm} \gamma_i x_i L_i \quad (2)$$

subject to:

$$x_i \in D_i \quad D_i = \{d_{i,1}, d_{i,2}, \dots, d_{i,r}\} \quad (3)$$

$$\delta_{min} \leq \delta_i \leq \delta_{max} \quad i = 1, 2, \dots, nn \quad (4)$$

$$\sigma_{min} \leq \sigma_i \leq \sigma_{max} \quad i = 1, 2, \dots, nm \quad (5)$$

$$\sigma_i^b \leq \sigma_i \leq 0 \quad i = 1, 2, \dots, nc \quad (6)$$

where X is a vector containing the design variables; D_i is an allowable set of discrete values for the design variable x_i ; ng is the number of design variables or the number of member groups; r is the number of available discrete values for the i -th design variable; $W(X)$ is the cost function which is taken as the weight of the structure; nn is the number of nodes; nm is the number of members forming the structure, nc is the numbers of compression elements, γ_i is the material density; L_i is the length of the member i ; σ_i and δ_i are the stress and nodal displacement, respectively; min and max mean the lower and upper bounds of constraints, respectively, σ_i^b is the allowable buckling stress in member i when it is compression.

5 EXAMPLE

The double-layer barrel vault has span $S = 48 \text{ m}$, length $L = 48 \text{ m}$ and supported on columns of height $H = 10 \text{ m}$. The orthogonal space truss is pin-supported at the corner and 3 middle point of boundary line (all number is 16). Cylindrical pipes made from hot-rolled steel with yield stress 235 MPa . The series of types for cross-sectional areas of truss bars containing 12 group (4 for top-, 4 for web- and 4 for bottom-layer).

Loading condition: dead load = 0.20 kN/m^2 , snow load = 0.74 kN/m^2 , wind load = 0.62 kN/m^2 . Solution was obtained in Autodesk Robot Structural Analysis Professional [8] with optimization module. Table 1 give solution for six models, where change depth (2, 3 or 4 m) and rise of structure (6 or 12 m).

Tab. 1: Weight for six models barrel vault

Model	Modules M	Modules N	Depth [m]	Rise [m]	Weight [kN]	Deflection [cm]
1	24	24	2	6	1025	4.0
2	16	16	3	6	432	4.4
3	12	12	4	6	641	4.6
4	24	24	2	12	990	3.1
5	16	16	3	12	836	2.8
6	12	12	4	12	777	2.6
						$L/250 = 19.2$

In Fig. 5 - 7 show normal force and cross-section areas for the best solution (minimum weight 432 kN) for three layer (top, web and bottom).

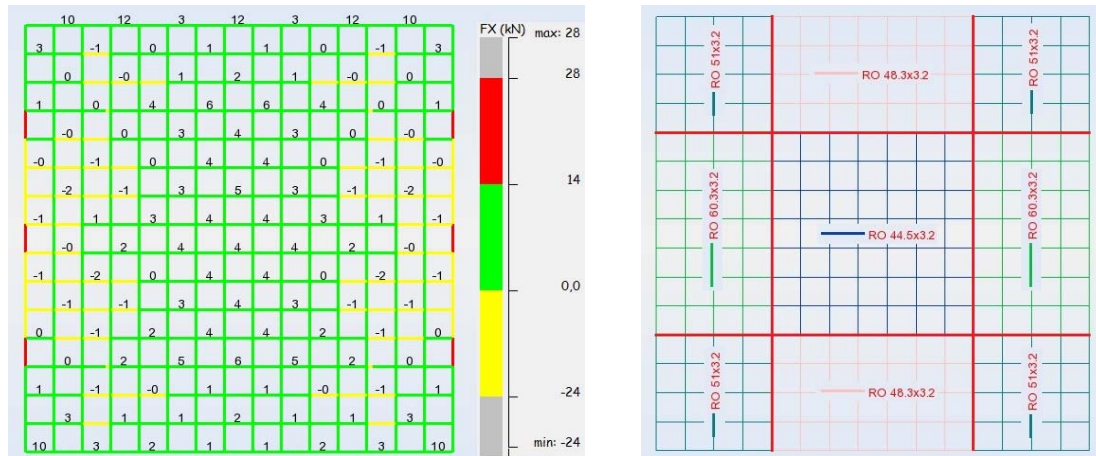


Fig. 5: Normal force (left) and profiles (right) in top layer for model 2

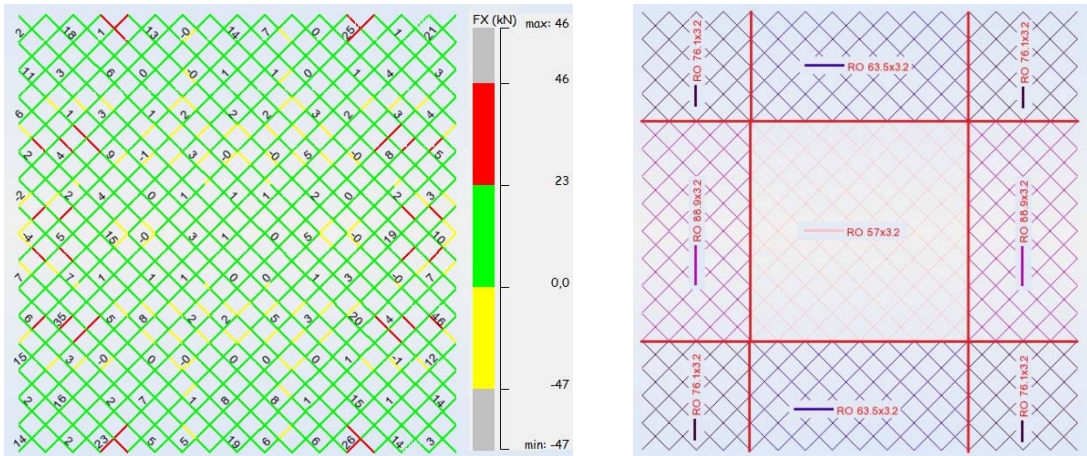


Fig. 6: Normal force (left) and profiles (right) in web layer for model 2

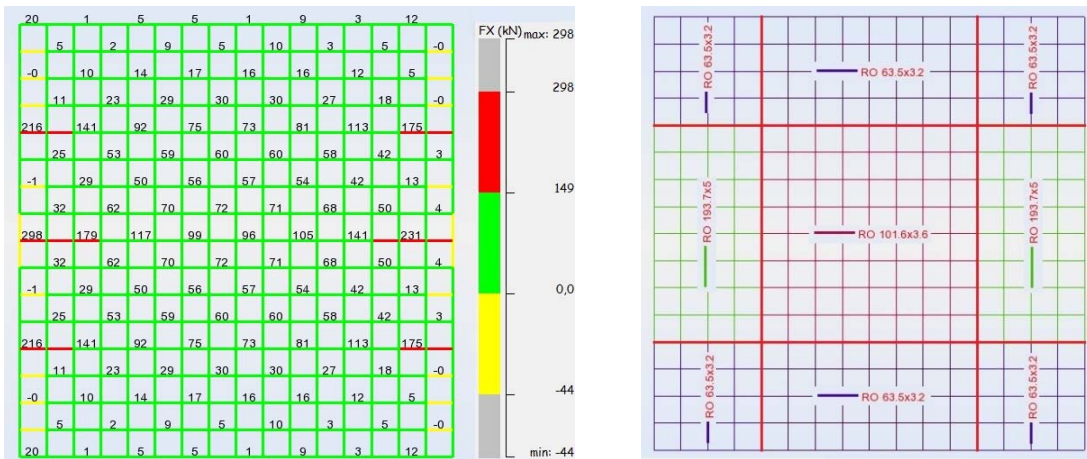


Fig. 7: Normal force (left) and profiles (right) in bottom layer for model 2

6 CONCLUSION

Space structure configurations are elegant and impressive but, unless the designer is equipped with suitable conceptual tools, the task of generation of geometry is extremely difficult. Formex algebra stands alone as an algebra which provides a powerful mathematical basis for new approach to data generation. Formex configuration is emerging branch of generation of geometry of skeletal structures. It complements the human imagination and allows mentally visualized configuration to express in a concise and elegant manner.

Formian software provides a platform for formex configuration processing, generation of various types of structure like braced domes, grids, barrel vaults, pyramid, towers. This software not only generated the geometry but can be also integrated with other software like Autodesk Robot Structural Analysis. In addition to this it is also generated the data exchange file (*.dxf), which can be imported in majority of CAD packages. This facility is efficiently exploited in present work.

Loads, grid size, depth and rise have some important effect on the optimization of barrel vaults also number group in every layer (in example no more than four).

The displacement in the best solution is 4.4 cm (limit 19.2 cm) suggests that the structure is very rigid and may seek even better solutions.

The greatest normal forces (298 kN) at bottom layer occurred close to boundary line. The number of supports on one side was 5 - increase their certainly will cause improvement solutions.

An increase in height (rise) will reduce forces in the structure but unfortunately results in an overall increase in length and weight increase.

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