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NUMERICAL ANALYSIS OF BUCKLING OF VON MISES PLANAR TRUSS

Abstract

A computational algorithm of a discrete model of von Mises planar steel truss is presented. The structure deformation is evaluated by seeking the minimal potential energy. The critical force invented by mathematical solution was compared with solution by computer algorithm. Symmetric and asymmetric effects of initial shape of geometric imperfection of axis of struts are used in model. The shapes of buckling of von Mises planar truss of selected vertical displacement of top joint are shown.

Keywords

Potential energy, von Mises planar truss, computer algorithm, discrete model, normal solution, bending solution, initial geometric imperfection.

1 INTRODUCTION

The von Mises truss is a static specific planar problem. This is a classic elastic system having numerous references in literature [1- 4]. The truss is created by these points, of one top joint, and two simple supports (with free rotation and fixed translation boundary conditions). The top joint is connected with the supports by means of straight bars. These bars were divided on forty-nine segments and fifty nodes. The discrete model is obtained in such a way. If the structure of this planar truss is loaded so that the vertical deflection will be imposed to top joint, our output will be individual shapes of buckling of the von Mises truss [5]. For a real structure, only one correct way of deformation will occur, but computation programmes need not be able to differentiate these shapes.

The present paper describes an algorithm which simulates the loading of top joint of the von Mises truss by deformation – deflection, calculates the force acting at the place of top joint and illustrates the shapes of buckling of the von Mises truss at individual deformation steps. The computation programme was applied to the model of von Mises truss consisting of bars formed by real profiles IPE 200. The values resulting from the programme were compared with mathematical solution.

The programme also takes into consideration the influence of geometrical imperfections which can occur in building practice.

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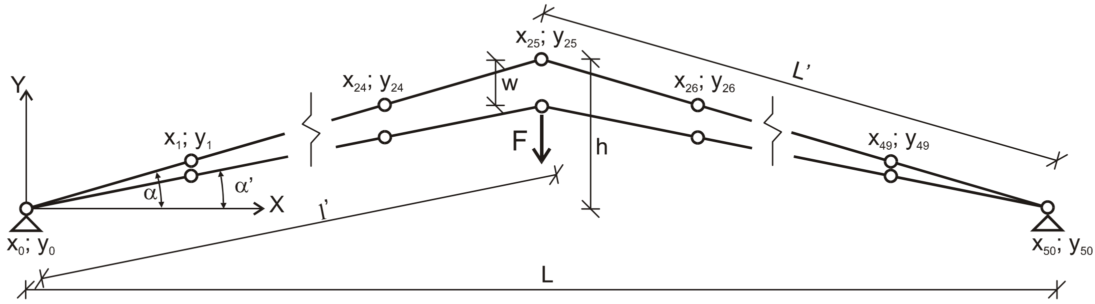

Fig. 1: Calculation model of von Mises truss

2 INPUT PARAMETERS OF CALCULATION MODEL

The scheme of calculation model of the von Mises truss is illustrated in Fig. 1. The loading is defined as a vertical deflection at top joint. The known parameters are the span L and the angle formed by the non-deformed von Mises truss with horizontal plane α. If the span L and angle α are known we can evaluate the height of von Mises Truss h, see (1).

$$h = \frac{L}{2} \cdot tg\alpha \tag{1}$$

Subsequently, the bar length (2) can be calculated applying the Pythagorean theorem and initial coordinates of individual mass points of the model are evaluated, see (3), (4).

$$L' = \sqrt{h^2 + (\frac{L}{2})^2} \tag{2}$$

$$x_i = i \cdot \frac{L}{n} \tag{3}$$

$$y_i = \frac{2h}{n} \cdot i \tag{4}$$

where:

i – represents the index of calculated node.

If coordinate y_i is evaluated for the top joint, then (4) is adjusted to the following equation stemming from (1) and (3):

$$y_i = h - (\frac{2h}{n} \cdot i - h) \tag{5}$$

3 DESCRIPTION AND MATHEMATICAL TESTING OF CALCULATION ALGORITHM

The model formed will be loaded, at top joint, by vertical deflection w. This deflection will be increasing by each iteration step k. In the initial step, the value $w = 0$ m. The size of initial deflection is $w = 0.01$ m for each iteration step. Small deformation d ($d = 1\text{x}10^{-6}$ m) according to the equation (6) will be imposed to the first (not the zero) node, the other nodes are included into the calculation without deformation (7). According to Fig. 1, at the beginning and at the end of the bar, it can be seen the form of simple supports, therefore, the deformation d will not be inserted in these nodes. Their coordinates will be then constant on the value $x_0 = y_0 = y_n = 0$ and $x_n = L$ during the whole loading time. The small deformation d is added (upper index U) and subtracted (upper index L) from the coordinate of counted node. In this way, four quadrate matrices are obtained:

$$x_i^U = x_i + d, y_i^U = y_i + d, x_i^L = x_i - d, y_i^L = y_i - d \tag{6}$$

$$x_i^U = x_i^L = x_i, y_i^U = y_i^L = y_i \tag{7}$$

3.1 Calculation of potential energy and solution of the problem of position by the Newton iteration method

For the so modified models, their potential energy can be calculated for corresponding coordinates modified by the small deformation d:

$$\begin{aligned} Ep_x^U &= \frac{1}{2}\cdot\left(K_u\cdot\sum\left(u_x^U\right)^2 + K_\varphi\cdot\sum\left(\varphi_x^U\right)^2\right), Ep_y^U = \frac{1}{2}\cdot\left(K_u\cdot\sum\left(u_y^U\right)^2 + K_\varphi\cdot\sum\left(\varphi_y^U\right)^2\right) \\ Ep_x^L &= \frac{1}{2}\cdot\left(K_u\cdot\sum\left(u_x^L\right)^2 + K_\varphi\cdot\sum\left(\varphi_x^L\right)^2\right), Ep_y^L = \frac{1}{2}\cdot\left(K_u\cdot\sum\left(u_y^L\right)^2 + K_\varphi\cdot\sum\left(\varphi_y^L\right)^2\right) \end{aligned} \tag{8}$$

where:

K_u – is axial toughness of segments connecting individual nodes [Nm^{-1}]

$$K_u = \frac{EA}{L'} \tag{9}$$

K_φ – is bending toughness of a unit formed by a node connecting two neighbouring segments [Nm]

$$K_\varphi = \frac{EI}{L'} \tag{10}$$

E – Young's modulus of material elasticity [Pa],

A – size of cross section area [m^2],

I – second moment of area [m^4],

u – size of distance between node i and (i+1) [m],

$$\begin{aligned} u_x^U &= \sqrt{\left(x_{i+1}^U - x_i^U\right)^2 + \left(y_{i+1} - y_i\right)^2} - \sqrt{\left(x_{i+1} - x_i\right)^2 + \left(y_{i+1} - y_i\right)^2} \\ u_y^U &= \sqrt{\left(x_{i+1} - x_i\right)^2 + \left(y_{i+1}^U - y_i^U\right)^2} - \sqrt{\left(x_{i+1} - x_i\right)^2 + \left(y_{i+1} - y_i\right)^2} \end{aligned} \tag{11}$$

φ – size of rotation between node i and (i+1) [rad]

$$\begin{aligned} \varphi_x^U &= \varphi_{i+1}^U - \varphi_i^U = arctg\left(\frac{y_{i+2} - y_{i+1}}{x_{i+2}^U - x_{i+1}^U}\right) - arctg\left(\frac{y_{i+1} - y_i}{x_{i+1}^U - x_i^U}\right) \\ \varphi_y^U &= \varphi_{i+1}^U - \varphi_i^U = arctg\left(\frac{y_{i+2}^U - y_{i+1}^U}{x_{i+2} - x_{i+1}}\right) - arctg\left(\frac{y_{i+1}^U - y_i^U}{x_{i+1} - x_i}\right). \end{aligned} \tag{12}$$

Analogously, $u_x^L, u_y^L, \varphi_x^L, \varphi_y^L$ will be obtained. The small deformation d will be inserted in the following node and the calculation will be repeated. The Newton iteration method was applied to find extremes of these potential energies:

$$J\left(x_i^k\right)\cdot v\left(x_i^k\right) = -f\left(x_i^k\right), J\left(y_i^k\right)\cdot v\left(y_i^k\right) = -f\left(y_i^k\right) \tag{13}$$

where:

$J\left(x_i^k\right), J\left(y_i^k\right)$ – is the Jacobi's matrix of partial derivations of the vector of function f in the given iteration step k for node i with coordinate x, or, as the case may be, y,

$v\left(x_i^k\right), v\left(y_i^k\right)$ – vector of unknown deflections of coordinate x, or, as the case may be, y and

$f(x_i^k), f(y_i^k)$ – numerical central derivations for vectors of potential energies, as given in relation (14)

$$f(x_i^k) = \frac{Ep_x^U - Ep_x^L}{2d}, f(y_i^k) = \frac{Ep_y^U - Ep_y^L}{2d}. \tag{14}$$

Assuming the matrix $J(x_i^k), J(y_i^k)$ to be regular, the vector of unknown deflections $v(x_i^k), v(y_i^k)$ gets just only one solution. The resulting values can be obtained, e.g. by applying the Gaussian elimination method, and so, new positions of individual nodes of the model can be determined:

$$x_i^{k+1} = v(x_i^k) + x_i^k, y_i^{k+1} = v(y_i^k) + y_i^k \tag{15}$$

In such a way, new coordinates for displacement of nodes are calculated. In next step of calculation, the model is loaded by increasing size of initial deflection w = 0.01 m for each iteration step k and calculation is continued from equations (6 and 7).

$$y_{(midpoint\,index)} = h - k \cdot w \tag{16}$$

The outputs are the diagram of dependence of the force on vertical deflection of top joint and buckling shapes of the von Mises truss at individual iteration steps. This method was published in [6].

3.2 Comparison of computer algorithm with mathematical solution

The model of von Mises truss of span L = 10 m and angle α = 45° was chosen for the present study. The model was divided into 50 nodes, 49 segments with axial tension stiffness and 48 bending units with bending stiffness. The cross section for straight trusses was the profile IPE 200 loaded by bending about the minor principal axis. The objective was to find the critical force F_{cr} at changing vertical deflection w and to compare this value of critical force with mathematical solution. The mathematical solution [7] is based on the pair of cases in Fig. 2.

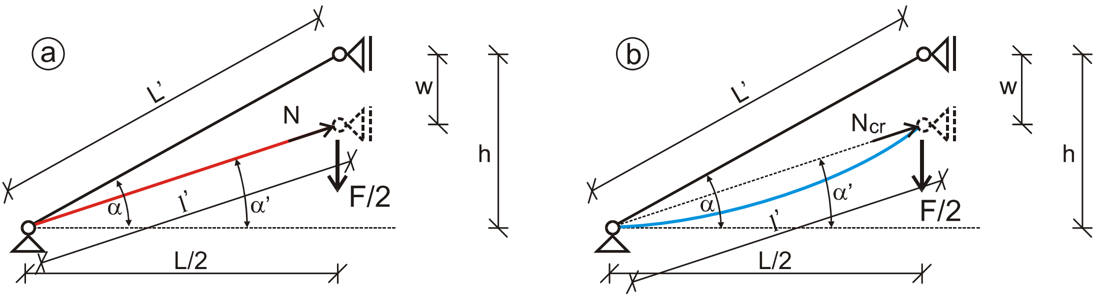

Fig. 2: Cases: a) Longitudinal solution b) Bending solution

One of these cases is the assumption of bar with bending stiffness equal to infinity. Such truss cannot buckle when loaded by vertical deflection of top joint, it remains straight after deformation and thus $I \to \infty$. Its elongation or shortening will be obtained from the relation (17)

$$\Delta = L' - l' = \frac{NL'}{EA}. \tag{17}$$

According to the equilibrium condition in the middle joint, it holds:

$$F = 2N \cdot \sin\alpha' = 2N \cdot \frac{h-w}{l'}. \tag{18}$$

The equation (19) was derived with used (17) and (18):

$$F_1 = 2\frac{L' - l'}{L'} \cdot EA \cdot \frac{h-w}{l'}. \tag{19}$$

where:

l' – is the length obtained from the Pythagorean theorem [m].

The equation (17) will be then modified. This solution was derived in [8]:

$$F_1 = 2\frac{EA}{L'} \cdot (h-w) \cdot \frac{L' - \sqrt{L'^2 + w^2 - 2hw}}{\sqrt{L'^2 + w^2 - 2hw}} \tag{20}$$

For the second case, a straight bar with high axial stiffness $A \to \infty$ and real bending stiffness EI/L' is considered. Due to this fact, only buckling of the bar can occur, elongation or compression does not take place. The problem of plane buckling is concerned, where the value of the Euler critical force can be written as $N = N_{cr} = \pi^2 EI/L'^2$. The situation is presented in Fig. 2b. By substituting $N = N_{cr}$ into the relation (18), it will be obtained:

$$F_2 = 2\frac{\pi^2}{L'} \cdot EI \cdot \frac{h-w}{l'} \tag{21}$$

Dependence of the force F_2 on vertical deflection was expressed. To find the critical vertical deflection w_{cr}, it will suffice to compare the two cases, $F_1 = F_2$:

$$w_{cr} = h - \sqrt{h^2 - \pi^2 \cdot \frac{I}{A}\left(2 - \frac{\pi^2 I}{L'^2 A}\right)} \tag{22}$$

By substitution into the equation for the problem examinated, $w_{cr} = 9.84 \times 10^{-4}$ m will be obtained, and subsequently, the studied critical force $F_{cr} = F_1 = F_2 = 83.236$ kN will be calculated. Maximum force reached according to the computation programme was $F_{max} = 83.126$ kN. The calculation deviation differs by 0.13%. It can be concluded from this fact that the calculation carried out on behalf of a computer algorithm is relatively accurate. This method was published in [9].

4 NUMERICAL NONLINEAR ANALYSIS OF IMPERFECT TRUSSES

As real building structures are influenced by geometrical imperfections, the chosen problem will be enlarged by their introduction into a computer programme. In Fig. 3, there can be observed some shapes of buckling of trusses when changing the vertical deflection w, as it was described in [10]. The studied von Mises planar truss has an angle $\alpha = 15°$. Both models are loaded by geometrical imperfections.

The model on the left is loaded by symmetrical imperfection, which having shape of one half-wave sine function with amplitude $L'/1000$. The deformation of the form of model is the same, contrary to the model on the right. There, the imperfection with amplitude $L'/1000$ is introduced with amplitude only into the left bar, the right bar remains straight without influence of imperfection. The very small bow imperfection $L'/1000$ of left bar has made significant asymmetry deformation of von Mises planar truss during loading step, see Fig. 3. It can be seen that the models with symmetrical imperfection deform less than the models with asymmetrical imperfection. To make the difference in curvature of axes evident, an enlarged scale had to be used.

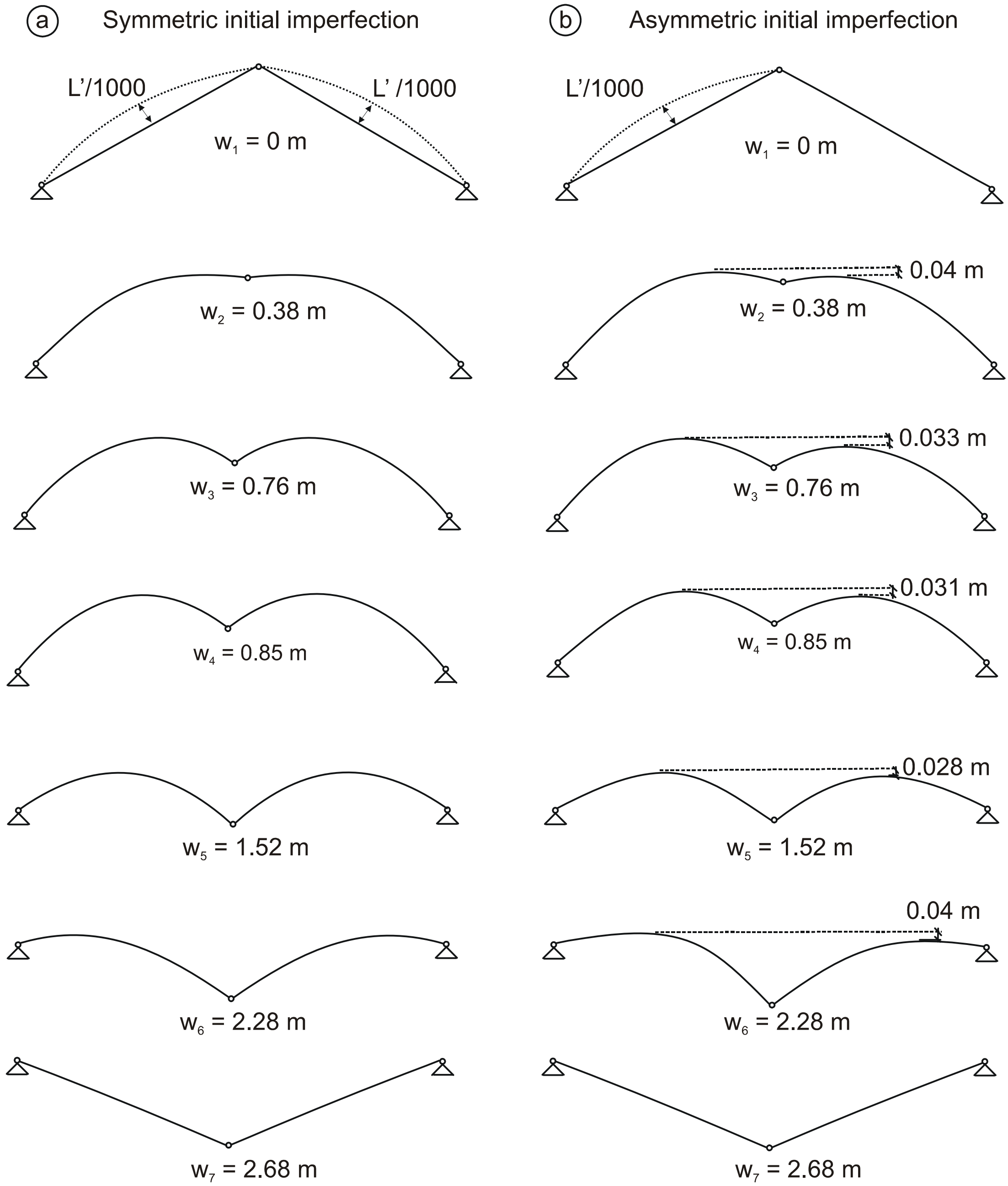

Fig. 3: Deformation of the von Mises truss with initial imperfection: a) Symmetrical b) Asymmetrical

5 CONCLUSION

In the present paper, the algorithm was described which simulates the loading of top joint of the von Mises truss by deformation – by deflection, the value of force loading on the top joint is calculated, and the shapes of buckling of the von Mises truss are illustrated at individual deformation steps. The programme output was compared on behalf of mathematical solution. Some shapes of buckling of the model of the von Mises truss with the influence of the symmetrical and asymmetrical initial imperfections were presented. According to the study, it can be assumed that the models with asymmetrical imperfection deform more than the models with symmetrical imperfection. For the future, it is planned to find how these model behave when introducing other initial geometrical imperfections. Results have shown that the computer programme can be also useful for studying load-deflection curves of asymmetrical trusses with random imperfection.

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REFERENCES

[1] VON MISES, R. *Über die Stabilitätsprobleme der Elastizitätstheorie*, ZAMM 3, 1923, Pp. 406-422.

[2] VON MISES, R. & RATZERSDORFER, J. *Die Knicksicherheit von Fachwerken*, ZAMM 5, 1925, Pp. 218-235.

[3] KWASNIEWSKI, Z. *International Journal of Non-Linear Mechanics* 44, 2009, Pp. 19-26.

[4] RAVINGER, J. *Stability and vibration*, Slovenská technická univerzita v Bratislave, Bratislava, 2012, p. 104.

[5] FRANTÍK, P. R*ozbor existence řešení dokonalého symetrického vzpěradla*. 2004. Ostrava: VŠB-TU Ostrava, 2004. Mezinárodní konference modelování v mechanice 2004. ISBN 80-248-0546-4.

[6] KALINA, M. *Static task of von Mises planar truss analyzed using potential energy. 11th International Conference of Analysis and Applied Mathematics 2013: ICNAAM 2013, Rhodes, Greece, 21-27 September 2013.* li, 2604 pages. Pp. 2107-2110, ISBN 978-0-7354-1185-2.

[7] FRANTÍK, P. *Řešení vzpěradla*. Uveřejněno na internetu na adrese http://www.kitnarf.cz/publications/2003/2003.01.www_vzperadlo/2003.01.www_vzperadlo.html. Leden 2003.

[8] GALAMBOS, T. V. *Stability Design Criteria for Metal Structures*, John Willey & Sons, New Jersey, 1998, p. 20.

[9] KALA, Z., KALINA, M. & FRANTÍK, P. *Buckling and post-buckling of von Mises planar truss. 12th International Conference of Analysis and Applied Mathematics 2014: ICNAAM 2014, Rhodes, Greece, 22-28 September 2014,* ISBN 978-0-7354-1287-3.

[10] KALINA, M. KALA, Z. & FRANTÍK, P. *Elastic buckling and post-buckling of von Mises planar trusses, International Conference on New Trends in Statics and Dynamics of Building, Bratislava, Slovenská republika, 16-17 October 2014,* ISBN 978-80-227-4259-7.

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