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# COMPARATIVE ANALYSIS OF STRESS IN HYPERELASTIC MOONEY-RIVLIN AND ZAHORSKI MATERIALS USING ADINA SOFTWARE

**Abstract**

The paper presents a comparative analysis of stress in hyperelastic Mooney-Rivlin and Zahorski materials. The comparison was performed in ADINA software with the example of a cube loaded with linear displacement of side walls in y and z directions with coefficient of strain $\lambda = 2$. The conclusions from the analysis were presented in the Conclusion section.

**Keywords**

Mooney-Rivlin, Zahorski, FEM, Adina, nonlinear hyperelastic material.

## 1 INTRODUCTION

Advances in measurement techniques have contributed to the development of the research studies on wave phenomena in material continuum, including the continuum modelled as a continuous compressible or incompressible hyperelastic material. Currently available numerical software also allows for a more comprehensive analysis of wave phenomena which are the consequence of dynamic effects, with division into simple and more complex elastic structures.

Analysis of behavior of non-linear hyperelastic materials can be supported with the use of numerical software which utilizes finite elements method (FEM). There are many publications describing the topic of finite element method use in civil engineering and mechanics, for example [1-3]. The most of numerical programs that are based on the FEM methodology feature libraries of selected models of materials, including hyperelastic material models. Choosing one of them allows for a detailed numerical analysis of non-linear behaviour of the components designed (see [4]). There are a variety of pieces of software that are based on commonly known elastic potentials. The most frequently used programs for computation of this type include: ANSYS, ABAQUS, MARC, NASTRAN, ALGOR, ADINA. Unfortunately, Zahorski material is not analysed in this group of programs. This study compares this incompressible hyperelastic material, using the author's solution for the ADINA software [5], with the commonly used Mooney-Rivlin material in the range of stress distribution.

## 2 HYPERELASTIC ZAHORSKI MATERIAL

The constitutive relationship for hyperelastic Zahorski material (see [6]) is described by the following formula

$$W(I_1, I_2) = C_1(I_1 - 3) + C_2(I_2 - 3) + C_3(I_1^2 - 9) \tag{1}$$

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where:

$W$ is elastic energy, $C_1, C_2, C_3$ – are material constants while $I_1, I_2$ – denote deformation tensor invariants.

As results from the condition above, elastic energy for the incompressible, isotropic and hyperelastic Zahorski material is non-linearly dependent on the deformation tensor invariants. Constitutive equations, also termed physical relationships, describe the behaviour of a medium affected by various external factors. Therefore, the choice of the material model depends on the factors which have the most essential importance to behaviour of the analysed medium and represent the subjective choice. The relationship (1) was used for analysis of wave phenomena concerning propagation of the disturbance in e.g. works [7], [8] and [9]. The constitutive equation proposed by Zahorski allows for a fuller analysis of wave phenomena that propagate in hyperelastic incompressible materials compared to Mooney-Rivlin material. The non-linear term in the equation $C_3(I_1^2 - 9)$ allows for a more precise analysis and obtaining individual qualitative components useful in the description of wave processes. If $C_3 = 0$, Mooney-Rivlin material is obtained, commonly used for modelling rubber and rubber-like materials [10]. It is noticeable that the constitutive Zahorski relation reflects behaviour of rubber for the principal strain, even for $\lambda = 3$, whereas for Mooney-Rivlin material, the satisfactory results are obtained for $\lambda = 1.4$ [11]. For strain greater than $\lambda = 2$, Mooney-Rivlin material should not be used.

The diagram (Fig. 1) for Zahorski material and Mooney – Rivlin material is generated from ADINA software having considered the author's modifications introduced into material libraries [5].

a) b)

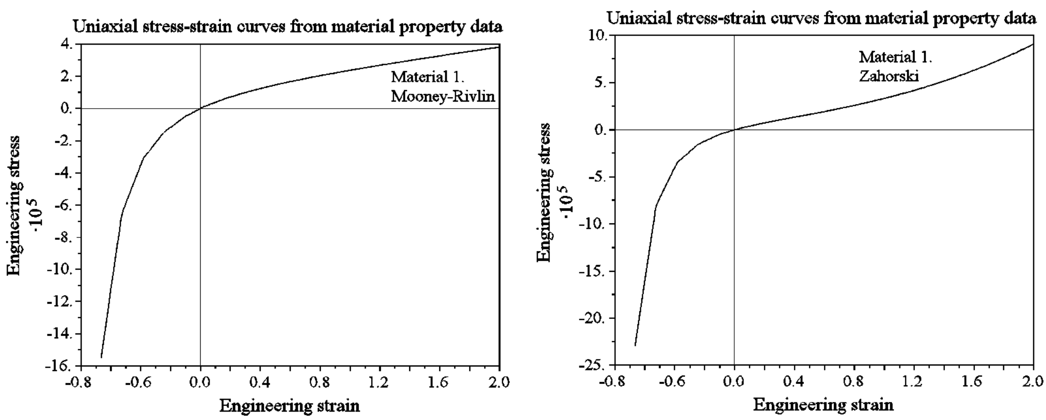


Fig. 1: Stress - strain diagram for analyzes rubber. a) Mooney - Rivlin material, b) Zahorski material

The Tab. 1 presents elastic constants for rubber considered in the work. The values presented in the table are based on the study [12].The values of the constants were obtained by computing per SI system.

Tab. 1: Constants $C_1$, $C_2$, $C_3$

| **Constants** | $C_1$ [Pa] | $C_2$ [Pa] | $C_3$ [Pa] |
|---|---|---|---|
| **Rubber "A" (OKA-1)** | $6.278 \cdot 10^4$ | $8.829 \cdot 10^3$ | $6.867 \cdot 10^3$ |

The Tab. 2 presents strength parameters for analyzes rubber OKA-1 (see [12]) and the Tab. 3 presents components of this rubber.

Tab. 2: Strength parameters for analyzes rubber [12]

| | |
|---|---|
| Time of vulcanization in minutes 3at. 143°C | 5 |
| Strength in kN/m$^2$ at 500 mm/min | ~$3.01 \cdot 10^4$ |
| Strength in kN/m$^2$ at 5 mm/min | ~$1.05 \cdot 10^4$ |
| Elongation in % at 5 mm/min | ~$5.81 \cdot 10^4$ |

Tab. 3: Components of analyzes rubber [12]

| Components in parts by weight | Rubber "A" (OKA-1) |
|---|---|
| Natural rubber, first grade | 10 |
| Stearin | 1 |
| Zinc oxide | 5 |
| Mercapto | 1 |
| Thiuram | 0.1 |
| Furnal black | - |
| Sulphur | 2 |

## 3 EXAMPLE OF CALCULATION

A cube with side length equal 10 cm was adopted in the study, with load represented by linear forced displacement of side walls along the directions of y and z. Coefficients of strain were adopted as $\lambda = 2$. This strain is obtained for the model analysed in the study during static analysis in time equal 10 using the automated time step (ATS).

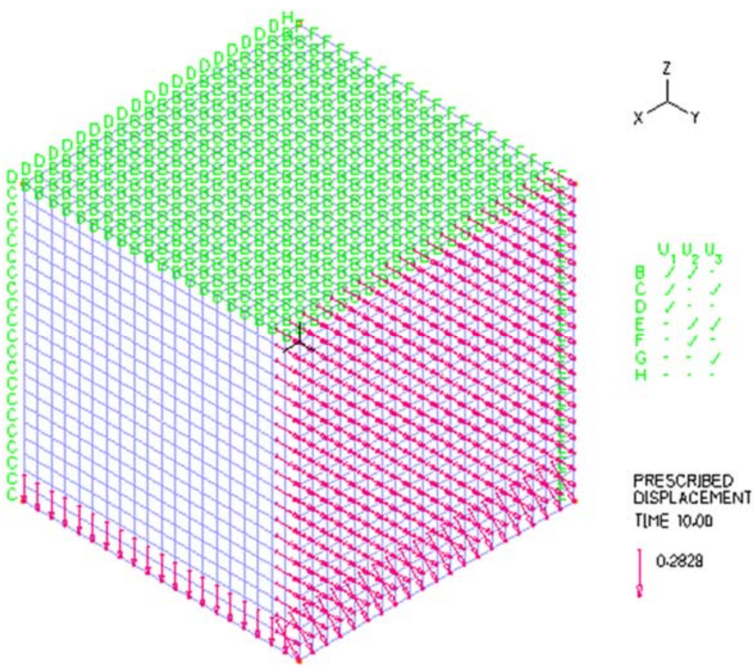


Fig. 2: Discrete model of the layer - ADINA program. $U_1$, $U_2$, $U_3$ - bonds

The constraints were added to the model adopted at the directions of x, y and z according to the Fig. 2. Discretization of the object analysed was obtained using 8-node components (cubes), adopting 20 linear components on each consecutive direction.

Comparison of the results obtained is presented in figs. 3 to 6. These comparisons offer opportunities for demonstrating the differences in the distribution of stress generated in Mooney-Rivlin and Zahorski materials. This represents the basis for the analysis of the results of numerical computations and, consequently, formulation of the conclusions contained in the Conclusion section.

Fig. 3 shows that maximum effective stress for Zahorski material was 5.764 MPa, whereas for Mooney-Rivlin material, maximum effective stress is 2.529 MPa. This means that, compared to Mooney-Rivlin material, the effective stress is by 2.28 times greater in Zahorski material.

Figs. 4 and 5 illustrate comparisons of distribution of XX and XY stresses, respectively, in the range from -3.6 kPa to 3.6 kPa, showing significant differences. Similar differences were presented in Figs. 6 and 7, with the distribution of XZ and YZ stresses presented respectively for the same range.

a) b)

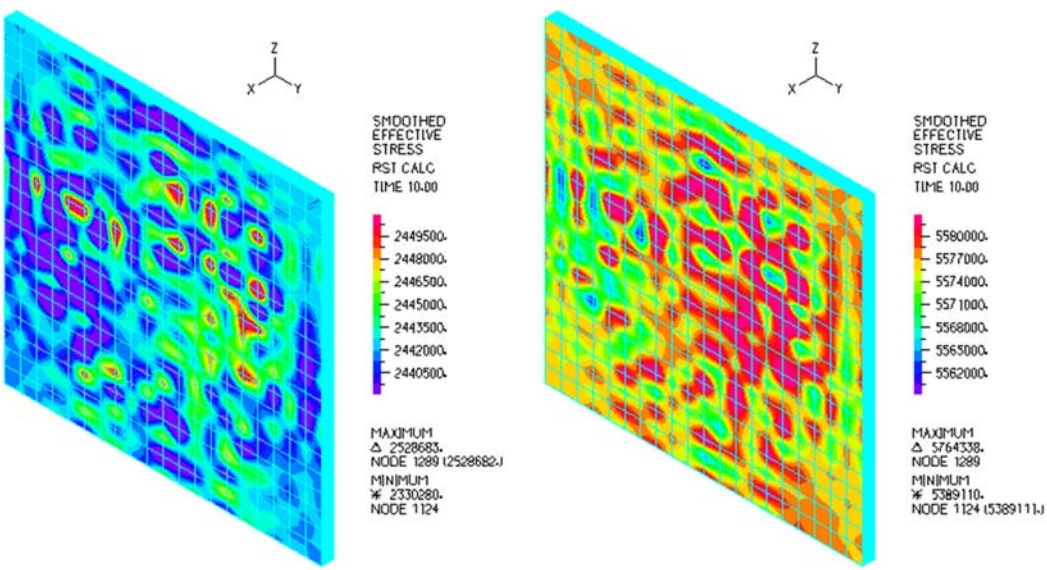


Fig.3: Comparison of the effective stress distribution [Pa] for the rubber discussed.
a) Mooney – Rivlin material, b) Zahorski material

a) b)

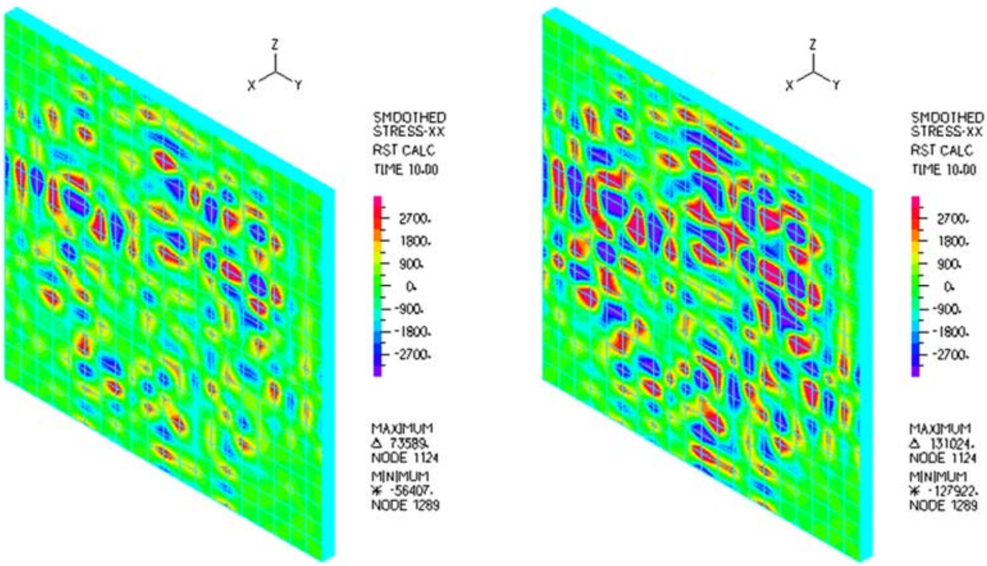


Fig. 4: Comparison of distribution of XX stress in the range from -3600 to 3600 [Pa] for the rubber analysed in the study. a) Mooney – Rivlin material, b) Zahorski material

a) b)

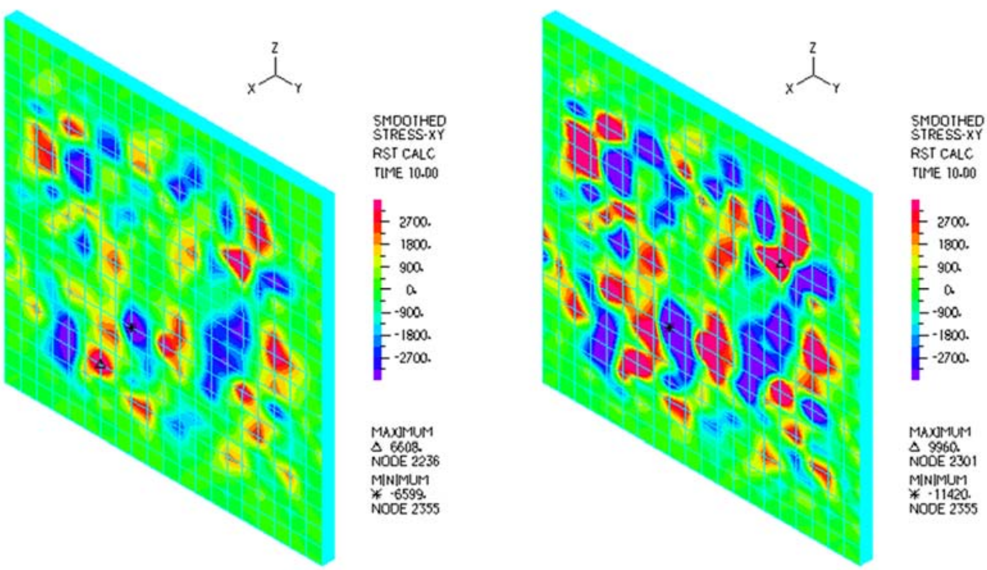


Fig. 5: Comparison of distribution of XY stress in the range from -3600 to 3600 [Pa] for the rubber analysed in the study. a) Mooney – Rivlin material, b) Zahorski material

a) b)

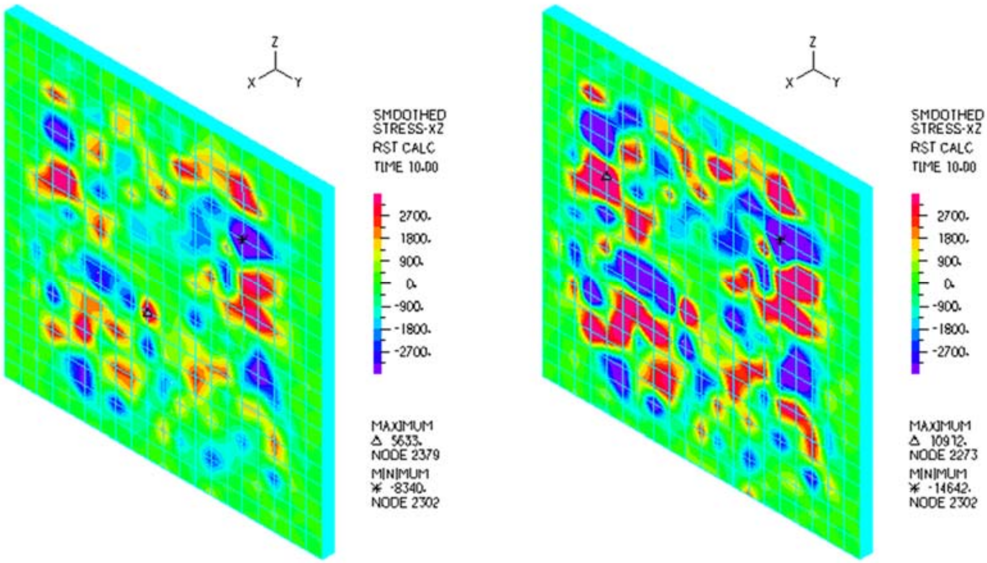


Fig. 6: Comparison of distribution of XZ stress in the range from -3600 to 3600 [Pa] for the rubber analysed in the study. a) Mooney – Rivlin material, b) Zahorski material

Distribution of YY and ZZ stresses in both incompressible hyperelastic materials used in the study were presented in Fig. 8 and 9. The results obtained correspond to the distribution of the effective stress presented in Fig. 3. Furthermore, the qualitative similarity and quantitative differences are noticeable in the distribution of the values obtained in Mooney-Rivlin and Zahorski materials.

a) b)

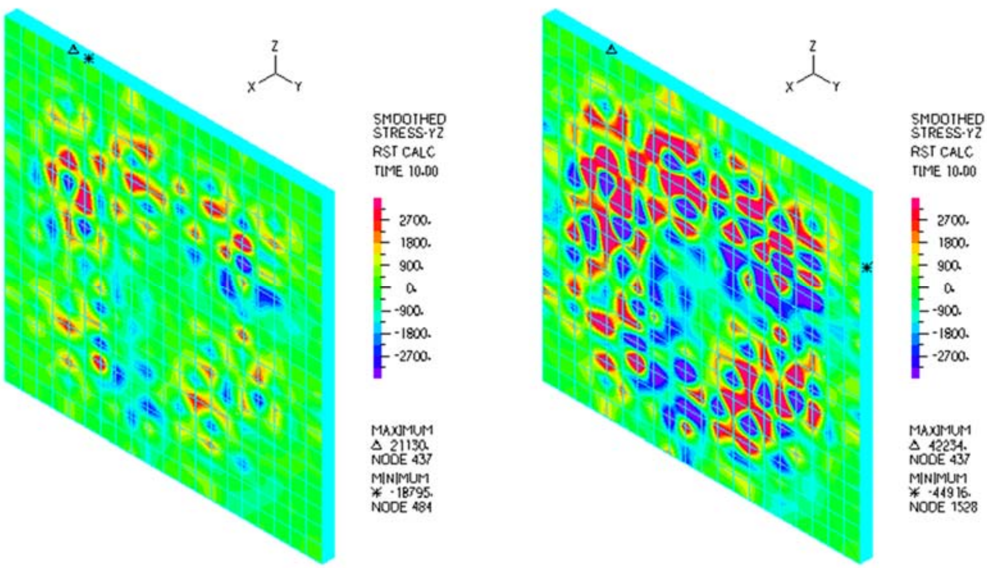


Fig. 7: Comparison of distribution of YZ stress in the range from -3600 to 3600 [Pa] for the rubber analysed in the study. a) Mooney – Rivlin material, b) Zahorski material

a) b)

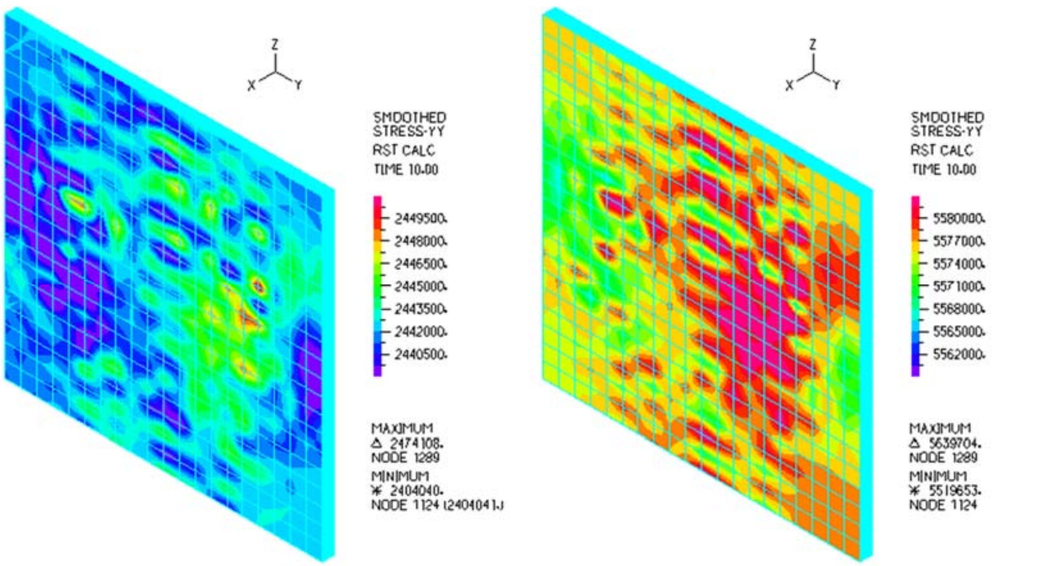


Fig. 8: Comparison of distribution of YY for the rubber analysed in the study.
a) Mooney – Rivlin material, b) Zahorski material

Maximal YY and ZZ stresses for Mooney-Rivlin material are 2.474 MPa and 2.470 MPa, respectively, whereas for Zahorski material, these value are 5.64 MPa and 5.633 MPa. This shows that, compared to Mooney-Rivlin material, maximal YY and ZZ stresses are (similarly to the effective stresses - see Fig. 3) by ~2.28 greater for Zahorski material.

a) b)

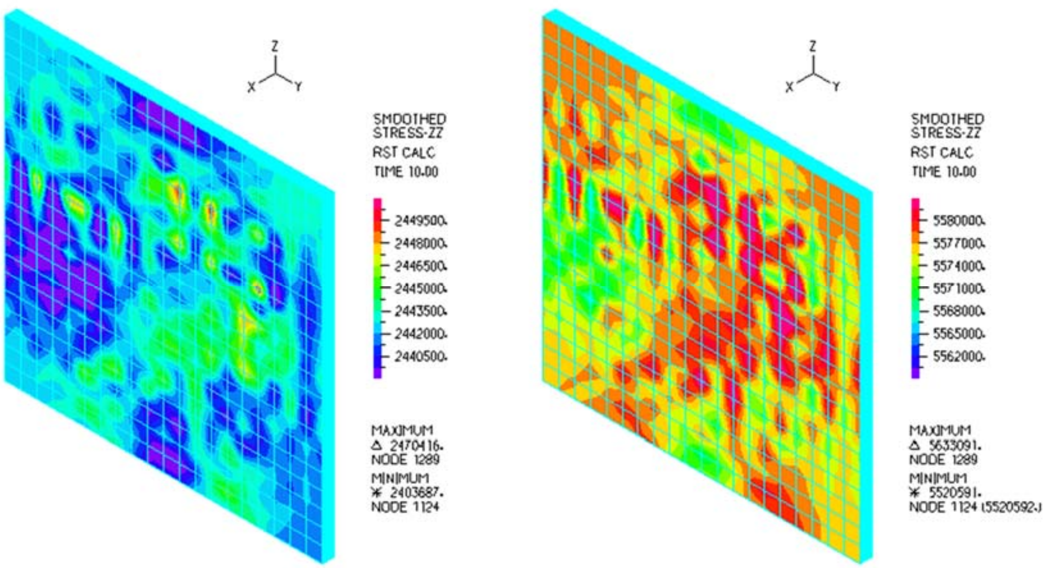


Fig. 9: Comparison of distribution of ZZ for the rubber analysed in the study.
a) Mooney – Rivlin material, b) Zahorski material

## 4 CONCLUSION

Detailed knowledge about behaviour of the rubber modelled as a hyperelastic incompressible Zahorski material i.e. material continuum described with hyperelastic Zahorski potential, is conducive to the development of technology and design of rubber products in the contemporary industry, where such products are a precondition for development of modern technical and technological solutions. With the example presented in this study, one can notice that adoption of Zahorski material for numerical modelling of rubber and rubber products allows for identification of detailed relationships between laboratory examinations and the process of extrusion, calandering and preparation of the mixtures.

This might substantially contribute to supplementation of the methods of evaluation of technological processes and determination of the parameters for development of new technologies. Rubber and rubber-like materials have been commonly used for reduction of the dynamic load that act on building structures. Therefore, the use of Zahorski material might contribute to evaluation of wave phenomena with new quality features and improve the efficiency of reduction of the load on structures as a consequence of disturbance propagation.

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