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**Eva KORMANÍKOVÁ<sup>1</sup>****GROUND SUPPORTED SHORT FIBER REINFORCED CONCRETE INDUSTRY PLATE****Abstract**

The present paper describes the elastic theory and Losberg Yield Line theory with plastic approach of solution. The finite element method for modeling of fiber reinforced concrete plates placed on Winkler foundation was used. There is presented Tsai and Pagano model within micromechanics of composite materials with short fibers. As the application problem it was used example of fiber-reinforced concrete industry plate. The strength obtained from elastic and plastic approach was compared and discussed in the conclusion of the paper.

**Keywords**

Short fiber composite, concrete, steel fibers, micromechanics, foundation plate.

**1 INTRODUCTION**

Several methods are used for analysis of interaction the concrete plate on soil-subgrade. The theory of Winkler formulation is one of them. The Winkler theory assumes that the deflection at every point of the plate on elastic soil is proportional to the pressure applied at that point and it is independent of the pressure acting at nearby point of the plate. Also the finite element method can be used for solution of the Winkler formulation of interaction foundation plate and soil-subgrade [1].

Fiber reinforced concrete (Fig. 1) can be defined as composite materials. Specifically short fiber-reinforced composites are one of the most widely used man-made composite materials; they are constituted by reinforcing fibers embedded in a matrix material (Fig. 2).

Modeling can play an important role in the analysis and design of fiber-reinforced composite materials. Their mechanical properties can be predicted early during the design stage using effective modeling techniques [2 - 4].

Many analytical techniques of homogenization are based on the equivalent eingstrain method.



Fig. 1: Fiber-reinforced concrete

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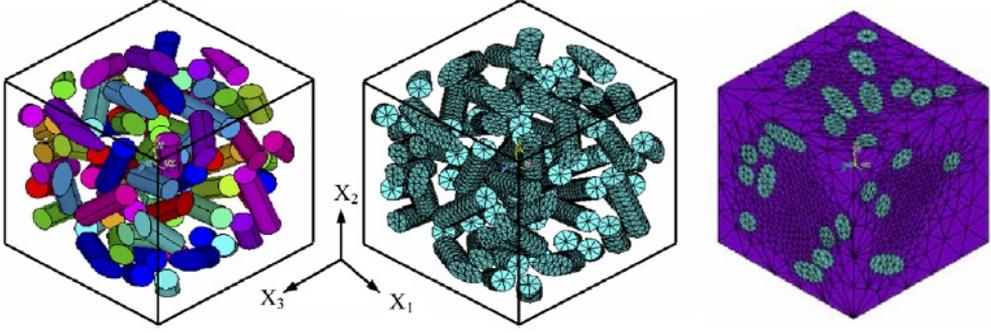


Fig. 2: RVE with randomly distributed fibers [3]

## 2 MICROMECHANICS OF COMPOSITE MATERIALS WITH SHORT FIBERS

### Halpin Tsai model

The classical laminate theory is the most commonly used theory for analysis of composites with randomly oriented short fibers. The laminates with the orientation of angles  $[0/\pm 45/90]$  and  $[0/\pm 60]$  are particularly very suitable for practical applications. In order to predict the strength of this type of composite, it is best to use the maximum strain criterion and then the strength of a composite with randomly-oriented short fibers can be determined by using the properties of unidirectionally reinforced composites with short fibers [5].

For determination of material characteristics of fiber-reinforced composite the Halpin Tsai equations were considered [6]

$$\begin{aligned}
 E_1 &= E^{(m)} \frac{1 + \frac{l}{d} \xi_E \eta_L \xi}{1 - \eta_L \xi}, & E_2 &= E^{(m)} \frac{1 + \xi_E \eta_T \xi}{1 - \eta_T \xi}, \\
 G_{12} &= G^{(m)} \frac{1 + \xi_E \eta_G \xi}{1 - \eta_G \xi}, & \nu_{12} &= \nu^{(m)} \frac{1 + \xi_E \eta_\nu \xi}{1 - \eta_\nu \xi},
 \end{aligned} \tag{1}$$

$$\begin{aligned}
 \eta_L &= \frac{\frac{E^{(f)}}{E^{(m)}} - 1}{\frac{E^{(f)}}{E^{(m)}} + \xi_E \frac{l}{d}}, & \eta_T &= \frac{\frac{E^{(f)}}{E^{(m)}} - 1}{\frac{E^{(f)}}{E^{(m)}} + \xi_E}, \\
 \eta_G &= \frac{\frac{G^{(f)}}{G^{(m)}} - 1}{\frac{G^{(f)}}{G^{(m)}} + \xi_E}, & \eta_\nu &= \frac{\frac{\nu^{(f)}}{\nu^{(m)}} - 1}{\frac{\nu^{(f)}}{\nu^{(m)}} + \xi_E}.
 \end{aligned} \tag{2}$$

where  $\xi$  is fiber volume fraction and parameter  $\xi_E$  is obtained from experiment. The parameter  $\xi_E = 2.5$  for circular cross-section area.

In order to evaluate the elastic matrix  $E$  of a composite, the simplified relations are defined by Tsai and Pagano as

$$\begin{aligned}
U_1 &= (3(E_{11})_L + 2(E_{12})_L + 3(E_{22})_L + 4(E_{66})_L) / 8, \\
U_2 &= ((E_{11})_L - (E_{12})_L) / 2, \\
U_3 &= ((E_{11})_L - 2(E_{12})_L + (E_{22})_L - 4(E_{66})_L) / 8, \\
U_4 &= ((E_{11})_L + 6(E_{12})_L + (E_{22})_L - 4(E_{66})_L) / 8, \\
U_5 &= ((E_{11})_L - 2(E_{12})_L + (E_{22})_L + 4(E_{66})_L) / 8.
\end{aligned} \tag{3}$$

where the subscript L means the local coordinate system.

The components of elastic matrix in global coordinate system can be calculated as

$$\begin{aligned}
E_{11} &= U_1 + U_2 \cos 2\theta + U_3 \cos 4\theta, & E_{22} &= U_1 - U_2 \cos 2\theta + U_3 \cos 4\theta, \\
E_{12} &= U_4 - U_3 \cos 4\theta, & E_{24} &= \frac{1}{2} U_2 \sin 2\theta - U_3 \sin 4\theta, \\
E_{14} &= \frac{1}{2} U_2 \sin 2\theta + U_3 \sin 4\theta. & E_{44} &= U_5 - U_3 \cos 4\theta.
\end{aligned} \tag{4}$$

### 3 STRENGTH OF COMPOSITE WITH SHORT FIBERS

The main characteristic essentially changed is the strength of a composite with short fibers. In the absence of more accurate information, the average and characteristic value of an equivalent flexural tensile strength for steel wire fibres can be calculated as follows [7]

$$R_{e_{m,150}} = \frac{180W_f \lambda_f d_f^{1/3}}{180C + W_f \lambda_f d_f^{1/3}}, \tag{5}$$

with  $C=20$  for hooked-end steel fibres under the trade name of Dramix, where  $W_f$  is the fibre content (in  $\text{kg}/\text{m}^3$ ),  $d_f$  is the diameter of steel fibres, and  $\lambda_f$  is the ratio between the length and the diameter of steel fibres. The flexural tensile strength  $f_{fctm,eq}$  and characteristic flexural tensile strength  $f_{fctk,eq}$  are given as

$$f_{fctm,eq} = \frac{R_{e_{m,150}} f_{fctm,fl}}{100}, \tag{6}$$

$$f_{fctk,eq} = 0.7 f_{fctm,eq}. \tag{7}$$

### 4 EXAMPLE OF APPLICATION

Let us have a foundation plate on the Winkler formulation, dimension of 24 m x 36 m and thickness of 0.2 m. Soil modulus is given  $K = 1 \cdot 10^5 \text{ kNm}^{-3}$ . Distances of dilatations are made of 6 x 6 m. The concrete foundation plate on soil-subgrade is solved by application of Finite Element Method. The loads on the plate were made according with STN EN 1991-1-1. The fiber-reinforced concrete consists of C 25/30 and Dramix RC-65/60-BN fibers.

Loads: dead weight of the plate, weight of the shelves with maximum reaction of one strut 100 kN at the place 0.2 m x 0.2 m. Dimensions of one shelf are 1.1 m x 2.8 m, distance between shelves is 0.3 m. It was taken into account vehicle FL6 and uniform distributed load caused by weight of materials of  $100 \text{ kN}/\text{m}^2$  at the place 5 m x 15 m.

The maximum load combination is location of vehicle between edge of the plate and set of shelves. The ultimate limit state was use for design of short fiber reinforced concrete plate.

The Fig. 3 shows distribution of loads on the industry plate. In the Fig. 4 can be seen the yield line model used in design of short fiber reinforced concrete ground plate.

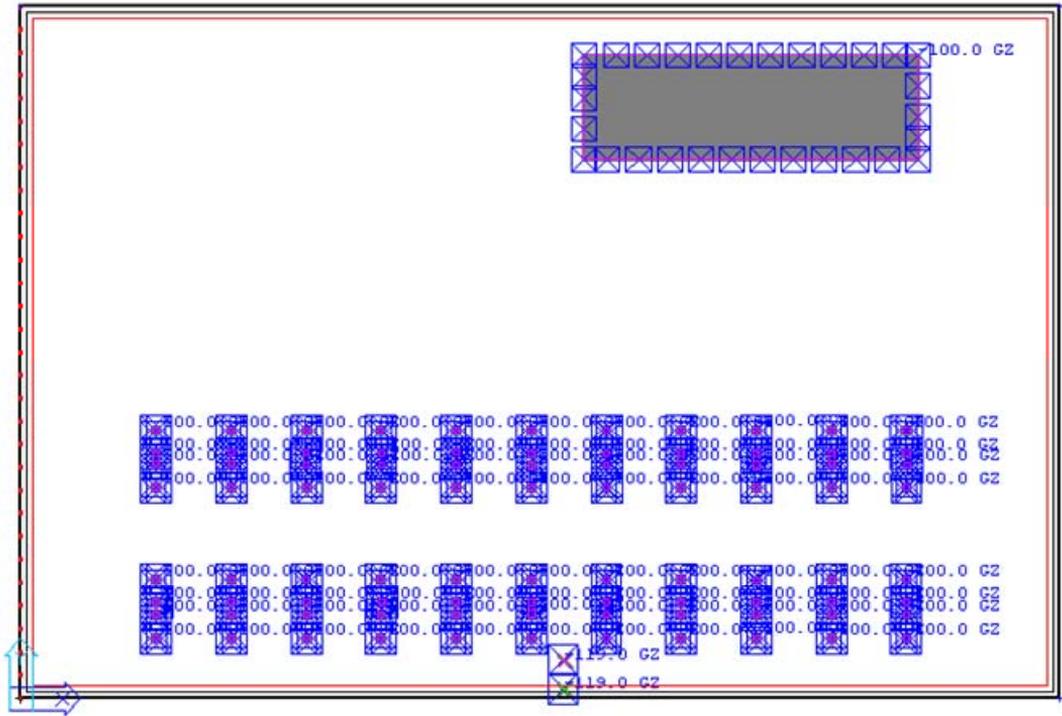


Fig. 3: Loads of the ground plate

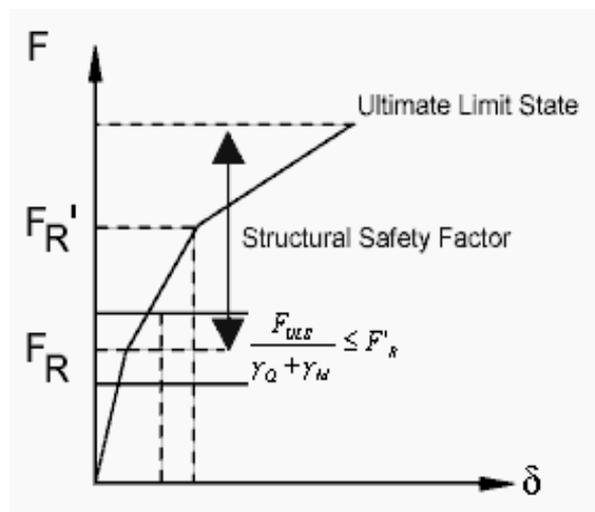


Fig. 4: Yield line model [7]

Tab. 1: Normal stresses of the ground plate

$\sigma_{\max}$ [MPa]	$\sigma_{\max}$ [MPa]
Linear analysis, Winkler	Losberg Yield Line model
4.033	3.260

## 5 CONCLUSIONS

The parameters controlling the design of ground-supported slabs at the ultimate limit state are not as clear-cut as for general reinforced concrete design, where ultimate refers to the strength of the structure and serviceability to the limitation of crack widths, deflections etc. For ground-supported slabs, two ultimate strength modes of failure of the concrete slab are possible, namely flexure (bending) and local punching. Slab design for flexure at the ultimate limit state is based on yield line theory, which requires adequate ductility to assume plastic behaviour. At the ultimate limit state, the bending moment along the sagging yield lines may be assumed to be the full plastic value. However, a principal requirement in the design of ground-supported slabs is the avoidance of cracks on the upper surface.

There were compared the results obtained from linear FEM analysis and Losberg Yield Line model in the paper [8]. For given industry plate (see example of application) it was designed the amount of fibers 39 kg/m<sup>3</sup> by Losberg Yield Line model. From linear analysis (39 kg/m<sup>3</sup>) the maximum normal stress is 4.033 MPa. The value 3.26 MPa was obtained with the help of Losberg Yield Line model (Tab. 1). The smaller value of stress was obtained by Losberg Yield Line model with plastic approach of solution. The linear solution causes the overdesigned and uneconomical design of industrial plates that is not suitable for design of industrial floor.

The design of ground supported industrial plate according the Losberg Yield Line model was solved by collaboration with Company Bekaert.

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