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POSTBUCKLING ANALYSIS OF A RECTANGULAR PLATE LOADED IN COMPRESSION

Abstract

The stability analysis of a thin rectangular plate loaded in compression is presented. The non-linear FEM equations are derived from the minimum total potential energy principle. The peculiarities of the effects of the initial imperfections are investigated using the user program. Special attention is paid to the influence of imperfections on the post-critical buckling mode. The FEM computer program using a 48 DOF element has been used for analysis. Full Newton-Raphson procedure has been applied.

Keywords

Stability, post-buckling, initial imperfection, finite element method.

1 INTRODUCTION

In the presented paper has been explained the behaviour of a thin plate loaded in compression [1]. The geometrically nonlinear theory represents a basis for the reliable description of the postbuckling behaviour of the plate. Influence of initial imperfection on the load-displacement path is researched. The results of the numerical solution are presented as graphs showing the dependency of the amount of load tendency of displacement (according to the buckling and for initial deflection mode with respect to minimum total potential energy principle). Solution from the user program is compared with results gained using FEM program ANSYS.

2 THEORY

Let us assume a rectangular plate simply supported along the edges (Fig. 1) with thickness t. The displacements of the point of the neutral surface are denoted $\boldsymbol{q}=\left[u,v,w\right]^T$ and the related load vector is $\boldsymbol{p}=\left[p_x,0,0\right]^T$. By formulation of the strains, non-linear terms have to be taken into account. Then it can be written as

$$\boldsymbol{\varepsilon}=\boldsymbol{\varepsilon}_m+\boldsymbol{\varepsilon}_b,\ \boldsymbol{\varepsilon}_m=\boldsymbol{\varepsilon}_l+\boldsymbol{\varepsilon}_n \tag{1}$$

where $\boldsymbol{\varepsilon}_l=\left[u_{,x}\ ,\ v_{,y}\ ,\ u_{,y}+v_{,x}\right]^T$, $\boldsymbol{\varepsilon}_n=\dfrac{1}{2}\left[w_{,x}^2\ ,\ w_{,y}^2\ ,\ 2w_{,x}w_{,y}\right]^T$, $\boldsymbol{\varepsilon}_b=-z\cdot\boldsymbol{k}=-z\cdot\left[w_{,xx}\ ,\ w_{,yy}\ ,\ 2w_{,xy}\right]^T$,

the indexes denote the partial derivations and w represents the global displacement.

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The initial displacements have been assumed as the out of plane displacements only. Then

$$\boldsymbol{\varepsilon}_0 = \boldsymbol{\varepsilon}_{0n} + \boldsymbol{\varepsilon}_{0b} \tag{2}$$

where $\boldsymbol{\varepsilon}_{0n} = \frac{1}{2}\left[w_{0,x}^2\,,\, w_{0,y}^2\,,\, 2w_{0,x}w_{0,y}\right]^T$, $\boldsymbol{\varepsilon}_{0b} = -z\cdot\boldsymbol{k}_0 = -z\cdot\left[w_{0,xx}\,,\, w_{0,yy}\,,\, 2w_{0,xy}\right]^T$ and w_0 is the part related to the initial displacement.

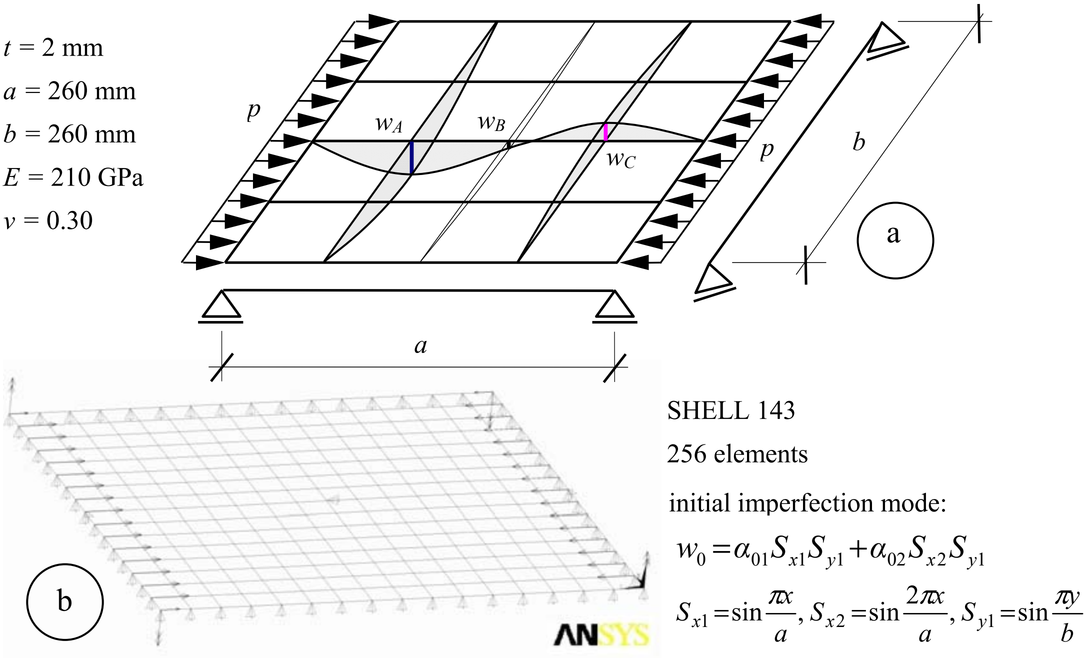


Fig. 1: Thin plate a) Notation of quantities, b) FEM model

The linear elastic material has been assumed

$$\boldsymbol{\sigma} = \boldsymbol{D}\cdot(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_0) \tag{3}$$

where $\boldsymbol{D} = \frac{E}{1-v^2}\begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \frac{1-v}{2} \end{bmatrix}$. E, v are the Young´s modulus and Poisson´s ratio.

The total potential energy can be expressed as

$$U = U_i + U_e = \int_V \frac{1}{2}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_0)^T \boldsymbol{\sigma}\, dV - \int_A \boldsymbol{q}^T \boldsymbol{p} dA \tag{4}$$

After modification Eq. (4) can be written as

$$U = \int_A \frac{1}{2}(\boldsymbol{\varepsilon}_m - \boldsymbol{\varepsilon}_{0n})^T t\boldsymbol{D}(\boldsymbol{\varepsilon}_m - \boldsymbol{\varepsilon}_{0n})dA + \int_A \frac{1}{2}(\boldsymbol{k} - \boldsymbol{k}_0)^T \frac{t^3}{12}\boldsymbol{D}(\boldsymbol{k} - \boldsymbol{k}_0)dA - \int_A \boldsymbol{q}^T \boldsymbol{p}\, dA \tag{5}$$

where $\boldsymbol{\varepsilon}$, $\boldsymbol{k}$ are strains and curvatures of the neutral surface; $\boldsymbol{\varepsilon}_0$, $\boldsymbol{k}_0$ are initial strains and curvatures; $\boldsymbol{q}$, $\boldsymbol{p}$ are displacements of the point of the neutral surface, related load vector.

The system of conditional equations can be obtained from the condition of the minimum increment of the total potential energy [2]

$$\delta \Delta U = 0 \tag{6}$$

This system can be written as

$$\boldsymbol{K}_{inc}\,\Delta\boldsymbol{\alpha} + \boldsymbol{F}_{int} - \boldsymbol{F}_{ext} - \Delta\boldsymbol{F}_{ext} = \boldsymbol{0} \tag{7}$$

where:

$\boldsymbol{K}_{inc} = \left[\begin{array}{c|c} \boldsymbol{K}_{incD} & \boldsymbol{K}_{incDS} \\ \hline \boldsymbol{K}_{incSD} & \boldsymbol{K}_{incS} \end{array}\right]$ - is the incremental stiffness matrix,

$\boldsymbol{F}_{int} = \left\{\begin{array}{c} \boldsymbol{F}_{intD} \\ \hline \boldsymbol{F}_{intS} \end{array}\right\}$ - is the vector of the internal forces,

$\boldsymbol{F}_{ext} = \left\{\begin{array}{c} \boldsymbol{F}_{extD} \\ \hline \boldsymbol{F}_{extS} \end{array}\right\}$ - is the vector of the external load of the plate,

$\Delta\boldsymbol{F}_{ext} = \left\{\begin{array}{c} \Delta\boldsymbol{F}_{extD} \\ \hline \Delta\boldsymbol{F}_{extS} \end{array}\right\}$ - is the increment of the external load of the plate,

$\boldsymbol{q} = \boldsymbol{B}\cdot\boldsymbol{\alpha} = \left[\begin{array}{c|c} \boldsymbol{B}_D & \\ \hline & \boldsymbol{B}_S \end{array}\right]\left\{\begin{array}{c} \boldsymbol{\alpha}_D \\ \hline \boldsymbol{\alpha}_S \end{array}\right\}$ and $\Delta\boldsymbol{q} = \boldsymbol{B}\cdot\Delta\boldsymbol{\alpha}$. For more details see [3].

Considering the structure in equilibrium $\boldsymbol{F}_{int} - \boldsymbol{F}_{ext} = 0$, the incremental steps can be defined as $\boldsymbol{K}_{inc}\Delta\boldsymbol{\alpha} = \Delta\boldsymbol{F}_{ext} \Rightarrow \Delta\boldsymbol{\alpha} = \boldsymbol{K}_{inc}^{-1}\Delta\boldsymbol{F}_{ext}$ and $\boldsymbol{\alpha}^{i+1} = \boldsymbol{\alpha}^{i} + \Delta\boldsymbol{\alpha}$. The Newton-Raphson iteration can be considered in the following way: taking into account that $\boldsymbol{\alpha}^{i}$ does not represent the exact solution, the residua are $\boldsymbol{F}_{int}^{i} - \boldsymbol{F}_{ext}^{i} = \boldsymbol{r}^{i}$. The corrected parameters are $\boldsymbol{\alpha}^{i+1} = \boldsymbol{\alpha}^{i} + \Delta\boldsymbol{\alpha}^{i}$, where $\Delta\boldsymbol{\alpha}^{i} = -\boldsymbol{K}_{inc}^{-1}\boldsymbol{r}^{i}$.

The identity of the incremental stiffness matrix with the Jacobian of the system of the nonlinear algebraic equations $\boldsymbol{J} \equiv \boldsymbol{K}_{inc}$ has been used in analysis. To be able to evaluate the different paths of the solution, the pivot term of the Newton-Raphson iteration has to be changed during the solution.

3 NUMERICAL RESULTS

Illustrative examples of compressed steel plate considered in Fig. 1 are presented as load – displacement paths for different amplitudes of initial geometrical imperfection. From Figs. 2 and 3 it is obvious that two almost identical modes of initial imperfection at the beginning of the process offer two different solutions in postbuckling mode. Due to the mode of the initial imperfection the nodal displacements denoted w_A and w_C have been taken as the reference values (see Fig. 1a). The thick lines in Fig. 2a and Fig, 3a represents displacement of node *A* and the thin lines represents displacement of node *C*. Shape of the plate in buckling and in postbuckling is also displayed.

The FEM computer program using a 48 DOF element (4 nodes, 12 DOF at each node) [4] has been used for analysis. FEM model consists of 4x4 finite elements (Fig. 1a). Full Newton-Raphson procedure, in which the stiffness matrix is updated at every equilibrium iteration, has been applied.

Obtained results were compared with results of the analysis using ANSYS system, where 16x16 elements model was created (Fig. 1b). Element type SHELL143 (4 nodes, 6 DOF at each node) was used.

Fig. 2 shows the solution for the initial displacement parameters $\alpha_{01} = 0.05\,\mathrm{mm}$ and $\alpha_{02} = 0.33\,\mathrm{mm}$. It can be seen that the fundamental path is in the postbuckling phase in 1st mode of buckling.

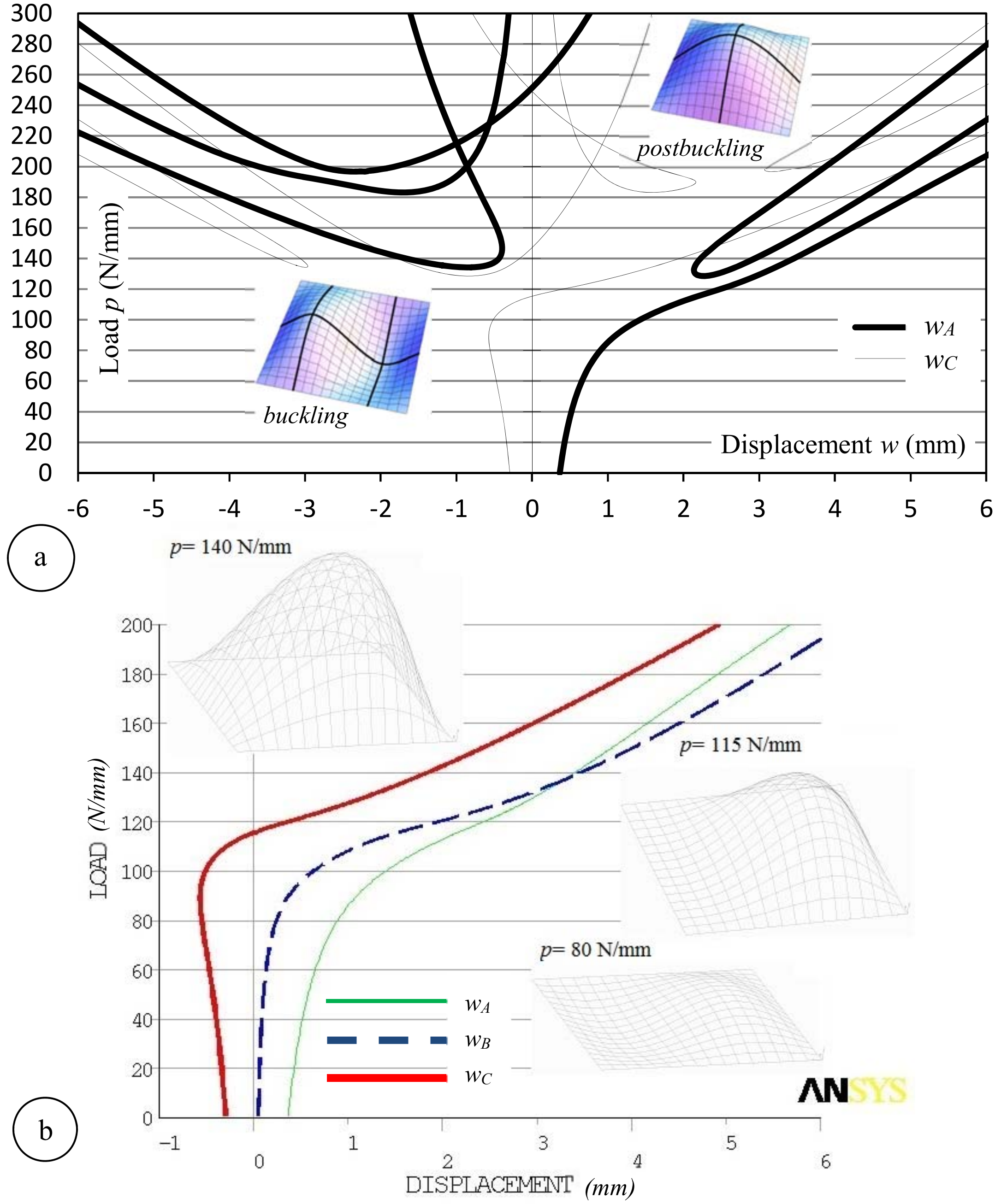
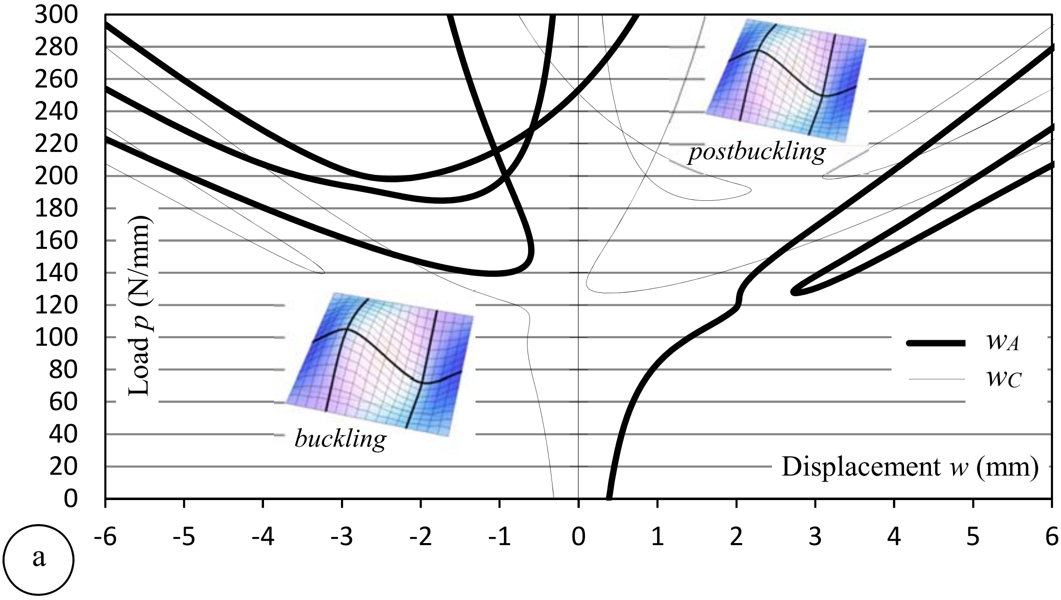


Fig. 2: The postbuckling of the thin plate with the initial displacement:

$w_0 = 0.05 \cdot \sin\frac{\pi x}{a} \cdot \sin\frac{\pi y}{b} + 0.33 \cdot \sin\frac{2\pi x}{a} \cdot \sin\frac{\pi y}{b}$, a) user program [3] [5], b) ANSYS

Increasing the effect of the 2nd mode in the shape of the initial displacement ($\alpha_{01} = 0.05$ mm and $\alpha_{02} = 0.35$ mm) the postbuckling mode of the thin plate is 2nd mode (Fig. 3).



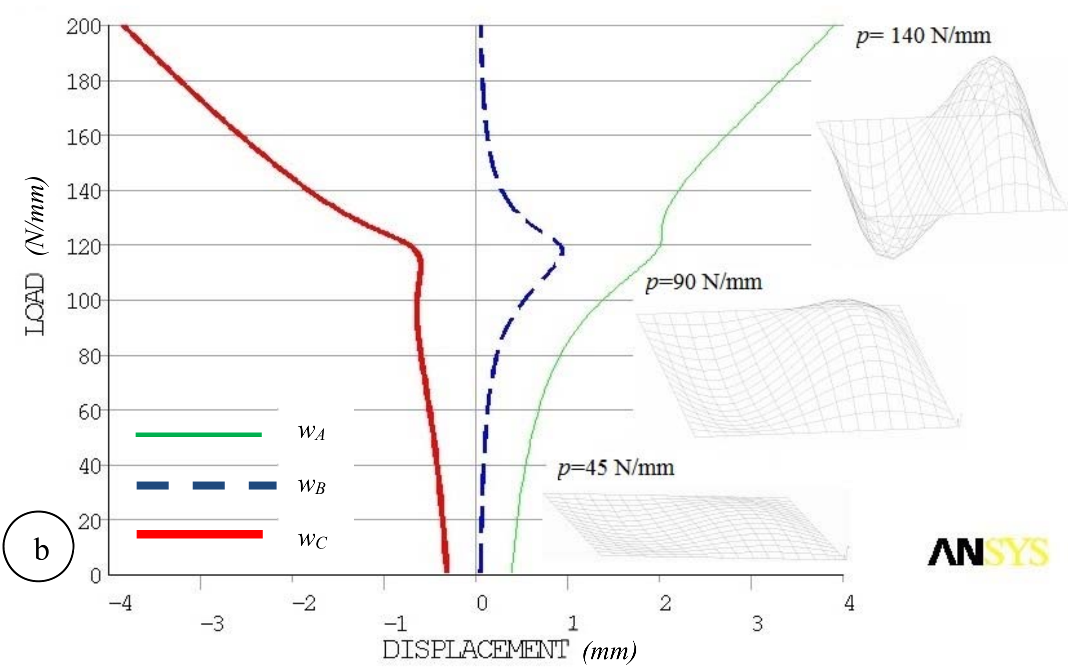


Fig. 3: The postbuckling of the thin plate with the initial displacement:

$w_0 = 0.05 \cdot \sin\frac{\pi x}{a} \cdot \sin\frac{\pi y}{b} + 0.35 \cdot \sin\frac{2\pi x}{a} \cdot \sin\frac{\pi y}{b}$, a) user program [3] [5], b) ANSYS

4 CONCLUSIONS

The influence of the amplitude and of the initial geometrical imperfection mode to the postbuckling behaviour of the thin plate is presented. Finite elements created for special purposes of thin plates stability analysis, enable high accuracy and speed convergence of the solution at less density of meshing. The possibility on an interactive affecting of the calculation within the user code makes it possible to investigate all load – displacement paths of the problem.

It can be seen that two almost identical modes of initial imperfection at the beginning of the loading process offers two different solutions in postbuckling mode. For solving models of thin plate, it is necessary to take into account initial geometrical imperfections.

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