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CALCULATION OF CHARACTERISTICS DESCRIBING THE PROPERTIES OF DYNAMICAL SYSTEMS

Abstract

There are characteristics that uniquely define the properties of dynamical systems from the point of its dynamical response. For example, natural frequencies and natural modes or frequency response functions can be assigned to these characteristics. Determination of these characteristics is fixed on the selection of computational model and on the means of structure excitation. This contribution discusses about analysis of such characteristics.

Keywords

Dynamic characteristics, natural frequencies and natural modes, frequency response functions, Fourier transform.

1 INTRODUCTION

Dynamical analysis of the structures depends on the choice of computational model and on the means of structure excitation. In practice it is often used the discrete computational model, because the motion equations for such model have character of ordinary differential equations. Computational model can be chosen on the basis of classical dynamics or on the basis of finite element method. Structure excitation can be force or kinematic. When choosing discrete computational model and the force excitation with variable frequency content, it is useful to use frequency response functions as characteristics describing properties of dynamical system. The presented article is devoted to the analysis of such characteristics.

2 FOURIER TRANSFORM AND FREQUENCY RESPONSE FUNCTION

For the transmission from time to frequency domain the Fourier transform can be used [1]. Fourier image of a time function $v(t)$ we denote by $V(q)$, where q is real number, in our considered case $q = \omega$, where ω is angular frequency in [rad/s]. The complex Fourier transform is defined as

$$V(q) = \int_{-\infty}^{+\infty} v(t) \cdot \mathrm{e}^{-\mathrm{i} \cdot q \cdot t} \cdot \mathrm{d}t \,. \tag{1}$$

The function $v(t)$ and its derivatives will be transformed as following

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$$k \cdot v(t) \quad \rightarrow \quad k \cdot V(q)\,,$$
$$\dot{v}(t) \ for \ v(\pm\infty) = 0 \quad \rightarrow \quad \mathrm{i} \cdot q \cdot V(q)\,,$$
$$\ddot{v}(t) \ for \ v(\pm\infty) = \dot{v}(\pm\infty) = 0 \quad \rightarrow \quad -q^2 \cdot V(q)\,. \tag{2}$$

Frequency response of a linear system (frequency response function $H(p)$ for $p = \mathrm{i}\cdot\omega$) is defined as the ratio of steady state response and harmonic excitation [2]

$$H(\mathrm{i}\cdot\omega) = r_{sst} / F \cdot \mathrm{e}^{\mathrm{i}\cdot\omega\cdot t}\,. \tag{3}$$

If the input value (e.g., excitation force) is periodical with unit amplitude

$$F(t) = F \cdot f(t) = 1 \cdot \mathrm{e}^{\mathrm{i}\cdot\omega\cdot t}\,, \tag{4}$$

it is possible to write the output value (e.g., deflection) as

$$v(t) = H(\mathrm{i}\cdot\omega) \cdot \mathrm{e}^{\mathrm{i}\cdot\omega\cdot t}\,. \tag{5}$$

Graphic representation of a frequency response is said to be frequency characteristic. Graphic representation of a dependence of absolute value (module) of frequency response function on the frequency of harmonic excitation is said to be amplitude characteristic. The phase characteristic is a graphic representation of a dependence of response function argument (phase) on the frequency of harmonic excitation. The frequency response $H(\mathrm{i}\cdot\omega)$ is a complex function and it can be calculated as a vector sum of real $\mathrm{Re}[H(\mathrm{i}\cdot\omega)]$ and imaginary $\mathrm{Im}[H(\mathrm{i}\cdot\omega)]$ parts.

3 FREQUENCY RESPONSE FUNCTIONS UPON FORCE EXCITATION

Let us assume discrete computational model with n degrees of freedom (Fig. 1) excited by discrete forces at points of lumped masses. The motion equation describing the forced un-damped oscillations of system can be written in the form

$$[m]_D \cdot \{\ddot{v}(t)\} + [k] \cdot \{v(t)\} = \{F(t)\} \tag{6}$$

where $[m]_D$ is diagonal mass matrix, $[k]$ is stiffness matrix, $\{v(t)\}$ is vector of unknown deflections of mass points and $\{F(t)\}$ is vector of exciting forces. Derivatives with respect to time are denoted by dot over the dependent variable symbol [3].

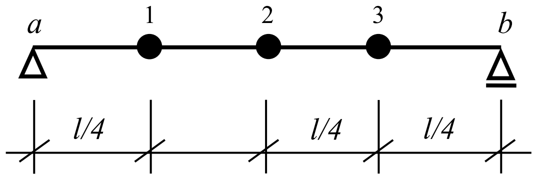

Fig.1: Discrete computational model of the structure

Let us apply the Fourier transform to the equation (6). Fourier images of functions $\{v(t)\}$ and $\{F(t)\}$ are denoted $\{V(q)\}$ and $\{F(q)\}$, respectively, where $q = \omega$. Equation (6) is then transformed to

$$-q^2 \cdot [m]_D \cdot \{V(q)\} + [k] \cdot \{V(q)\} = \{F(q)\}. \tag{7}$$

Let us suppose that only k-th function of vector $\{F(q)\}$ is nonzero and all others are equal zero. Now it is possible to define n^2 frequency responses for $i = 1 \div n$ and $k = 1 \div n$. For the frequency response $\bar{v}_{i,k} \equiv \bar{v}_{i,k}(q)$ it is fulfilled

$$\bar{v}_{i,k} \equiv \bar{v}_{i,k}(q) = \frac{V_i(q)}{F_k(q)}\,. \tag{8}$$

This way it is possible to obtain n systems of equations for k = 1 ÷ n, for the calculation of n frequency responses $\bar{v}_{i,k}$ at each step of the solution, for i = 1 ÷ n

$$-\omega^2\cdot[m]_D\cdot\{\bar{v}\}+[k]\cdot\{\bar{v})\}=\{F_k\},$$
$$([k]-\omega^2\cdot[m]_D)\cdot\{\bar{v})\}=\{F_k\}. \tag{9}$$

Vector $\{F_k\}$ for the k-th system of equations is zero except the k-th row, where it is number one.

4 NUMERICAL SOLUTIONS

Numerical calculations were applied to discrete computational model of real bridge structure with one span, made from bridge prefabricated elements I73 with the following parameters: span l = 29,0 m, elastic modulus E = 3,85·10^{10} N/m^2, quadratic moment of the cross-section I = 2,391711 m^4, intensity of mass μ = 19 680 kg/m. Masses of discrete model with 3 degrees of freedom are following $m_1 = m_2 = m_3 = \mu\cdot l/4 = 19680\cdot 29/4 = 142680$ kg.

Stiffness matrix elements are

$k_{11} = k_{33} = 2{,}38180641961540\cdot 10^9$ N/m,

$k_{22} = 3{,}31381762729099\cdot 10^9$ N/m,

$k_{12} = k_{21} = k_{23} = k_{32} = -2.27824961876256\cdot 10^9$ N/m, $k_{13} = k_{31} = 9{,}32011207675590\cdot 10^9$ N/m.

Natural frequencies of the model are

$f_{(1)} = 4{,}0389$ Hz , $f_{(2)} = 16{,}0432$ Hz , $f_{(1)} = 34{,}0633$ Hz .

Frequency response functions computations were realized in the frequency range 0 ÷ 40 Hz with step 0,1 Hz and there are displayed in the form of amplitude and phase characteristics. For the excitation in point k = 1 there are displayed in figures 2, 3, 4, for the excitation in point k = 2 there are displayed in figures 5, 6, 7 and for the excitation in point k = 3 there are displayed in figures 8, 9, 10.

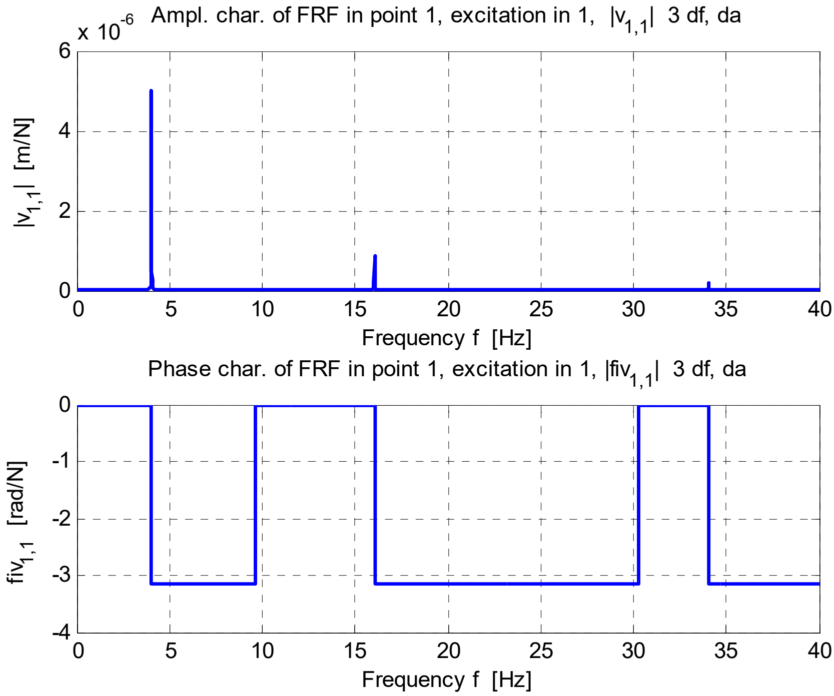

Fig.2: Amplitude and phase characteristics of FRF in point 1, excitation in 1

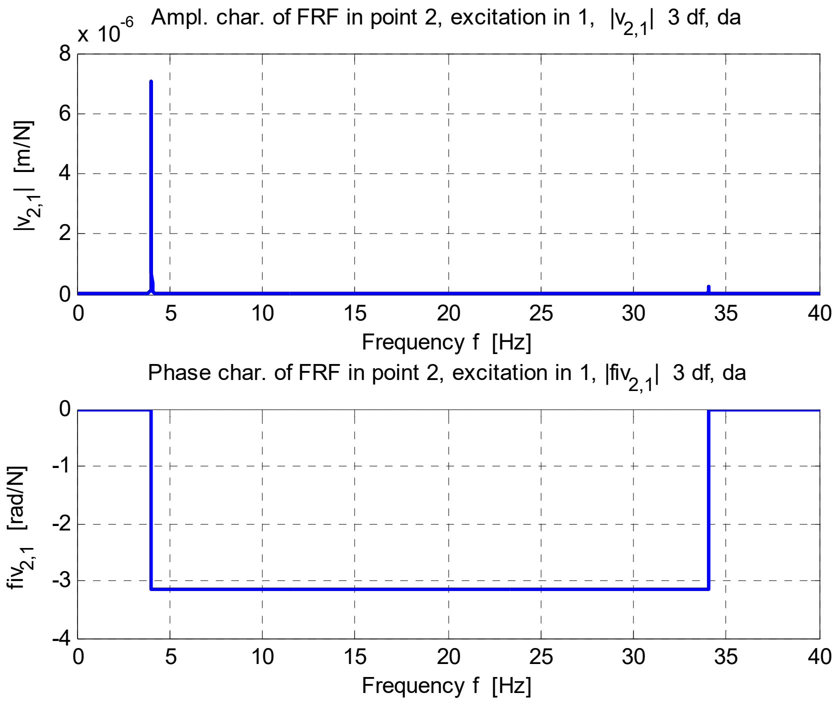

Fig. 3: Amplitude and phase characteristics of FRF in point 2, excitation in 1

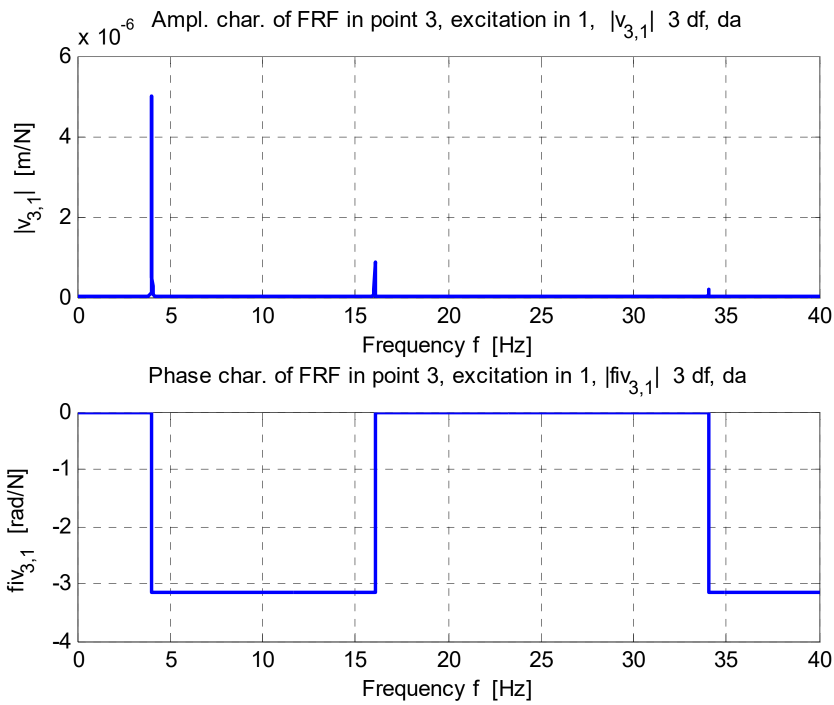

Fig. 4: Amplitude and phase characteristics of FRF in point 3, excitation in 1

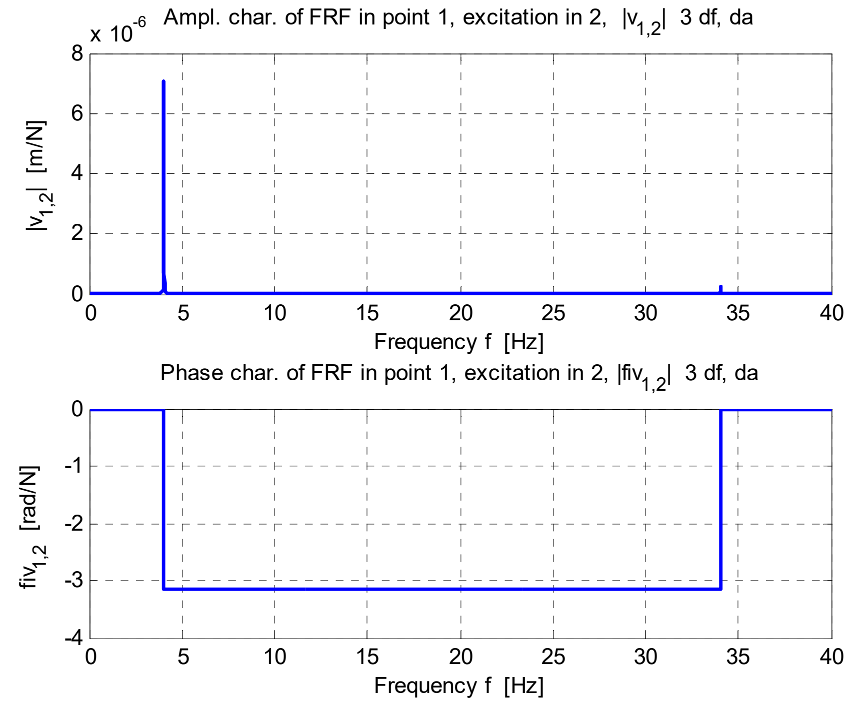

Fig. 5: Amplitude and phase characteristics of FRF in point 1, excitation in 2

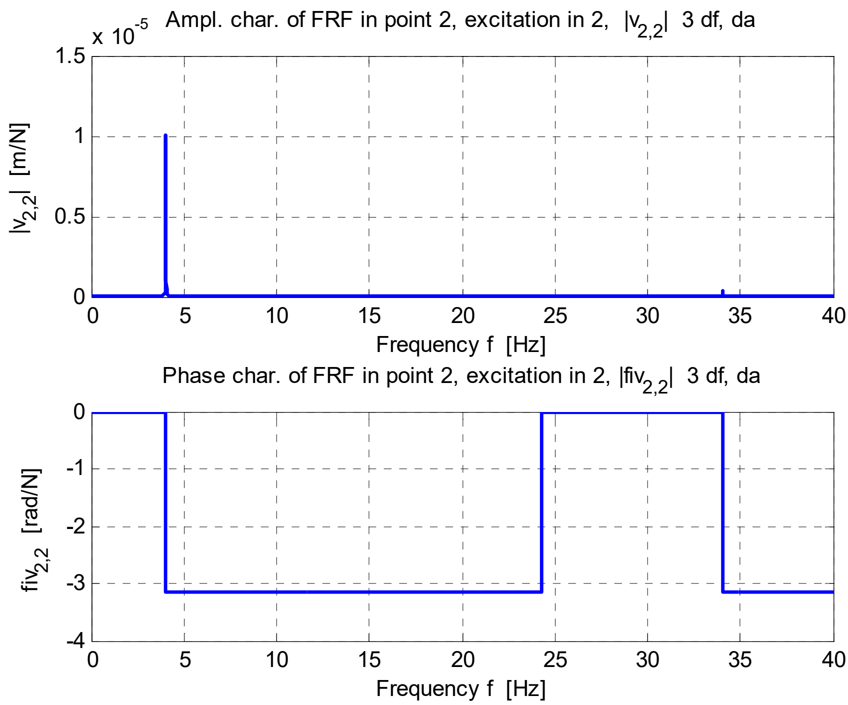

Fig. 6: Amplitude and phase characteristics of FRF in point 2, excitation in 2

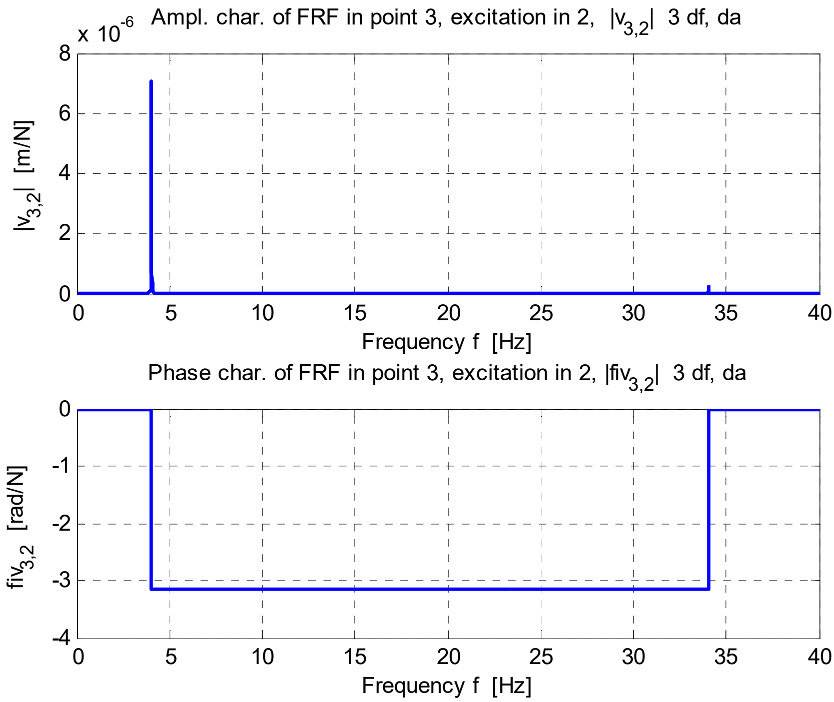

Fig. 7: Amplitude and phase characteristics of FRF in point 3, excitation in 2

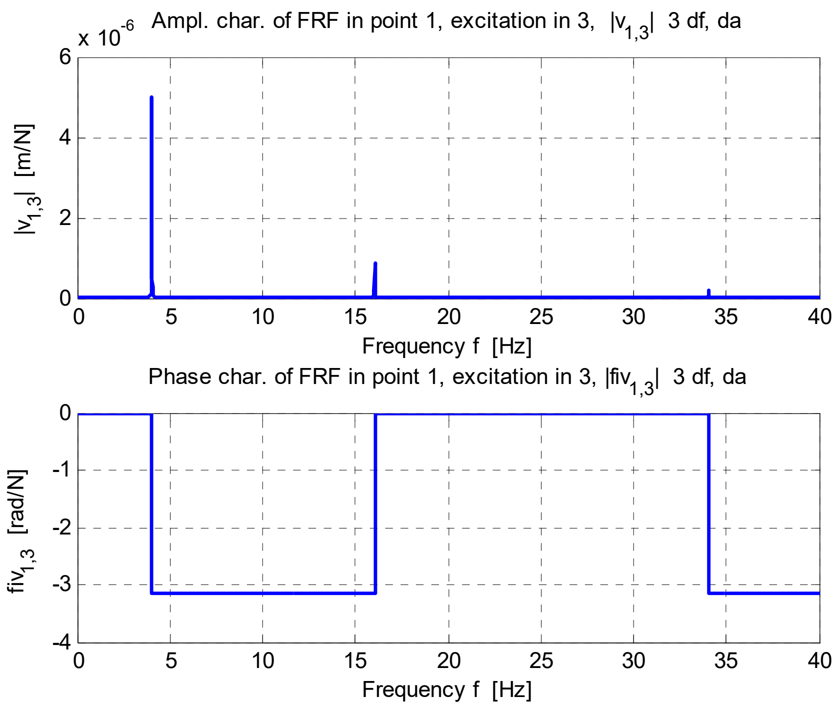

Fig. 8: Amplitude and phase characteristics of FRF in point 1, excitation in 3

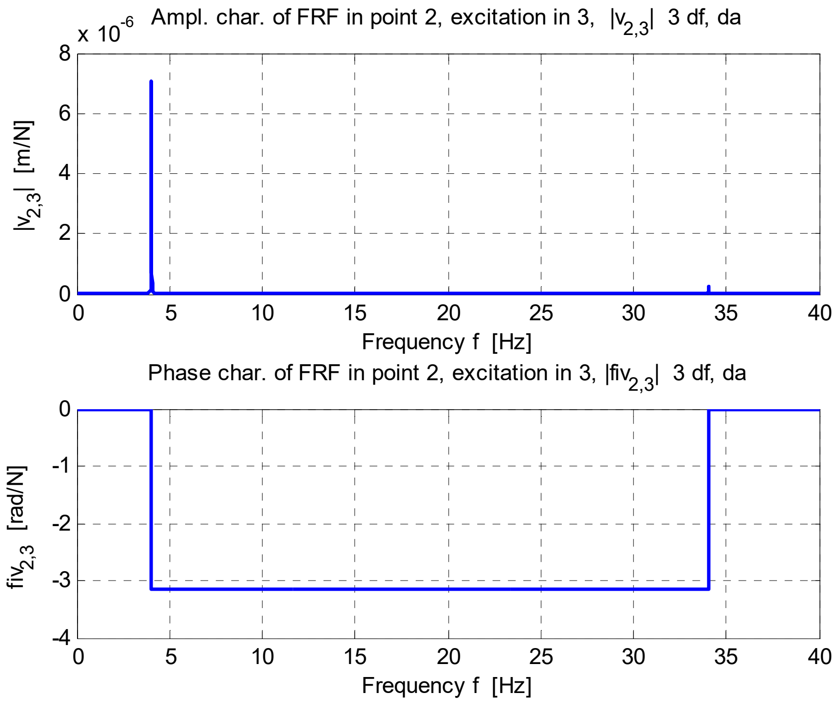

Fig. 9: Amplitude and phase characteristics of FRF in point 2, excitation in 3

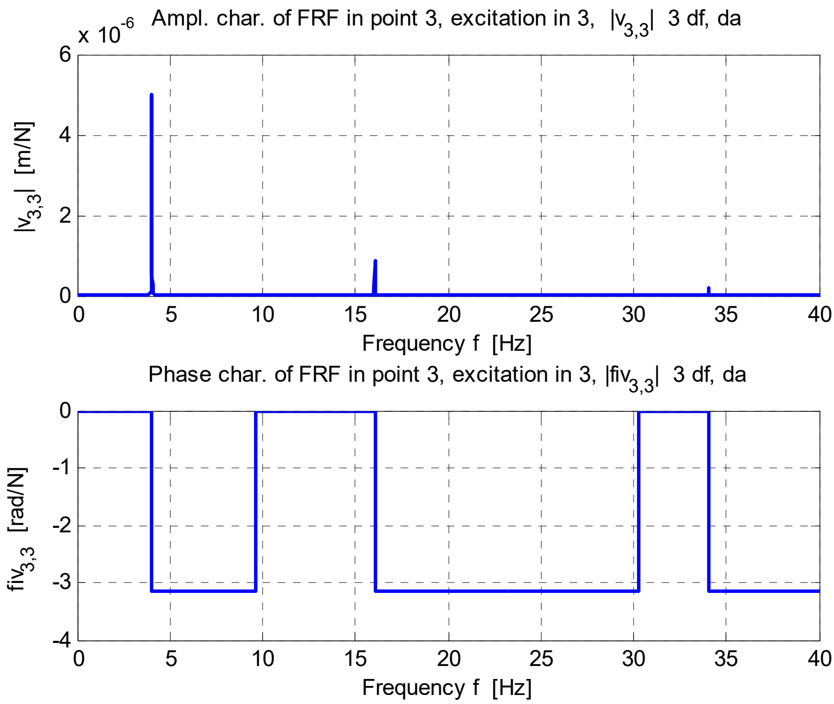

Fig. 10: Amplitude and phase characteristics of FRF in point 3, excitation in 3

5 CONCLUSIONS

Frequency response functions (FRF) are important characteristics that uniquely define properties of dynamical systems. They are linked to the choice of computational model and the means of structure excitation. It is possible to compute them in several ways, e.g., to use the Fourier transform for the transmission from time to frequency domain. Using of presented technique is possible when solving various dynamical problems [4-9]. From the presented results it can be seen that values of dominant frequencies are $f_{(1)} = 4{,}04$ Hz, $f_{(2)} = 16{,}04$ Hz, $f_{(3)} = 34{,}06$ Hz. From the theoretical point of view the amplitude characteristic of un-damped system at natural frequencies can reach infinity large values, what can cause problems in numerical solutions. Un-damped system oscillates in the phase or in anti-phase. Angle of phase deviation is then 0 or π. On the basis of Maxwell principle of reciprocal deflection $\left|\overline{v}_{ik}\right| = \left|\overline{v}_{ki}\right|$. In the case in question we considered symmetric structure with axis of symmetry in point 2. Because of symmetry it is valid $\left|\overline{v}_{11}\right| = \left|\overline{v}_{33}\right|$.

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