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OPTIMIZATION OF SINGLE-LAYER BRACED DOMES

Abstract

The paper deals with discussion of optimization problem in civil engineering structural space design. Minimization of mass should satisfy the limit state capacity and serviceability conditions. The cross-sectional areas of bars and structural dimensions are taken as design variables. Variables are used in the form of continuous and discrete. The analysis is done using the Structural and Design of Experiments modules of Ansys Workbench v17.2. As result of the method a mass reduction of 46,6 % is achieved.

Keywords

Structural optimization, space truss, braced dome, single-layer.

1 INTRODUCTION

In this paper, a Goal Driven Optimization [1] is proposed to evolve the configuration design of domes aiming at the mass minimization of the structure. Discrete and continuous design variables are considered corresponding to the sizing of the cross-sectional areas of the bars, joint coordinates of the dome.

Braced domes formed of a single-layer network of slender members are widely accepted as effective lightweight solutions to large span applications. Earlier structures such as the 213 m span New Orleans Superdome and the 200 m span Texas Astrodome only weigh 0.24 and 0.22 kN/m2, respectively [3]. Hundreds of successful braced dome applications now exist all over the world covering public halls, places of worship and many other buildings [4].

It is possible to consider the structure as a three-dimensional truss and the bars presenting joints non-rigidly interconnected. The normal and buckling stresses arising from the axial forces in the bars, and the displacements at the nodes, are the values that affect the sizes of the members and the final cost of the structure. For such structures it is interesting to carry out a non-linear analysis to obtain the axial forces and displacements. Moreover, to best explore the advantages of the structural behavior, joints must be considered as rigidly interconnected and the structure modeled as a space frame. Usually it leads to a lighter structure. In this case, the interaction between axial forces and bending moments in members with a high slenderness coefficient requires a non-linear analysis in order to check the stability of the structure [5, 6].

2 BRACED DOMES

Domes are one of the oldest and well-established structural forms and have been used in architecture since the earliest times. They are of special interest to engineers as they enclose a maximum amount of space with a minimum surface and have proved to be very economical in terms of consumption of constructional materials. The stresses in a dome are generally membrane and

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compressive in the most part of the shell except circumferential tensile stresses near the edge and small bending moments at the junction of the shell and the ring beam. Most domes are surfaces of revolution. The curves used to form the synclastic shell are spherical, parabolic, or elliptical covering circular or polygonal areas. Out of a large variety of possible types of braced domes, only four or five types proved to be frequently used in practice. The rise of a braced dome can be as flat as 1/6 of the diameter or as high as 3/4 of the diameter which will constitute a greater part of a sphere. For diameter of braced domes larger than 60 m, double-layer grids are recommended. The ratio of the depth to the diameter is in the range of 1/30 to 1/50. For long spans, the depth can be taken as small as 1/100 of diameter.

3 GOAL DRIVEN OPTIMIZATION

In the last decade, structural optimization has become one of the most interesting branches of structural engineering and many meta-heuristic algorithms have been developed and applied for optimization of truss structures.

Goal Driven Optimization (GDO) is a set of constrained, multi-objective optimization techniques in which the "best" possible designs are obtained from a sample set given the objectives you set for parameters. The available optimization methods are: Screening, MOGA, NLPQL, MISQP, Adaptive Single-Objective, Adaptive Multiple-Objective.

The Screening, MISQP, and MOGA optimization methods can be used with discrete parameters. The Screening, MISQP, MOGA, Adaptive Multiple-Objective, and Adaptive Single-Objective optimization methods can be used with continuous parameters with manufacturable values [1].

The GDO process allows you to determine the effect on input parameters with certain objectives applied for the output parameters. For example, in a structural engineering design problem, you may want to determine combination of design parameters best satisfy minimum mass, maximum natural frequency, maximum buckling and shear strengths, and minimum cost, with maximum value constraints on the von Mises stress and maximum displacement.

The Shifted Hammersley optimization method (Screening) is the sampling strategy used for all sample generation. The conventional Hammersley sampling algorithm is a quasi-random number generator which has very low discrepancy and is used for quasi-Monte Carlo simulations. A low-discrepancy sequence is defined as a sequence of points that approximate the equidistribution in a multi-dimensional cube in an optimal way. In other words, the design space is populated almost uniformly by these sequences and, due to the inherent properties of Monte Carlo sampling, dimensionality is not a problem (i.e., the number of points does not increase exponentially with an increase in the number of input parameters).

4 OPTIMAL DESIGN OF STRUCTURES

Minimizing the structural mass W requires the selection of the optimum values of number cross-section D_i while satisfying the design constraints. The discrete optimal design problem of truss structure [2] may be expressed as

find: $$X = [x_1, x_2, \dots, x_{ng}] \tag{1}$$

to minimize: $$W(X) = \sum_{i=1}^{nm} \gamma_i \, x_i L_i \tag{2}$$

subject to:

$$x_i \in D_i \qquad D_i = \{d_{i,1}, d_{i,2}, \dots, d_{i,r}\} \tag{3}$$

$$\delta_{min} \le \delta_i \le \delta_{max} \qquad i = 1,2, \dots, nn \tag{4}$$

$$\sigma_{min} \le \sigma_i \le \sigma_{max} \qquad i = 1,2, \dots, nm \tag{5}$$

$$0 \le \sigma_i \le \sigma_i^b \qquad i = 1,2, \dots, nc \tag{6}$$

where X is a vector containing the design variables; D_i is an allowable set of discrete values for the design variable x_i; ng is the number of design variables or the number of member groups; r is the number of available discrete values for the i-th design variable; $W(X)$ is the cost function which is taken as the weight of the structure; nn is the number of nodes; nm is the number of members forming the structure, nc is the numbers of compression elements, γ_i is the material density; L_i is the length of the member i; σ_i and δ_i are the stress and nodal displacement, respectively; min and max mean the lower and upper bounds of constraints, respectively, σ_i^b is the allowable buckling stress in member i when it is compression.

5 NUMERICAL EXAMPLE

In this paper, in order to demonstrate the proposed solution method a braced dome structure is presented as a simple sizing problem with discrete design variables and second solution as a shape optimization with continuous design variables.

The dome is subjected to a downward vertical equipment loading of 60 kN at in crown (node 33) and simply supported at nodes 1-8. The geometry and nodal coordinates are presented in Figure 1 and in Table 1. Properties of applied material shown in Table 2. According to the structural symmetry, truss members are grouped into seven member-groups (see in Table 3), element is between two nodes. Initial start with seven groups but after few optimizations to finish only two (A1, A2) and four groups (A1, A2, A3 and A4).

Tab. 1: The geometry of the 80-bars truss dome

Nodes	X [m]	Y [m]	Z [m]
1	16.000	0.000	0.000
2	11.310	11.310	0.000
9	12.500	0.000	3.000
10	10.670	4.420	3.000
11	8.840	8.840	3.000
25	7.000	0.000	5.000
26	4.950	4.950	5.000
33	0.000	0.000	7.000

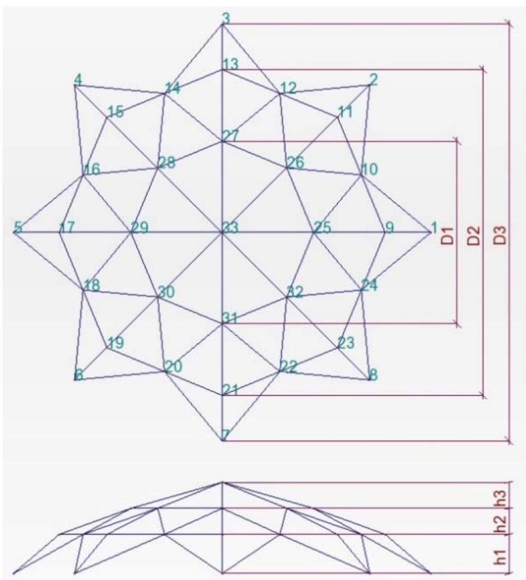

Fig.1: The layout of the 80-bars shallow truss dome

Tab. 2: Properties of the applied material

Modulus of elasticity	***E* = 210 000 MPa**
Material density	*ro* = 7850 kg/m^3
Stress constraints for tension	*sigma_t* = 100 MPa
Stress constraints for compression	*sigma_c* = 50 MPa

Tab. 3: Groups of dome elements

Groups	nodes	nodes	nodes	nodes	nodes	nodes	nodes	nodes
M1	1-9	2-11	3-13	4-15	5-17	6-19	7-21	8-23
M2	9-25	11-26	13-27	15-28	17-29	19-30	21-31	23-32
M3	25-33	26-33	27-33	28-33	29-33	30-33	31-33	32-33
M4	25-26	26-27	27-28	28-29	29-30	30-31	31-32	32-33
M5	1-10	2-10	2-12	3-12	3-14	4-14	4-16	5-16
	5-18	6-18	6-20	7-20	7-22	8-22	8-24	1-24
M6	9-10	10-11	11-12	12-13	13-14	14-15	15-16	16-17
	17-18	18-19	19-20	20-21	21-22	22-23	23-24	24-25
M7	10-25	10-26	12-26	12-27	14-27	14-28	16-28	16-29
	18-29	18-30	20-30	20-31	22-31	22-32	24-32	24-25

5.1 Sizing optimization

The algorithm searches for minimal mass of the dome changing the areas of the cross-section member (see Figure 2). The cross-sectional areas of truss bars for 2 group containing 12 sizes (see in Table 4) and for 4 group containing 13 sizes (in Table 5). The dome has not changing structure dimension: span $D3 = 32\ m$ length (inner rings $D2 = 25\ m$, $D1 = 16\ m$) and high $h = 7\ m$ ($h1 = 3\ m$, $h2 = 2\ m$ and $h3 = 2\ m$).

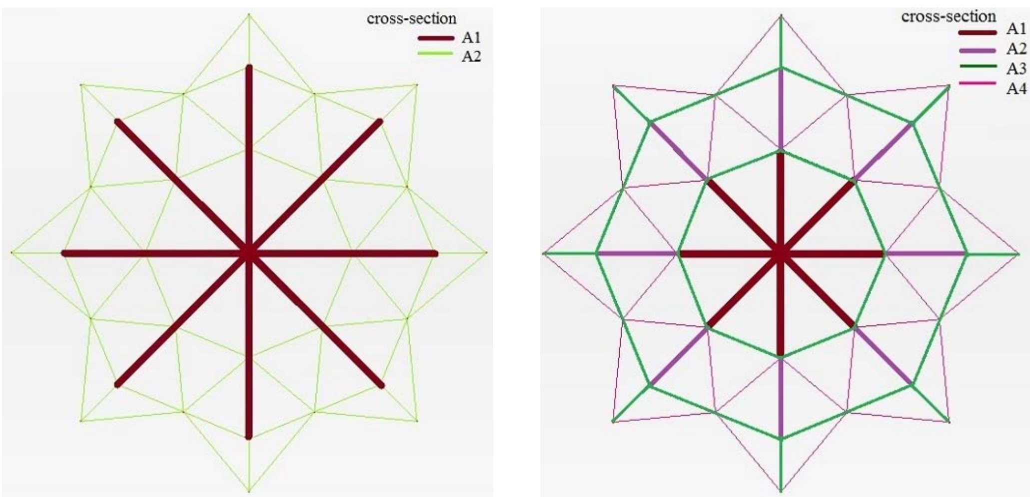

Fig.2: Configuration bars for (left) 2 groups and (right) 4 groups

Tab. 4: Cross-section of the discrete optimization for 2 groups

Groups	Initial		The	best	The	worst
	D [mm]	t [mm]	D [mm]	t [mm]	D [mm]	t [mm]
A1	90.0, 100.0, 110.0	4.0, 5.0	90.0	4.0	110.0	5.0
A2	60.0, 70.0, 80.0	3.0, 4.0	60.0	3.0	80.0	4.0
			Mass [kg]	**2507.2**	Mass [kg]	4232.9

Tab. 5: Cross-section of the discrete optimization for 4 groups

Groups	Initial		The	best	The	worst
	D [mm]	t [mm]	D [mm]	t [mm]	D [mm]	t [mm]
A1	90.0, 100.0	4.0, 5.0	90.0	4.0	100.0	5.0
A2	80.0, 90.0	4.0, 5.0	80.0	4.0	90.0	5.0
A3	60.0, 70.0	3.0, 4.0	60.0	3.0	70.0	4.0
A4	20.0	2.0	20.0	2.0	20.0	2.0
			Mass [kg]	**1710.6**	Mass [kg]	2400.3

5.2 Shaping optimization

The algorithm searches for minimal mass of the dome changing the structure dimension. Result for the structure dimension for 2 group (see in Table 6) and for 4 group (in Table 7). The dome has not changing the cross-section (diameter and thickness): A1: $90x4$, A2: $80x4$, A3: $60x3$, A4: $20x2$.

Tab. 6: Structure dimension of the continuous optimization for 2 groups cross-section

Groups	Range	The best	The worst
D1 [m]	12.0 – 16.0	14.900	12.756
D2 [m]	23.0 – 27.0	23.316	26.296
h1 [m]	2.0 – 4.0	2.370	3.910
h2 [m]	1.0 – 3.0	1.572	2.963
	Mass [kg]	**2398.2**	2664.9

Tab. 7: Structure dimension of the continuous optimization for 4 groups cross-section

Groups	Range	The best	The worst
D1 [m]	12.0 – 16.0	12.212	15.636
D2 [m]	23.0 – 27.0	23.514	26.592
h1 [m]	2.0 – 4.0	2.610	3.790
h2 [m]	1.0 – 3.0	1.947	2.213
	Mass [kg]	**1640.9**	1803.9

6 CONCLUSIONS

In this paper a Goal Driven Optimization algorithm was used to solve the shape, and sizing design for mass minimization of a dome structure. The algorithm searches for the areas of the cross-sectional members (discrete), and the node coordinates (continuous). Result are shown in Table 8. More sensitivity to mass structure is change the areas of the cross-section then the node coordinates.

Tab. 8: Results of optimization mass the dome

Bar members	Sizing optimization	Shaping optimizatio	Reduction mass
2 groups	2507.2 kg	2398.2 kg	4.5 %
4 groups	1710.6 kg	1640.9 kg	4.2 %
	46.6 %	46.1 %	

REFERENCES

[1] *ANSYS Workbench User's Guide*. 2009. SAP IP, Inc. 120 p.

[2] GRZYWIŃSKI, M. Optimization of Double-Layer Braced Barrel Vaults, *Transaction of the VŠB – Technical University of Ostrava, Civil Engineering Series*, 2015, vol. 15, issue 2, paper #6, (6 p). ISSN 1804-4824.

[3] MAKOWSKI, Z. S. *Analysis, Design and Construction of Braced Domes.* 1984. Nichols Pub. Co. ISBN 0-89397-191-X. 701 p.

[4] NARAYANAN, S. *Space Structures: Principles and Practice*, 2006. Multi-Science Publishing Co. Ltd. ISBN 0-906522-42-0. 844 p.

[5] SAKA, M.P. and M. ULKER. Optimum design of geometrically non-linear space trusses. *Computers and Structures*, 1991, vol. 41, pp. 1387-1396 (10 p). ISSN 0045-7949.

[6] SAKA, M.P. and E. S. KAMESHKI. Optimum design of nonlinear elastic framed domes. *Advances in Engineering Software*, 1997, vol. 29, issue 7–9, pp. 519-528 (10 p). ISSN 0965-9978.