

Katarína TVRDÁ¹**OPTIMAL DESIGN, RELIABILITY AND SENSITIVITY ANALYSIS OF FOUNDATION PLATE****Abstract**

This paper deals with the optimal design of thickness of a plate rested on Winkler's foundation. First order method was used for the optimization, while maintaining different restrictive conditions. The aim is to obtain a minimum volume of the foundation plate. At the end some probabilistic and safety analysis of the deflection of the foundation using LHS Monte Carlo method are presented.

Keywords

Reliability, probability, foundation plate, optimal design, thickness, Winkler's theory, LHS Monte Carlo method, minimum volume, safety.

1 INTRODUCTION

The importance of optimizing certain parameters of the structures at the global level continually increases. Many world leading universities deal with this problem, in particular when proposing the topic tasks for the practice, in particular in the air force industry, the construction of ships as well as building and civil engineering structures. The importance of optimization of structural stiffness is also increasing in civil engineering, particularly in optimization of structures stored on the elastic foundation [2-10]. Thickness of the foundation plate rested on Winkler's foundation is reduced to the minimum prescribed thickness depending on the objective function and the optimization of parameters to be determined in the optimization analysis by using ANSYS software package [1]. In the design of structures or parts there is one of the most important tasks of assessing the reliability of the structure, and its ability to retain the confidence of the means required for the entire time of the technical properties of life. The quality of the design and methods used for the assessment of structures are being developed with increasing levels of theoretical and practical knowledge and, of course, by increasing the quality of the computer equipment. Reliability assessment method of construction can be divided into deterministic and probabilistic analysis, or simulation, half-probabilistic analysis, and others. Probability calculation takes into account the effect of the variability of the thickness of the structure, the load effect, the material parameters, the geometric characteristics and variability of soil compressibility modules. It also allows an assessment of structure in terms of better security and reliability of the structure. Calculation on probability sampling is concerned with a number of authors (see [11-14]).

2 OPTIMIZATION OF PLATE THICKNESS AT CHOSEN POINTS

This section deals with an example of optimizing plates using first-order method, which uses except unknown function and its first derivative. We calculate the plate rested on Winkler foundation, below the object, which has seven above-ground and one underground floor. The plate is made

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of concrete C25/30 and it is stored on moderate fever gravel and sand ground. Material properties are: $E_{xl} = 31 \text{ GPa}$, $\nu = 0.2$ and the compressibility modulus of the soil $k = 70 \text{ MN/m}^3$.

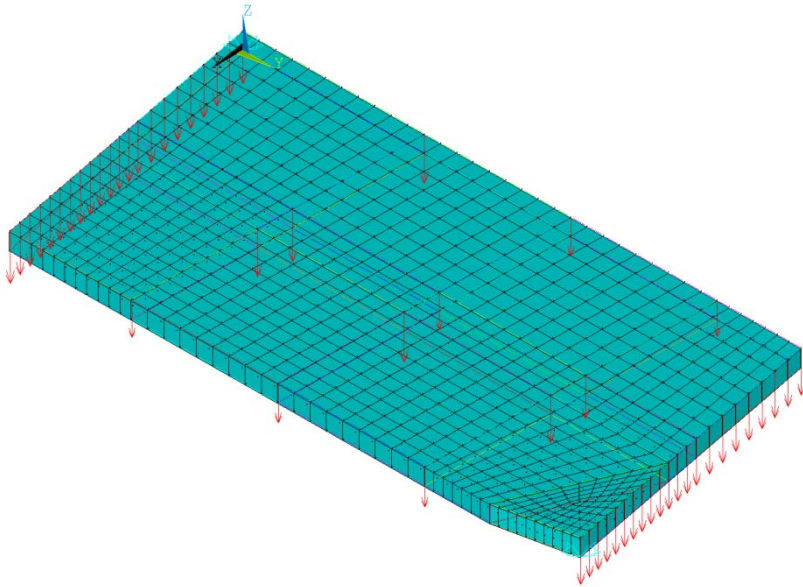
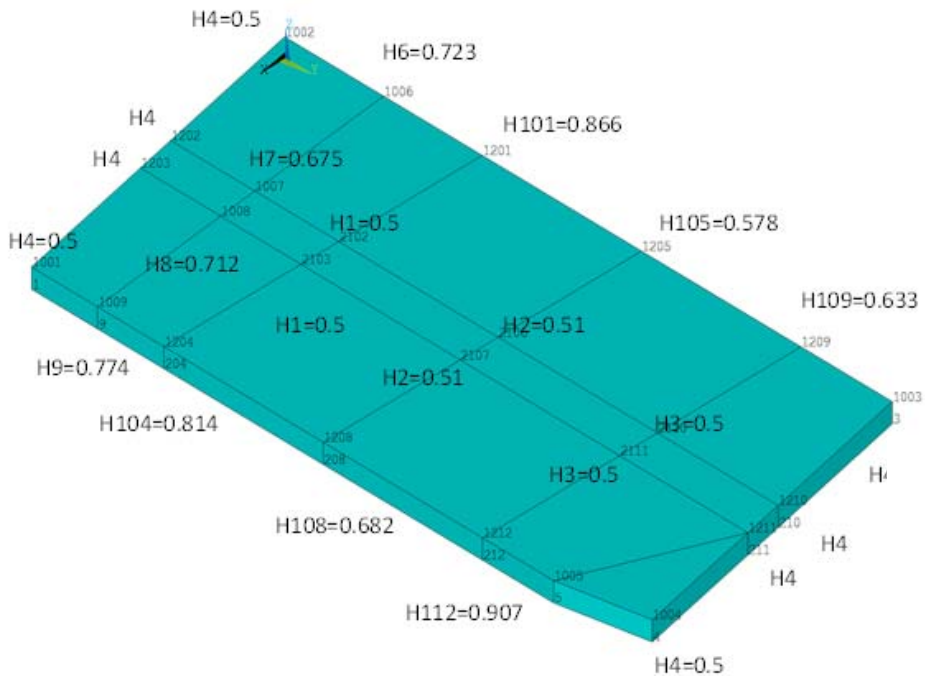


Fig. 1: Statics model of foundation plate

Optimization of the plate is realized on to a static model (Fig. 1). The goal of the optimization is to minimize the volume of the foundation plate by the thicknesses in the individual points H. Foundation plate thickness varied from 0.5 m to 1.5 m, and such in the points marked as H4 was the same thickness.



SOLID185 elements were used for modelling plate and SURF154 elements for modelling ground with stiffness subsoil in ANSYS software [1]. SOLID185 element was selected because of the optimization procedure (advantageous to optimize volume).

Table 1: Table of the final thicknesses

parameter	Name	Design 1/set 16	Design 2/set 19
STRS – stress /kPa/	SV	24755	24939
DEFL – deflection /m/	SV	7.866 e^{-3}	7.723 e^{-3}
H1– thickness /m/	DV	0.530	0.500
H2	DV	0.500	0.509
H3	DV	0.532	0.500
H4	DV	0.500	0.500
H6	DV	0.500	0.723
H7	DV	0.500	0.675
H8	DV	0.500	0.712
H9	DV	0.500	0.774
H101	DV	0.814	0.866
H104	DV	0.780	0.814
H105	DV	0.674	0.578
H108	DV	0.682	0.682
H109	DV	0.709	0.633
H112	DV	0.934	0.907
TVOL -volume	OBJ	303.03	306.22

The set von Misses stress equal 25 MPa was chosen as the limit parameter. The problem was solved using the First order optimization method. Optimization was carried out in ten or fourteen points. (See Tab. 1, Fig. 2). The best optimization design was proposed in the 19th iteration step in the design Nr. 2.

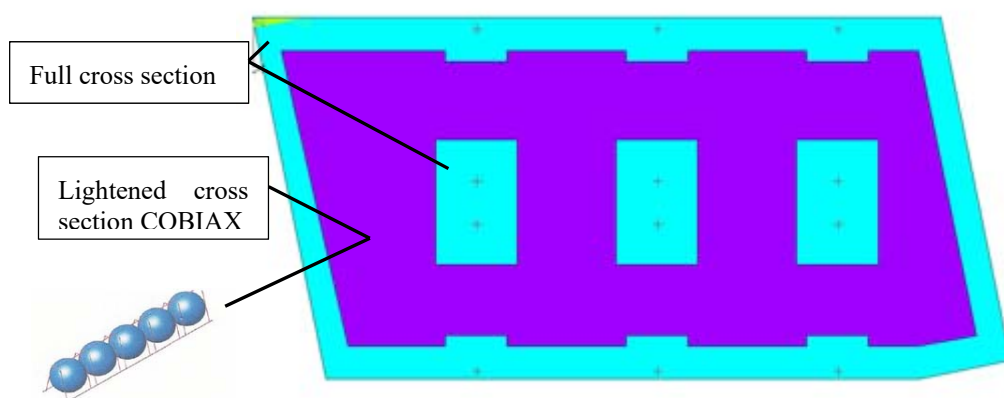


Fig. 3: The final draft of the plate thickness after optimization procedure

The final thickness of the plate determined by the optimization procedure was 950 mm made of two layers. The first layer thickness of 350 mm was made earlier. The Cobiach balls with diameter $\varphi = 450$ mm, put on this layer, form the second layer of 600 mm thickness. In the area nearest to the columns, lightweight is not implemented as a result of resistance to puncture plate (Fig. 3).

3 PROBABILITY CALCULATION

To determine the reliability of the design we used probability calculation to determine the reliability of the criteria on the basis of a functional relationship between the first n input variables, called bases random variables X_i , where $i = 1, 2, \dots, n$. This relationship is called a function of reliability (security, usability, feature or function, or function failure reliability reserves) and is marked as

$$F_s = g(X_1, X_2, \Lambda, X_n) \quad (1)$$

A functional dependency $g(X_1, X_2, \dots, X_n)$ is a computational model that is based on simplifying assumptions and represents the idealization of physical reality.

In deterministic calculation we entered the input parameters as the fixed constants. When we used probability calculation, the input parameters specified in the range were accidentally due to inaccuracies in manufacture and the determination of material characteristics. The individual parameters varied according to the diagrams. Five mutually independent random variables were used as the stochastic inputs: h_{var} , q_{var} , $EX1_{\text{var}}$, $EX2_{\text{var}}$, K_{var} and w_{VAR} (see Tab. 2). The foundation plate was loaded with singular forces from columns and the linear loads. The forces of the columns are $F = 3769 - 5515$ kN and the linear load is between $q_{\text{det}} = 65-113$ kN / m (Fig.4).

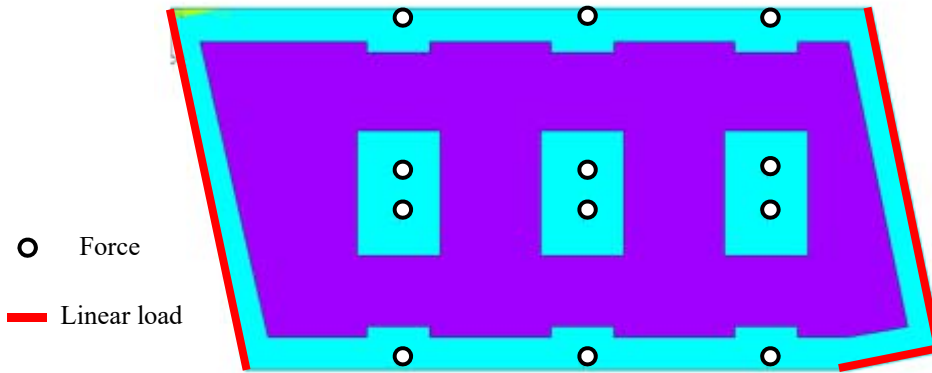


Fig. 4: Location forces and linear load

Table 2: Tables of deterministic and stochastic inputs

Inputs	deterministic	Par1	Par2	type
H1 [m]	H1 = 0.95	0.94	0.96	uniform
q [kN/m]	q	1	0.1	lognormal
EX1 [kPa]	31 e6	2.759e7	1.3795e6	gauss
EX2 [kPa]	0.85*31e6	2.759e7	1.3795e6	gauss
K [kN/m ³]	70000	70000	3500	gauss

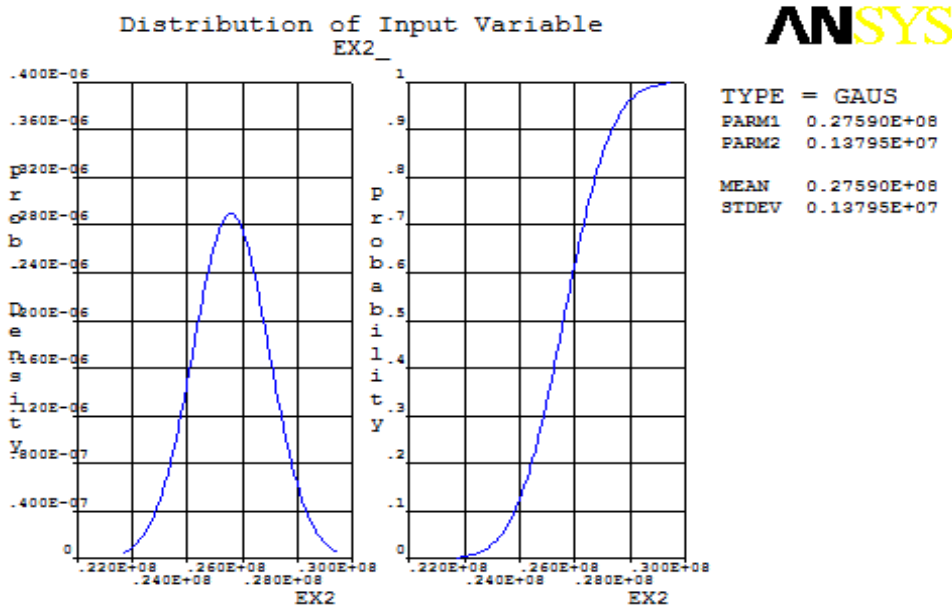


Fig. 5: Distribution of input variable modulus of elasticity of plate EX1_, EX2_

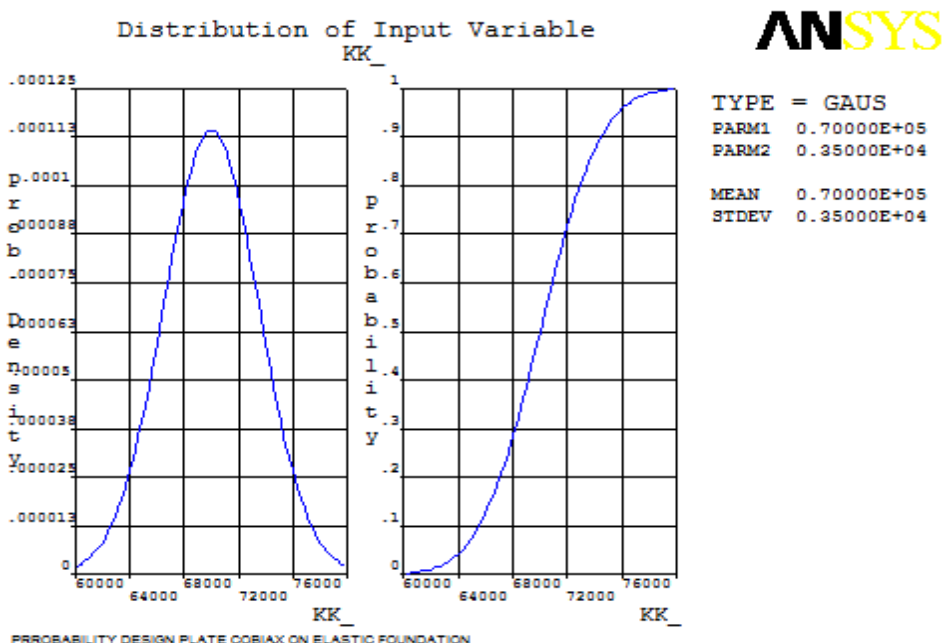


Fig. 6: Distribution of input variable foundation stiffness K_

These input parameters (geometry, material properties, and boundary conditions) are defined in the model.

The variation of these input parameters are defined as random input variables and are characterized by their distribution type (Gaussian – Fig. 5, Fig. 6, lognormal – Fig. 7, uniform – Fig. 8, etc.) and by their distribution parameters (mean values, standard deviation, etc.).

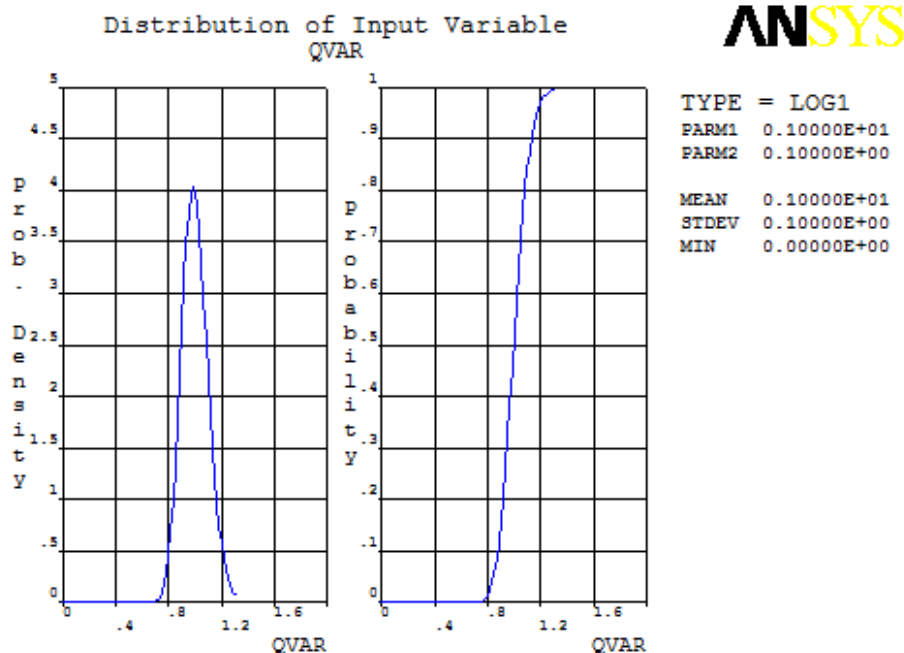


Fig. 7: Distribution of input variable load Qvar

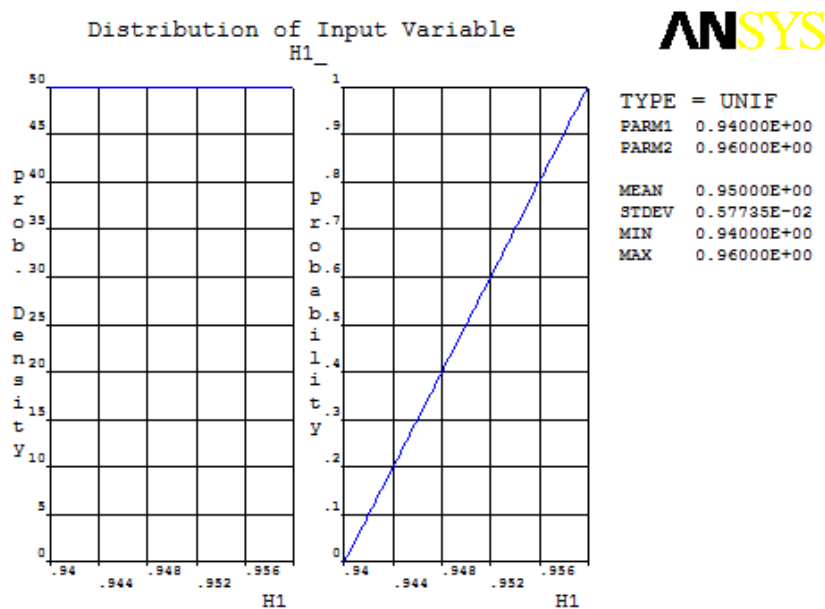


Fig. 8: Distribution of input variable thickness H1_

During a probabilistic analysis ANSYS executes multiple analysis loops to compute the random output parameters as a function of the set of random input variables. The values for the input variables are generated either randomly using Monte Carlo simulation with Latin Hypercube Sampling. The number of samples was set to 1000. The important results are defined as random output parameters (Table 3.).

Table 3: Result Set VYSL_MONTECARLO_LHS - Statistics of the Random Output Parameters

Name	Mean	Standard Deviation	Skewness	Kurtosis	Minimum	Maximum
MAX_MX	704.0	76.17	0.3248	0.1760	498.9	976.5
MAX_MY	698.9	74.25	0.3326	0.1614	500.1	963.3
NAP	1.5351E+04	1553.	0.3012	0.1286	1.1193E+04	2.1223E+04
SPOL	1.411	0.4923	0.3623	-1.872	1.000	2.000
PRIEH	-4.4134E-03	4.6917E-04	-0.3462	0.2723	-6.1133E-03	-3.1680E-03

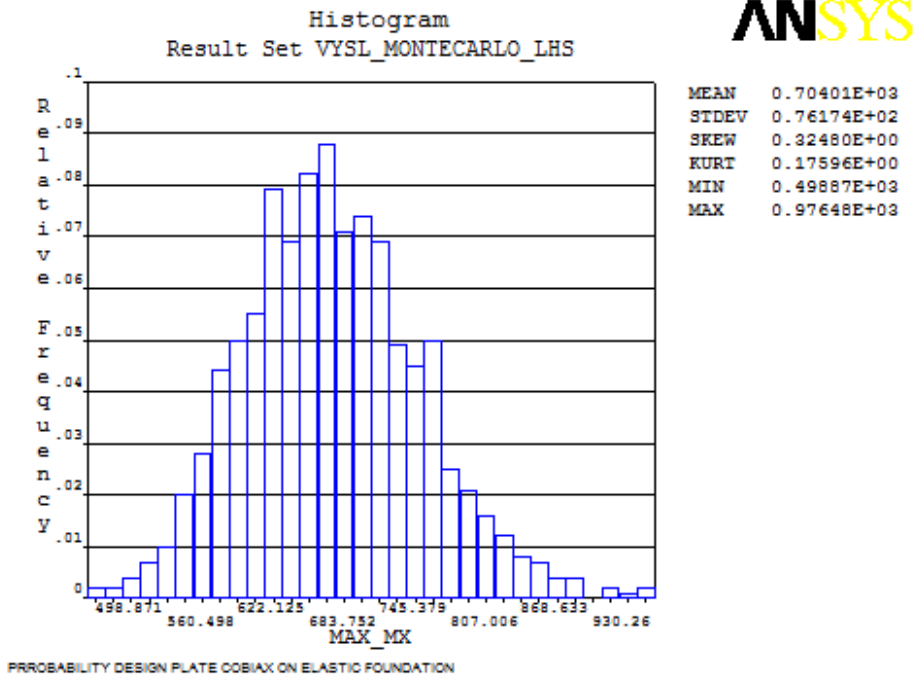


Fig. 9: Histogram of output variable MAX_Mx

The cumulative distribution curve is plotted with either a total number of 100 points or using the sample size as the number of points, whichever is lower. If the sample size is less than 100, all samples are represented in the plot. If the sample size is larger than 100, the probabilistic design tool classifies the sample into 100 classes of appropriate size.

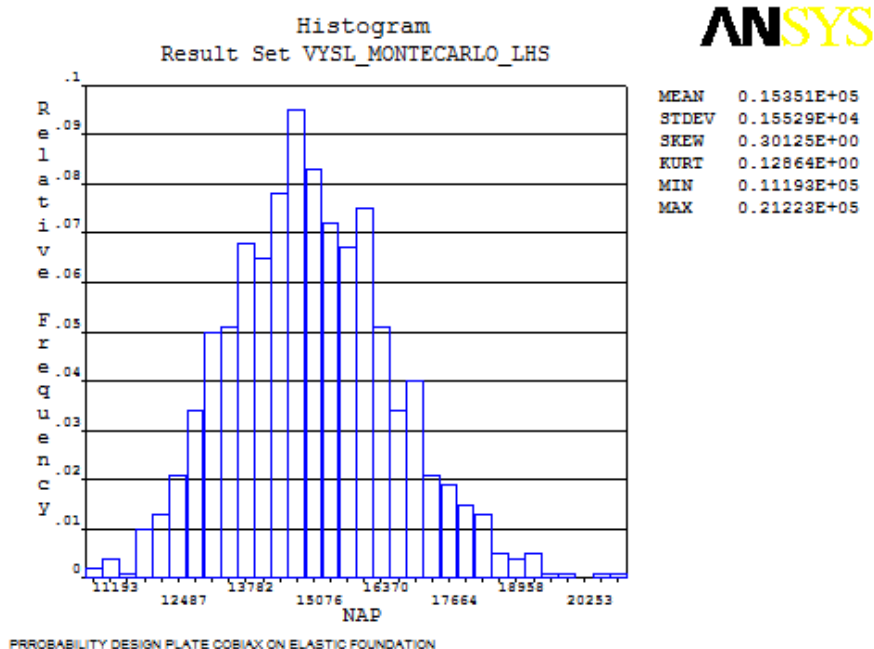


Fig. 10: Histogram of output variable Sigma ekv

For printing the report no confidence level has been specified. Therefore, a default confidence level of 95.00% will be used. For Monte Carlo simulation methods, a confidence level of 95.000% is used to plot confidence bounds around the cumulative distribution function.

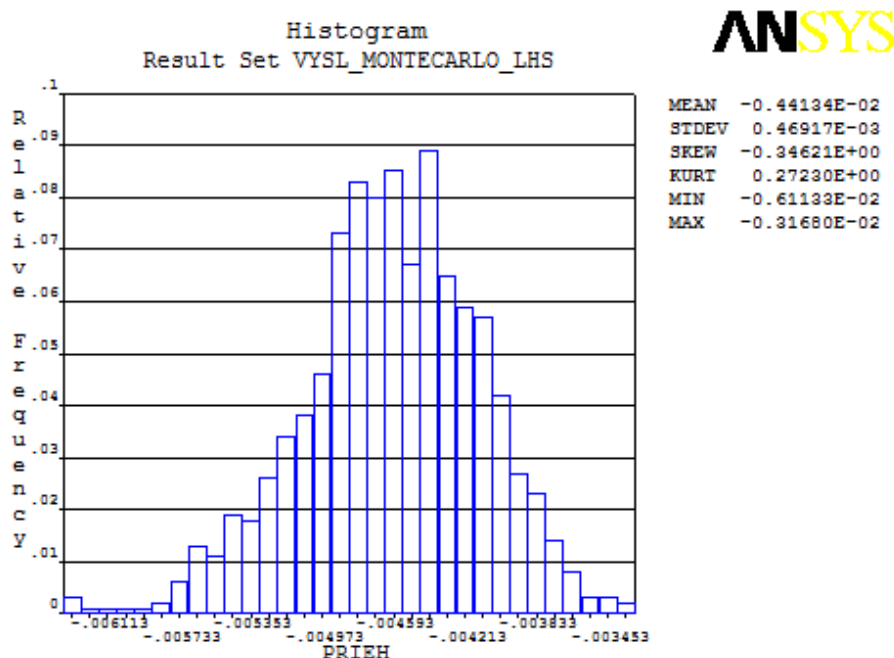


Fig. 11: Histogram of output variable PRIEH - deflection

For response surface methods, the accuracy of the results is determined by the goodness of the response surface fit and not by the confidence level. Therefore, the curves for lower and upper confidence bounds are not plotted for response surface method results.

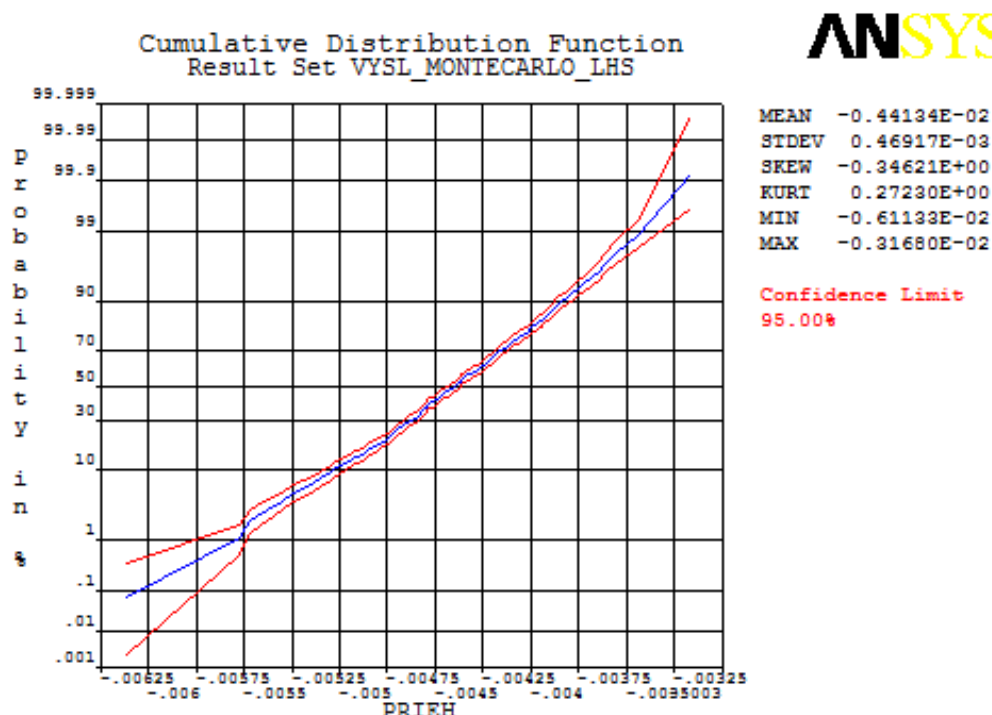


Fig. 12: Histogram of cumulative distribution function PRIEH - deflection

According to the CDF (Fig. 12) we can determine probability of the corresponding parameter PRIEH (the maximum value of deflection).

The output shows the probability that max deflection is less than -0.006 m, representing that the design is at $3.47e^{-3}$ unreliable.

4. PROBABILISTIC SENSITIVITY

The evaluation of the probabilistic sensitivities is based on the correlation coefficients between all random input variables and a particular random output parameter. Either Spearman rank order correlation coefficients or Pearson linear correlation coefficients may be used based on user's specifications.

To plot the sensitivities of a certain random output parameter, the random input variables are separated into two groups: those that are significant (important) and those that are insignificant (not important) for the random output parameter. The sensitivity plots will only include the significant random input variables.

Sensitivity analysis (Fig. 13) showed that the input parameter QVAR – load, EX1_, EX2_ module of elasticity, KK_ foundation stiffness most affected on the output parameter Max_Mx – specific bending moment.

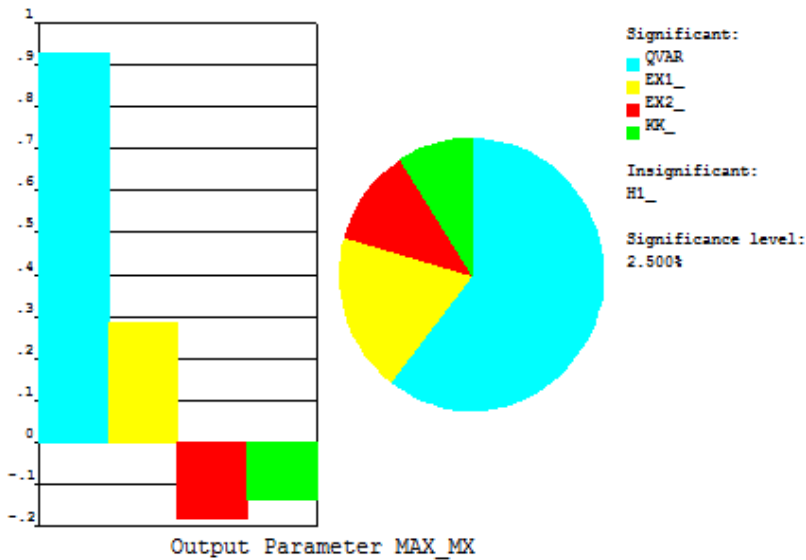


Fig. 13: Rank-order correlation sensitivities of output variable MaX_MX – specific bending moment

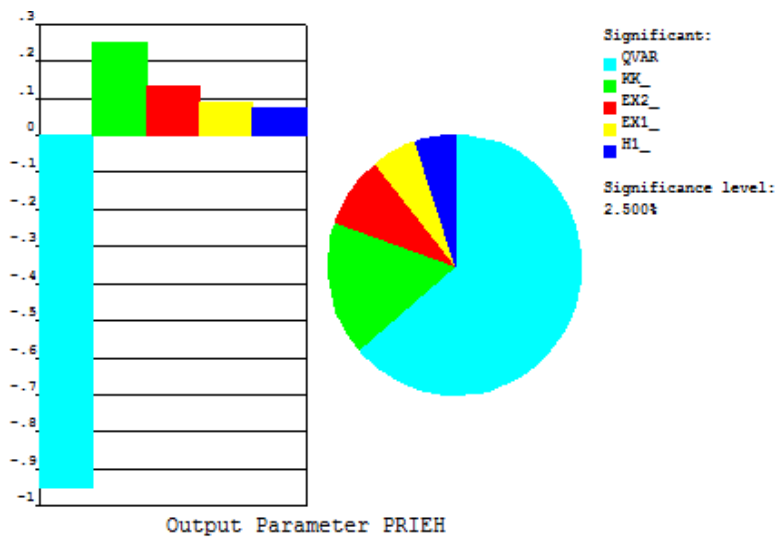


Fig. 14: Rank-order correlation sensitivities of output variable PRIEH - deflection

4. CONCLUSION

The aim of this analysis was to determine the probability of the failure of structure, and then to determine its reliability depending on the input parameters. In our case, there has been a failure ($3.47e-3$), if we have exceeded the limit deflection -0.006 m. The statistics of the random output parameters were computed using the ANSYS results and illustrate the properties of the output parameters using histogram plots, cumulative distribution curves, and/or history plots. The influence of random input variables on individual output parameter is known as the "sensitivity".

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