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BUCKLING AND POSTBUCKLING OF AN IMPERFECT PLATE SUBJECTED TO THE SHEAR LOAD

Abstract

The stability analysis of an imperfect plate subjected to the shear load is presented. To solve this problem, a specialized computer program based on FEM has been created. The nonlinear finite element method equations are derived from the variational principle of minimum of total potential energy. To obtain the nonlinear equilibrium paths, the Newton-Raphson iteration algorithm is used. Corresponding levels of the total potential energy are defined. Special attention is paid to the influence of imperfections on the post-critical buckling mode. Obtained results are compared with those gained using ANSYS system.

Keywords

Stability, buckling, postbuckling, geometric nonlinear theory, initial imperfection.

1 INTRODUCTION

Solving stability of the thin plate, it is often insufficient to determine the elastic critical load from eigenvalue buckling analysis, i.e. the load, when perfect plate starts buckling. It is necessary to include initial imperfections of real plate into the solution and determine limit load level more accurately. The geometrically non-linear theory represents a basis for the reliable description of the postbuckling behaviour of the imperfect plate. The result of the numerical solution represents high number of load versus displacement paths.

2 THEORY

Restricting to the isotropic elastic material and to the constant distribution of the residual stresses over the thickness, the total potential energy can be expressed as:

$$U = \int_A \frac{1}{2}\left(\boldsymbol{\varepsilon}_m - \boldsymbol{\varepsilon}_{0m}\right)^T t\,\boldsymbol{D}\left(\boldsymbol{\varepsilon}_m - \boldsymbol{\varepsilon}_{0m}\right) dA + \int_A \frac{1}{2}\left(\boldsymbol{k} - \boldsymbol{k}_0\right)^T \frac{t^3}{12}\boldsymbol{D}\left(\boldsymbol{k} - \boldsymbol{k}_0\right) dA - \int_A \boldsymbol{q}^T \boldsymbol{p}\, dA \tag{1}$$

where:

$\boldsymbol{\varepsilon}_m, \boldsymbol{k}$ – are strains and curvatures of the neutral surface,

$\boldsymbol{\varepsilon}_{0m}, \boldsymbol{k}_0$ – are initial strains and curvatures,

$\boldsymbol{q}, \boldsymbol{p}$ – are displacements of the point of the neutral surface, related load vector.

The system of conditional equations [1] one can get from the condition of the minimum of the increment of the total potential energy $\delta\,\Delta U = 0$. This system can be written as:

$$\boldsymbol{K}_{inc}\,\Delta\boldsymbol{a} + \boldsymbol{F}_{int} - \boldsymbol{F}_{ext} - \Delta\boldsymbol{F}_{ext} = \boldsymbol{0} \tag{2}$$

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where:
$\boldsymbol{K}_{inc}$ – is the incremental stiffness matrix of the plate,
$\boldsymbol{F}_{int}$ – is the internal force of the plate,
$\boldsymbol{F}_{ext}$ – is the external load of the plate,
$\Delta\boldsymbol{F}_{ext}$ – is the increment of the external load of the plate.

Eq. (2) represents the base for the Newton-Raphson iteration and the incremental method as well. The Gauss numerical integration (5 points) was used to evaluate the stiffness matrices and the load vectors.

3 FEM NONLINEAR ANALYSIS

The FEM computer program using a 48 DOF element [2] has been created for analysis. Used FEM model [3] consists of 8x8 finite elements. Full Newton-Raphson procedure, in which the stiffness matrix is updated at every equilibrium iteration, has been applied [4]. The fundamental path of the solution starts from the zero load level and from the initial displacement. It means that the nodal displacement parameters of the initial displacements and the small value of the load parameter have been taken as the first approximation for the iterative process. To obtain other paths of the solution, random combinations of the parameters as the first approximation have been used. Interactive change of the pivot member during calculation is necessary for obtaining required number of load – displacement paths. Quality of presented paths has not been investigated in this paper.

Obtained results were compared with results of the analysis using ANSYS system, where 32x32 elements model was created (Fig. 1b). Element type SHELL143 (4 nodes, 6 DOF at each node) was used [5]. The arc-length method was chosen for analysis, the reference arc-length radius is calculated from the load increment. Only fundamental path of nonlinear solution has been presented. Shape of the plate in postbuckling has also been displayed.

4 ILLUSTRATIVE EXAMPLE

Illustrative example of steel plate loaded in shear (Fig. 1) is presented. Results of eigenvalue buckling analysis are presented first. These serve to prepare shapes of initial geometrical imperfection [6], [7] as a linear combination of eigenvectors. Also offer an image about location of critical points of nonlinear solution, help with settings in the management of nonlinear calculation process. Results of fully nonlinear analysis follow.

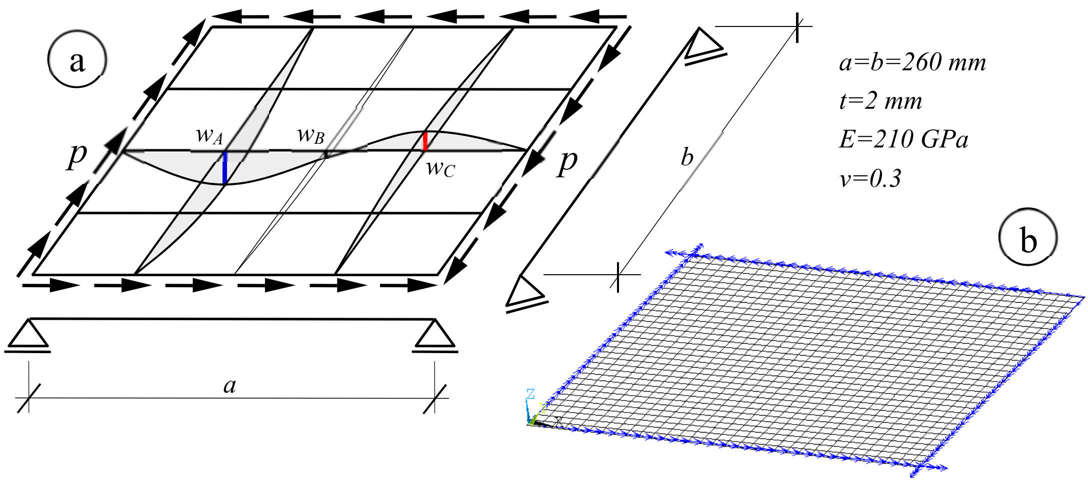
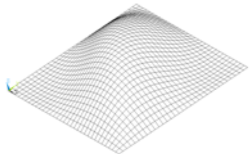
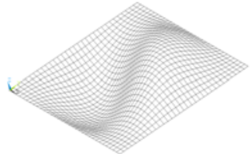
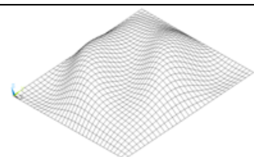
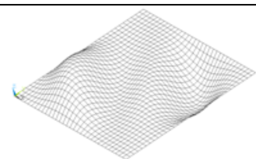
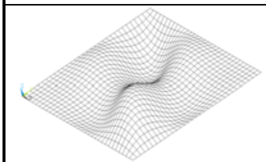
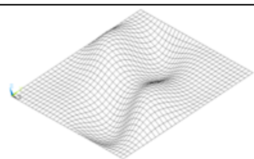
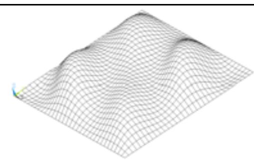
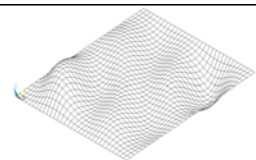


Fig. 1: a) Notation of the quantities of the plate loaded in shear, b) ANSYS FEM model

4.1 Eigenvalue buckling analysis

Eigenvalue buckling analysis predicts the theoretical buckling strength of an ideal linear elastic structure and is a problem of eigenvalues and eigenvectors [8]. Eigenvalues define the buckling load multipliers and the corresponding eigenvectors buckling mode shapes of the structure. Results for perfect plate [9] from Fig. 1 can be seen in the Table 1.

Tab. 1: Buckling loads p_{cr} [N/mm] and modes of buckling

209.07 [N/mm]	258.86	554.57	598.53
			
685.41	721.70	896.66	983.59
			

4.2 Nonlinear analysis

The geometrically nonlinear theory represents a basis for the reliable description of the postbuckling behaviour of the plate [10], [11]. The result of the numerical solution of steel plate loaded in shear is presented as load – displacement paths. The initial displacements were assumed as the out of plane displacements only [12] as a combination of buckling modes

$$d_0 = \sum \alpha_i * MODE_i \tag{3}$$

Parameters α_i are mentioned below. In order to better describe post-buckling shape of the plate, nodal displacements w_A, w_C have been taken as the reference nodes (see Fig. 1).

These presented nonlinear solutions of the postbuckling behaviour of the plate are divided into two parts. On the left side, there is load versus nodal displacement parameters relationship, on the right side the relevant level of the total potential energy is drawn [13]. (Unloaded plate represents a zero total potential energy level.)

Following Figures present two cases, in which the plate in a post-buckling mode buckles in the shape that is identical to a shape of initial imperfection (but different from the first buckling mode obtained from eigenvalue buckling analysis). The difference consists in a fact, that while in first case the fundamental path represents the path with minimum value of the total potential energy for a given load, in the second case there exists also a path with the total potential energy level lower than that of the fundamental path [14].

Figure 2 presents a nonlinear analysis of the plate with initial imperfection whose shape was formed from first three eigenmodes. According to (3), following parameters α were considered: α_1=0.3 mm, α_2=0.2 mm, α_3=0.1 mm. There are presented first three loading paths representing various forms of changes between buckling shapes. Displacement w_C has been plotted by a solid line, w_A by a dashed line. The Figure illustrates also shapes of the buckling area for particular paths and selected load values. In the right part, respective values of total potential energy can be seen. Fundamental path corresponds with the minimum value of total potential energy, thus there is no presumption of a snap-through.

For comparison with an analysis of the same plate using ANSYS software system, the fundamental path of solution is presented (see Figure 3). In the selected load levels, corresponding deformed shapes of the plate have been drawn along the paths. For greater clarity different scales were chosen for different load levels.

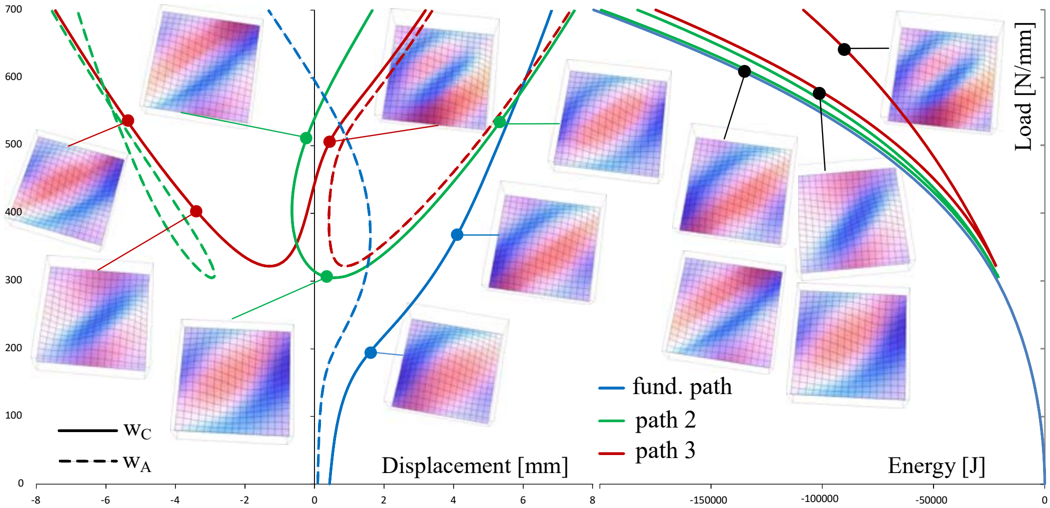


Fig. 2: Results for α_1=0.3 mm, α_2=0.2 mm, α_3=0.1 mm

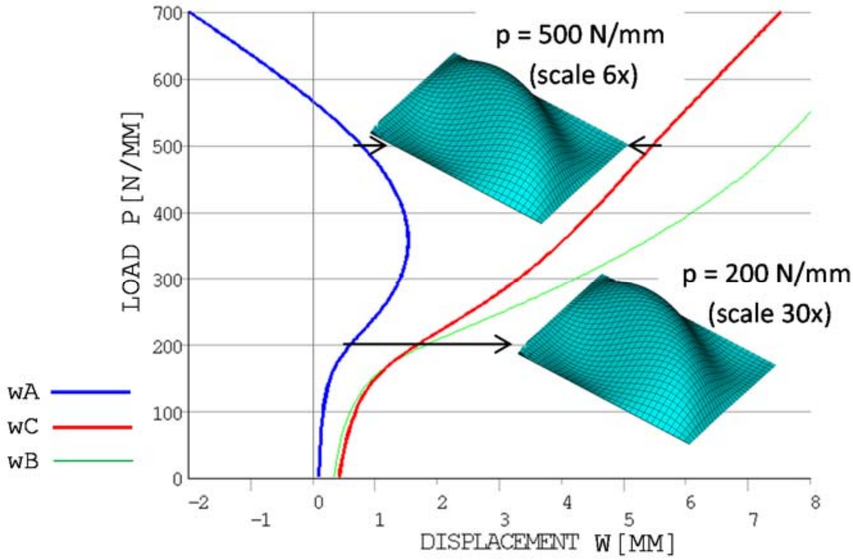


Fig. 3: Fundamental path for α_1=0.3 mm, α_2=0.2 mm, α_3=0.1 mm from ANSYS

In Figure 4 one can observe analysis of a thin plate with initial imperfection of a shape identical to a shape of the 2nd eigenmode. Parameter α_2 of a value 0.1 mm has been considered.

Displacement w_C has been plotted by a solid line, w_A by a dashed one again. Shapes of the buckling area are located next to the paths. On the right side of the Figure one can see, that the total potential energy for the fundamental path (blue line) is higher than energy for path 2 (red line). This path 2 represents buckling according to the 1st buckling mode, thus there is presumption of a snap-through.

For comparison with an analysis of the same plate using ANSYS software system, the fundamental path of solution is presented (see Figure 5). In the selected load levels, corresponding deformed shapes of the plate have been again drawn along the paths. For greater clarity different scales were chosen for different load levels.

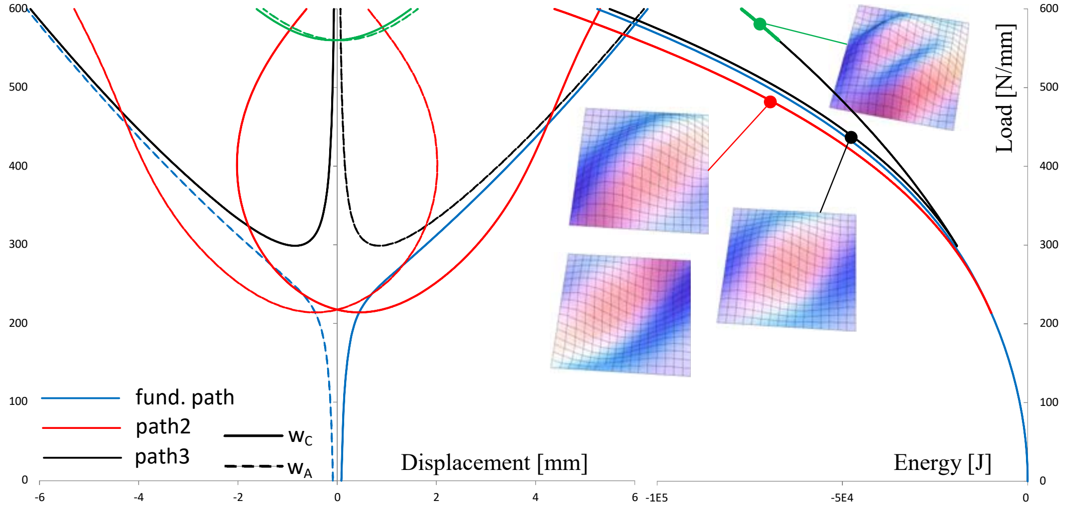


Fig. 4: Results for α_2=0.1 mm

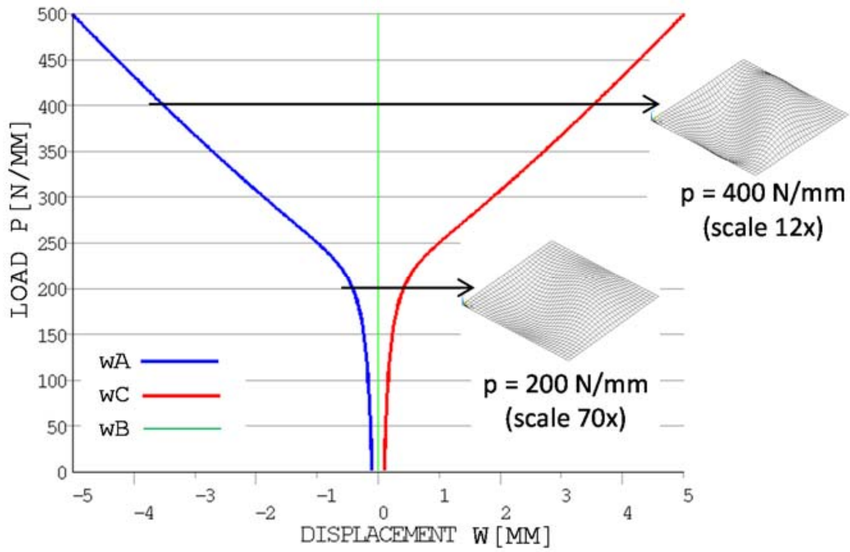


Fig. 5: Fundamental path for α_2=0.1 mm from ANSYS

5 CONCLUSIONS

The influence of the value of the amplitude and the mode of the initial geometrical imperfections on the postbuckling behaviour of the thin plate subjected to the shear load was presented. Finite elements created for special purposes of thin plates stability analysis, enable high accuracy and speed convergence of the solution at less density of meshing. The possibility on an interactive affecting of the calculation within the user code makes it possible to investigate all load – displacement paths of the problem.

As the important result one can note, that the level of the total potential energy of the fundamental stable path can be higher than the total potential energy of the secondary stable path.

This is the assumption for the change in the buckling mode of the plate. The evaluation of the level of the total potential energy for all paths of the non-linear solution is a small contribution to the investigation of the post buckling behaviour of thin plates.

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