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APPLICATION OF MORE COMPLEX RHEOLOGICAL MODELS IN CONTINUUM MECHANICS

Abstract

The paper deals with mathematical modeling of the structural materials representing their rheological properties. The materials are modeled by more complex models (enhancement of Voigt and Maxwell models). The constitutive equations are derived; the relationships within the creep and relaxation process are developed. The rheological behavior of the materials is introduced.

Keywords

Rheological models, differential operator form of the constitutive equations, relaxation, creep, retardation and relaxation time intervals, biorheological processes.

1 INTRODUCTION

Rheology studies the stress-strain response in the matters, and especially the rate of its change. We will treat the solid substance rheological properties; derive the relations among stress, strain and their time derivatives, their time integrals and time. [5, 6, 10]

The linear rheological modeling is based on a few elementary models that are assembled in the certain way in the complex that matches the physical situation in real material as much as possible. Then the properties of the entire complex model are compounded from the particular properties of elementary models involved. In the paper we deal with modeling of the rheological properties of the solid matters, where the viscoelasticity belongs.

We will further suppose that:

- investigated body is quasi-homogeneous viscoelastic continuum where each point of the body together with its arbitrary small neighborhood nicely represents the entire body
- the Boltzman criterion of superposition is in valid
- the axioms of constitutive equations phenomenological theory is in operation
- the linear theory of viscoelasticity goes into the consideration under isothermal conditions
- the impenetrability of the matter is warranted

2 ELEMENTARY RHEOLOGICAL MATTERS

The deformation properties will be expressed by the combination of stress – strain relations and their time derivatives of elementary rheological models.

The elastic matter is represented by a spring characterized by its spring constant *E.* Likewise in the case of a matter showing both elastic and viscous properties. In such a case we combine the

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model of Hook elastic matter (**H**) and Newton viscous liquid (**N**). Accordingly, if we want to incorporate the friction properties in the model, we use the St Venant matter (**StV**). The friction is operated by the threshold friction stress σ_T. Until this value not reached, the rheological model is stagnant. Once the stress reaches or exceeds σ_T , the mode actuates with the permanent friction resistance, see Fig. 2.1.

Also the viscous liquid resists the hydrostatic pressure, which causes the volume variation. This fact is evident from the working diagrams of the elementary matters (**H**) and (**N**).

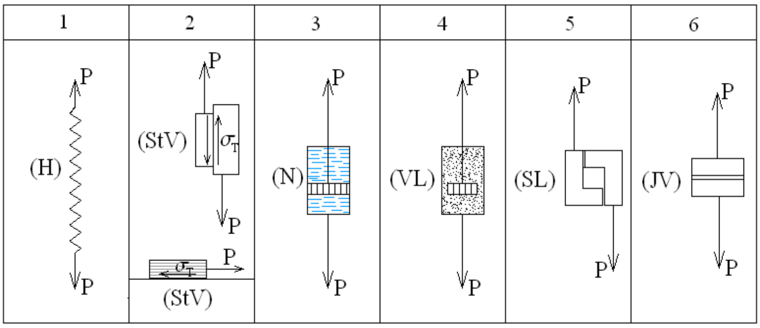

Fig. 2.1: Selected elementary rheological matters [10]

Legend:

1. Hook's solid matter (**H**)
2. St. Venant plastic (pliable) matter (**StV**)
3. Newton viscous liquid (**N**)
4. Soft (pliant) matter (**VL**)
5. Indurate matter (**SL**)
6. One – sided bound (**JV**)

3 RHEOLOGICAL MODELS IN VISCOELASTICITY

In [7] we introduce the Maxwell and Kelvin-Voigt models generated from the two elementary matters (**H**) and (**N**). The structural schemes are introduced therein, the mechanism of the constitutive equation is explained and reasoned, the creep test on both models and relaxation test on Maxwell model are performed. The relaxation test is not realizable on Kelvin-Voigt model, as it is impossible to impose the instantaneous deflection. Thus, but it is not the only reason, the enhancing of the models need arises. The enhanced (more than two-element) rheological models are more complex and they simulate real corresponding matters better. [2, 4]

3.1 Classical three-element rheological models

In the following the constitutive equation for selected three element rheological models (see the schemes in the Fig. 3.1) are justified and derived.

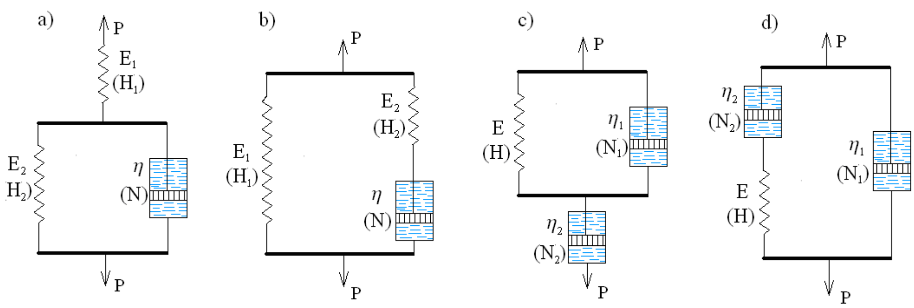

Fig. 3.1: Selected three element matters [10]

The structural scheme of the Poynting-Thompson matter (**PTh**), see Fig.3.1a), is

$$(\mathbf{PTh}) = (\mathbf{H}_1) - [(\mathbf{H}_2) \mid (\mathbf{N})] = (\mathbf{H}_1) - (\mathbf{K}) \tag{3.1}$$

where (**K**) is Kelvin model (parallel assemble of (**H**) and (**N**), see e.g. [7]

The system of the corresponding sectional constitutive equations is

$$\begin{aligned} &\varepsilon_N = \varepsilon_{H_2} && \eta\dot{\varepsilon}_N = \sigma_N \\ &\sigma_N + \sigma_{H2} = \sigma_{H1} = \sigma && E_1\varepsilon_{H_1} = \sigma_{H1} \\ &\varepsilon_{H_1} + \varepsilon_{H_2} = \varepsilon && E_2\varepsilon_{H_2} = \sigma_{H2} \end{aligned} \tag{3.2}$$

By eliminating of the indexed parameters (stresses and strains) we get a constitutive equation of (**PTh**) model in the form

$$E_1\eta\dot{\varepsilon} + E_1E_2\varepsilon = \eta\dot{\sigma} + (E_1 + E_2)\sigma \tag{3.3}$$

The rheological model in the Fig 3.1b) represents the Zener matter (**Z**). Its structure can be represented by the scheme

$$(\mathbf{Z}) = (\mathbf{H}_1) \mid [(\mathbf{H}_2) - (\mathbf{N})] = (\mathbf{H}_1) \mid (\mathbf{M}) \tag{3.4}$$

where (**M**) is Maxwell model, see e.g. [7]

Corresponding system of constitutive equations is now of the form

$$\begin{aligned} &\sigma_N = \sigma_{H_2} && \eta\dot{\varepsilon}_N = \sigma_N \\ &\sigma = \sigma_N + \sigma_{H1} && E_1\varepsilon_{H_1} = \sigma_{H1} \\ &\varepsilon = \varepsilon_{H_1} = \varepsilon_N + \varepsilon_{H_2} && E_2\varepsilon_{H_2} = \sigma_{H2} \end{aligned} \tag{3.5}$$

Accordingly, by eliminating of the indexed stress and strain values we get a constitutive equation of (**Z**) model

$$\eta(E_1 + E_2)\dot{\varepsilon} + E_1E_2\varepsilon = \eta\dot{\sigma} + E_2\sigma \tag{3.6}$$

The third rheological model, see the Fig 3.1c), can be shortened by the structural scheme

$$(\text{"3.1c"}) = (\mathbf{N}_2) - [(\mathbf{H}) \mid (\mathbf{N}_1)] = (\mathbf{N}_2) - (\mathbf{K}) \tag{3.7}$$

Corresponding system of sectional constitutive equations is now of the form

$$\begin{aligned} &\sigma_H + \sigma_{N_1} = \sigma = \sigma_{N_2} && \eta_1\dot{\varepsilon}_{N_1} = \sigma_{N1} \\ &\sigma_H = \sigma_{N_2} && E\varepsilon_H = \sigma_H \\ &\varepsilon_H + \varepsilon_{N_2} = \varepsilon && \eta_2\dot{\varepsilon}_{N_2} = \sigma_{N_2} \\ &\varepsilon_H = \varepsilon_{N_1} && \\ &\varepsilon_{N_1} + \varepsilon_{N_2} = \varepsilon && \end{aligned} \tag{3.8}$$

By eliminating of the indexed stress and strain values we get a constitutive equation of ("3.1c"):

$$\eta_1\eta_2\ddot{\varepsilon} + E(\eta_1 + \eta_2)\dot{\varepsilon} = E\sigma + \eta_2\dot{\sigma} \tag{3.9}$$

The 4th model, that one in the Fig 3.1d), is represented by the structural scheme

$$(\text{"3.1d"}) = (\mathbf{N}_1) \mid [(\mathbf{H}) - (\mathbf{N}_2)] = (\mathbf{N}_1) \mid (\mathbf{M}) \tag{3.10}$$

Corresponding system of sectional constitutive equations is now of the form

$$\begin{aligned} &\sigma = \sigma_{N_1} + \sigma_H + \sigma_{N_2} && \eta_1\dot{\varepsilon}_{N_1} = \sigma_{N1} \\ &\varepsilon = \varepsilon_{N_1} = \varepsilon_{N_2} + \varepsilon_H && \eta_2\dot{\varepsilon}_{N_2} = \sigma_{N2} \\ &\sigma_{N_2} = \sigma_H && E\varepsilon_H = \sigma_H \end{aligned} \tag{3.11}$$

By eliminating of the indexed stress and strain values we get a constitutive equation of ("3.1d") model

$$\eta_1\eta_2\ddot{\varepsilon} + E\eta_2\dot{\varepsilon} = E\sigma + (\eta_1 + \eta_2)\dot{\sigma} \tag{3.12}$$

The 3rd and 4th models depicted on the Fig. 3.1 c) and d) are represented by equations, cumbersome for mathematical treatment, though they do not simulate the matters better; hence it is optimal to use (**PTh**) and (**Z**). Both are formally of the same type and the corresponding $\sigma \sim \varepsilon$ relation can be expressed in the form

$$En\dot{\varepsilon} + H\varepsilon = n\dot{\sigma} + \sigma \tag{3.13}$$

where for (**PTh**), see Fig. 3.1 a), we take:

$$E = E_1,\ H = E_1E_2/(E_1 + E_2),\ n = \eta/(E_1 + E_2) \tag{3.14}$$

and for (**Z**), see Fig. 3.1 b):

$$E = E_1 + E_2,\ H = E_1,\ n = \eta / E_2 \tag{3.15}$$

In the relations (3.14) and (3.15) the variables E and H have their physical meaning: E represents the value of the immediate modulus and H is a long-lasting modulus. From the equation (3.13) can be seen that if the deformation rate and stress rate are relatively small in comparison to the deformation and stress magnitude, i.e. $|\dot{\varepsilon}| << |\varepsilon|$ and $|\dot{\sigma}| << |\sigma|$ then $\dot{\varepsilon}$ and $\dot{\sigma}$ can be neglected then, and, in such a case we come back to the classical Hook law $\sigma = E\varepsilon$. In the case $|\dot{\varepsilon}| >> |\varepsilon|$ and $|\dot{\sigma}| >> |\sigma|$ we get a modification of the Hook law (E being an instantaneous elastic modulus.)

$$\dot{\sigma} = E\dot{\varepsilon}, \tag{3.16}$$

Once we append the initial conditions for $t = 0$: $\varepsilon(0) = \varepsilon_0, \sigma(0) = \sigma_0$ alternatively to the equation (3.13), we can resolve it and find the exact solution of the arisen governing initial problem. Moreover, when the common initial conditions taken to the account for the matter, i.e. $\varepsilon_{0N} = 0$ for viscous element, for (**PTh**), see Fig. 3.1a) reads

$$\varepsilon_N = \varepsilon_{H_2} = \varepsilon - \varepsilon_{H_1} = \varepsilon - \frac{\sigma_{H_1}}{E_1} = \varepsilon - \frac{\sigma}{E_1} = \varepsilon - \frac{\sigma}{E} \tag{3.17}$$

If $\varepsilon_N = 0$ we get the simple relation $\varepsilon = \dfrac{\sigma}{E}$, and for initial time $t_0 = 0$ we acquire $\varepsilon_0 = \dfrac{\sigma_0}{E}$.

Hereinafter, regarding the Fig.3.1b) we obtain

$$\varepsilon_N = \varepsilon - \frac{\sigma_{H_2}}{E_2} = \varepsilon - \frac{\sigma - \sigma_{H_1}}{E_1} = \varepsilon\frac{E_1 + E_2}{E_2} - \frac{\sigma}{E_2} \tag{3.18}$$

Furthermore, if $\varepsilon_N = 0$ then in the time instant $t = t_0$ it is valid $\varepsilon_0 = \dfrac{\sigma_0}{E}$

Considering the usual initial conditions for the material satisfying the constitutive equation (3.13), it is evident that deformation obeys the Hook law for instantaneous elastic modulus.

<u>Creep test 1</u> (constant stress exposition): We are questing after the response of the rheological model in the case when we exposed it to the constant stress ($\sigma = const$). Let us recall, that the corresponding constitutive equation of the model is still given by (3.13). By solving of this equation with regard to the function $\varepsilon(t)$ we get the deformation dependence on stress

$$\varepsilon(t) = Ce^{-\frac{Ht}{En}} + \frac{\sigma}{H} \tag{3.19}$$

Onward, involving the initial condition

$$\varepsilon(0) = \frac{\sigma}{E} \tag{3.20}$$

we get the precised solution ($\varepsilon \sim \sigma$ relationship)

$$\varepsilon(t) = \frac{\sigma}{H} + \sigma\left(\frac{1}{E} - \frac{1}{H}\right)e^{-\frac{Ht}{En}} \tag{3.21}$$

The expression in the round brackets in (3.21) is negative due to $E > H$. A graphical representation of the (3.21) is performed on the Fig.3.2.

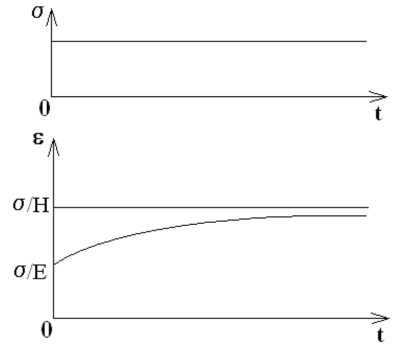

Fig. 3.2: Creep in time of the (**PTh**) and (**Z**) matters for the constant value of stress (above) with σ / E immediate deformation and σ / H long-lasting deformation magnitude highlighted

The initial deformation magnitudes (**PTh**) and (**Z**) matters are equal to the magnitude of the elastic deformation $\frac{\sigma}{E}$.

Creep test 2 (step function of stress rheological model exposition): Imposing the constant stress to (**PTh**) or (**Z**) within the time period $\langle 0, t_0 \rangle$ raises the entire deformation (let us denote it ε_0). Due to ceasing the load suddenly afterwards (in the instant $t_0, \sigma = 0$), the equation (3.19) within an interval $\langle t_0, t_1 \rangle, t_1 > t_0$ will acquire the form

$$\varepsilon(t) = Ce^{-\frac{Ht}{En}} \tag{3.22}$$

But the "initial value" ε_0 cannot be considered to be an initial condition after the unloading the model straightly, because due to sudden drop of the stress $(\sigma = 0)$ in the instant t_0, the deformation ε_0 happens down to the value

$$\varepsilon(t_0^+) = \varepsilon_0 - \frac{\sigma_0}{E} \tag{3.23}$$

where in disburdening time $t = t_0$ the deformation was ε_0 and the stress σ_0. The superscript + over the t_0 signalizes that it the discontinuity in that point should be better described by the limits.
Within the interval $\langle t_0, t_1 \rangle, t_1 > t_0$ the graph will continuously descend approaching to zero. By substituting (3.23) as the constant of integration to (3.22) we get

$$\varepsilon(t \geq t_0^+) = \left(\varepsilon_0 - \frac{\sigma_0}{E}\right)e^{-\frac{H(t-t_0)}{En}} \tag{3.24}$$

Now, we can collect and depict (3.21) and (3.24) in one graph, see Fig. 3.3.

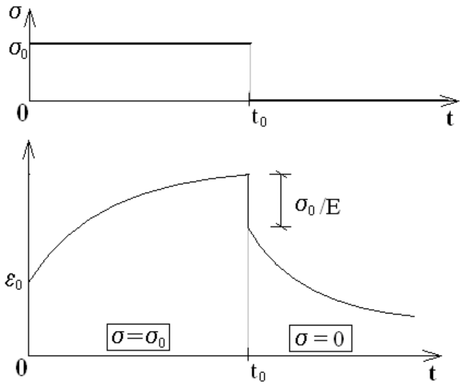

Fig. 3.3: Creep in time of the (**PTh**) and (**Z**) matters
with stress function being piecewise constant

3.2. Differential-operator form of the constitutive equations for complex rheological models

As performed above, the more complex rheological models are too laborious as soon as the constitutive equations concerns. For the sake of better convenience and clarity it is worthwhile to establish of the algebraic rearrangement symbolical method applied to the differential operators:

$$D^i(\bullet) \equiv \frac{d^i(\bullet)}{dt^i}, \quad \frac{d^k(\bullet)}{dt^k} \equiv \frac{d}{dt}\left(\frac{d^{k-1}(\bullet)}{dt^{k-1}}\right) \tag{3.25}$$

Herein, $(\bullet)$ represents a differentiable function up to the required order. $\sigma \sim \varepsilon$ relation involving symbol *D* represents the stiffness operator of the model. The stiffness operator can be completed by the all elements regarded as elastic, whereas the coefficient of "elasticity" of the viscous element is the operator ηD.

For the (**PTh**) model, see Fig. 3.1a), the entire stiffness operator of the network involving the parallelly connected elastic and viscous element will be

$$E_2 + \eta D = E' \tag{3.26}$$

Then the entire stiffness of the model is obtained from the relation

$$\frac{1}{E} = \frac{1}{E'} + \frac{1}{E_1} \tag{3.27}$$

which yields

$$\frac{1}{E} = \frac{E_1 + E'}{E_1 E'} \Rightarrow E = \frac{(E_2 + \eta D)E_1}{E_1 + E_2 + \eta D} = E(D) = \frac{\sigma(t)}{\varepsilon(t)} \tag{3.28}$$

or

$$(E_1 + E_2 + \eta D)\sigma(t) = (E_1 E_2 + E_1 \eta D)\varepsilon(t) \tag{3.29}$$

and after rearranging

$$E_1 \eta \dot{\varepsilon} + E_1 E_2 \varepsilon = (E_1 + E_2)\sigma + \eta \dot{\sigma} \tag{3.30}$$

It is can be seen that the equation (3.29) corresponds with the equation (3.3). For general rheological model involving elastic and viscous elements we can express the relative stiffness of the model in the shape of a rational function of the symbol *D*. [10, 3]

$$E(D) = \frac{\hat{A}(D)\varepsilon}{\hat{B}(D)\sigma} = \frac{A_0 + A_1 D + \ldots + A_i D^i}{B_0 + B_1 D + \ldots + B_k D^k} = \frac{\varepsilon(t)}{\sigma(t)} \tag{3.31}$$

where A_l and B_m, $(l = 1,\dots,i;\ m = 0,\dots,k)$ are the constant coefficients. Equation (3.31) is equivalent with an ordinary differential equation

$$A_0\varepsilon + A_1\dot{\varepsilon} + \cdots + A_i\varepsilon^{(i)} = B_0\sigma + B_1\dot{\sigma} + \cdots + B_k\sigma^{(k)} \tag{3.32}$$

In general, the order of the differential equation of the (3.32) type depends on the number of viscous elements. But on the other hand this order can be reduced due to the viscous elements ordering. Though the two viscous elements are connected in parallel or serially, see Fig.3.4, we treat them as just one element; likewise even when an elastic element is located between them.[9, 10]

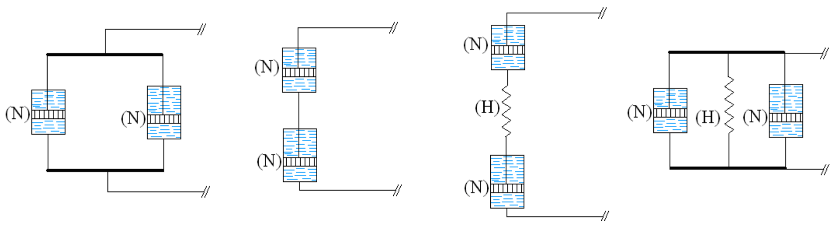

Fig. 3.4: Viscous elements ordering within a rheological model that can be formally treated as one element

This facility follows from the coequality of the deformation rates in the case of parallel connection, and in the case of serial connection the stresses and their rates will be the same. It means – if the viscous elements will be connected parallelly, then [1, 8]

$$\eta = \sum_i \eta_i \tag{3.33}$$

and for serial connection of several viscous elements

$$\eta^{-1} = \sum_i \eta_i^{-1} \tag{3.34}$$

As we look over the equation (3.31) we can claim that

- during the long-lasting constant load the mechanical properties of the system described by the corresponding rheological model stabilize ($\dot{\varepsilon} \to 0, \dot{\sigma} \to 0$) and the relative stiffness converges to the long-term elastic modulus $E(0)$
$$E(D = 0) = \frac{A_0}{B_0} = H \tag{3.35}$$
- when the instantaneous elastic modulus equals to zero, then the rheological model describing the mechanical properties of the material, is unusable.
- if $i = k$, then the instantaneous elastic modulus is finite. If $i > k$ then the elastic modulus raises out beyond all limits
- if the long-term elastic modulus equals to zero then if $\sigma =$ const acts sufficiently long to the construction, the deformations of an arbitrary magnitude can occur
- if the long-term elastic modulus $H \to \infty$, then the fully stiff construction goes into the consideration; which does not correspond to the reality, so it has to be $B_0 > 0$.
- when $\sigma \equiv 0$ then the solution to the homogeneous differential equation (3.32) can be expected in the form
$$\varepsilon(t) = \sum_{j=1}^{i} C_j e^{\lambda_j t} \tag{3.36}$$

where λ_j are the roots of the characteristic equation

$$\sum_{j=1}^{i} A_j \lambda^j = E(\lambda) = 0 \tag{3.37}$$

Since for $\lambda > 0$, the function $E(\lambda)$ is increasing and $E(0) \geq 0$. Hence, the roots of the (3.37) are negative, so the solution to the equation (3.36) is represented by the finite sum of decreasing exponential functions. The values $\left\{-\lambda_j^{-1}\right\}_{j=1}^{i}$ will determine the retardation time intervals. The retardation times values are collected in the set called the spectrum of the retardation times.

For $\varepsilon \equiv 0$ we attempt likewise. The general solution to the equation (3.32) for the unknown function $\sigma = \sigma(t)$ will be of the form

$$\sigma(t) = \sum_{j=1}^{k} C_j e^{\mu_j t} \tag{3.38}$$

where μ_j are the roots of the characteristic equation

$$\sum_{j=1}^{i} B_j \mu^j = E(\mu) = 0 \tag{3.39}$$

If μ_j runs from 0 to ∞, then the function $E^{-1}(\mu)$ descends. It is valid

$$E^{-1}(\mu) = \left\langle \begin{array}{ll} > 0 & \text{for } \mu = 0 \\ \leq 0 & \text{for } \mu \to \infty \end{array} \right. \tag{3.40}$$

The values of μ_j are called relaxation times and the sequence $\left\{-1/\mu_j\right\}_{j=1}^{k}$ is called the spectrum of the relaxation.

Example: Let us take a complex rheological model consisting of *k* ($k = 5$ in this example), elsewhere an arbitrary finite number) of parallelly connected basic Maxwell models, see Fig. 3.5.

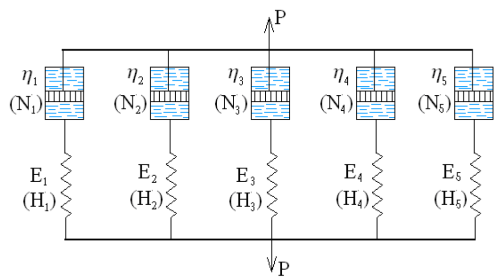

Fig. 3.5: Complex Maxwell model

The particular relaxation times are $n_i = \eta_i / E_i$. While solving the task we will apply the symbolic method for algebraic operations with differential operator *D*. Particular constitutive equation written for the i^{th} Maxwell model can be expressed as follows:

$$\begin{aligned} &\eta_i \dot{\varepsilon} = \sigma_i + n_i \dot{\sigma}_i \qquad (i = 1,2,\ldots,k) \\ &\sigma = \sum_{i=1}^{k} \sigma_i \end{aligned} \tag{3.41}$$

where n_i is a relaxation time of i^{th} particular model and the resultant stress is given by the sum of stress of particular models.

Hence, the relative stiffness for i^{th} will be

$$E_i(D) = \frac{\sigma_i}{\varepsilon} = \frac{\eta_i D}{1 + n_i D} \tag{3.42}$$

The lumping relative stiffness of the rheological model will be yielded from the relation

$$E(D) = \sum_{i=1}^{k} E_i(D) = \frac{\sigma}{\varepsilon} = D \sum_{i=1}^{k} \frac{\eta_i}{1 + n_i D} \tag{3.43}$$

Spectrum of the relaxation times can be completed by solving the equation

$$E(\mu) = \infty \Rightarrow \sum_{i=1}^{k} \frac{\eta_i}{1 + n_i \mu} = \infty \tag{3.44}$$

and

$$\mu_i = -\frac{1}{n_i} \qquad (i \approx 1, 2, ..., k) \tag{3.45}$$

In other words, the relaxation times spectrum $-\frac{1}{\mu_i}$ of analyzed model is a sequence of relaxation times n_i of all particular Maxwell models involved the investigated system.

While seeking the spectrum of retardation times we will start from the condition

$$E(\lambda) = 0 \Rightarrow \sum_{i=1}^{k} \frac{\eta_i}{1 + n_i \lambda} = 0 \tag{3.46}$$

$E(\lambda)$ is a decreasing function with a discontinuity of the second type in the points for $\lambda_i = -\frac{1}{n_i} \qquad (i = 1, 2, ..., k)$.

Among the values $\lambda_i, (i = 1, 2, ..., k)$ there are the roots of the equation (3.46), where the last value of the root k corresponds to the zero value of time retardation.

4 RHEOLOGY IN BIOMECHANICS

The human body consists of viscoelastic solid materials (bones, muscles, cartilages, ligaments, tendons, skin, etc. – mechanical behavior of all of them is dependent on the history of deformation) and over 75% of rheological liquids (circulating blood, urine, gas content of the lungs, etc.). The viscoelastic properties behavior of human body components provides the protection of the body from injuries.

It is interesting to observe e.g. the creep of skin while exposed to the stepped stress, especially after sudden unload the skin – due to changing of the viscous and elastic properties ratio, the rate of re-getting its original form strongly depends on the age of the one. As an example of the relaxation the tendon or muscles can be taken. As they are stretched gradually, without overloading, they relax in time and permanent exercising makes them more stretchy and flexible. The synovial liquids can be mentioned, acting as dashpot alleviating the hits outcome. The most focused liquid of the human body nowadays, and maybe even ever, is blood. Rheological behavior of the blood is caused by several factors. As mentioned in the chapters above, the viscosity of blood, which varies between 1 and 6 mPa.s, [11], plays the essential role in its rheological behavior. Among the blood viscosity influencing factors we can mention the easy deformability of red blood corpuscles, haematocrit, temperature, osmotic pressure difference on the both sides of the membrane of the cell, shear velocity affecting the erythrocytes clustering, etc.

Shear deformation of the red blood cells depends on the ratio of the intrinsic liquid viscosity and the viscosity of the outer ambient. Within the higher velocity range of the blood flow it was observed the significant decrease of the blood viscosity. This is caused by the profile deformation of the erythrocytes to the shape of lengthwise stretched ellipsoids, whereas the membrane of the red blood cell is rotating around the inner content of the cell. This phenomenon was observed at the shear stress magnitude over 0.5 Pa. [11]

5 CONCLUSION

The paper deals with the continuation of the previous research of the authors. It performs and elaborates the enhancements of the two – elements models described therein. The reason for the enhancement the two – elements models is further better possibility to model real material, especially biomaterials. The constitutive equations for the entire particular complex model are derived by using the constitutive equations of elementary models involving the geometrical relations arising from the configuration of compositions. The explicit stress – strain and strain – stress relations are performed, appropriate initial conditions involved. For the sake of better mathematical expressing convenience the differential operator forms are used.

Creep and relaxation tests are introduced; the mathematical treatment is described, the appropriated mathematical and graphical representations are included.

The last chapter of the paper is devoted to the examples of usage of the models in biomechanics, especially in biorheology.

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