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UNCERTAINTY IN CRACK WIDTH ESTIMATES ACCORDING TO FIB MODEL CODE 2010

Abstract

The contribution is focused on quantifying model uncertainty of crack width estimates for reinforced concrete beams. Predictions obtained by the model provided by the *fib* Model Code 2010 are compared with results of tests of beams having different longitudinal and shear reinforcement ratios and concrete cover. Trends of model uncertainty with basic variables are investigated.

Keywords

Crack width, model uncertainty, concrete cover, reinforcement ratio.

1 INTRODUCTION

The study is based on a limited database of 12 crack tests of reinforced concrete beams with different concrete covers, reinforcement ratios and stirrup spacing. Test data are obtained from the study by Caldentey [1]. The test results are compared with the predictions obtained by the simplified analytical model provided in the *fib* MC2010 [2]. The aim is to provide insights into quantification of model uncertainty in crack width predictions that is considered to be the key issue in reliability analysis with respect to cracking. Similar analysis [3] was performed for the EN 1992-1-1 model.

2 MODEL UNCERTAINTY

Model uncertainty should describe (1) random effects neglected in the model and (2) simplifications in mathematical relations on which the model is based. The uncertainties in resistance models are obtained from comparisons of physical tests and results of the theoretical model, accounting generally for all relevant sources of uncertainties. In this pilot study, model results as a dominating source of uncertainty are discussed only. However, the effect of the other uncertainties - test uncertainty and structure-specific conditions [4] - should be verified in future studies.

The model uncertainty is investigated for crack widths of practical interest – 0.2-1.0 mm. Model uncertainty θ is treated here as a random variable [5]:

$$\theta = w_{\mathrm{exp}} / w_{\mathrm{model}} \tag{1}$$

where:

w_{exp} – is the experimental crack width value and

w_{mod} the crack width calculated by a model.

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3 TEST DATABASE

Database of 12 tests [1] provides the basis for the pilot model uncertainty quantification. Notation and ranges of material and geometrical characteristics of the tested beams are given in Tab. 1. All beams were loaded until failure so that their serviceability could be fully explored.

Tab.1: Notation of basic variables and the ranges of their values.

Symbol	Description	Value
f_c	measured concrete compressive strength	26.9 MPa
f_{yk}	characteristic value of strength of longitudinal reinforcement	500 MPa
b	width of a cross-section in the tensile area	350 mm
h	height	450 mm
s	stirrup spacing	100 or 300 mm*
c	concrete cover	20 or 70 mm
ϕ_l	bar diameter	12 or 25 mm
A_{sl}	area of longitudinal reinforcement	452 or 1963 mm^2
$d = h - c - 0.5\ \phi_l$	effective depth	0.367-0.424 mm
$\rho_l = A_{sl} / (b\ d)$	longitudinal reinforcement ratio	0.305-1.5%

*Four beams without stirrups.

4 CRACK WIDTH MODEL BASED ON MC 2010

According to *fib* MC 2010, the maximum crack width is estimated using the following relationship:

$$w_{mod} = 2\ [k\ c + 0.25(f_{ct} / \tau_b)(\phi_l / \rho_{s,ef})\ ]\ [(\sigma_s - \beta\ \sigma_{sr}) / E_s] \quad (2)$$

where:

k – is the coefficient of influence of the concrete cover ($k = 1$),

f_{ct} / τ_b– is the ratio of tensile and bond strength ($f_{ct} / \tau_b \approx 1/1.8$),

σ_s – is the steel stress in a crack,

β – is the empirical coefficient depending on the type of loading ($\beta = 0.6$),

E_s – is the modulus of elasticity of reinforcement ($E_s = 200$ GPa) and

σ_{sr} – is the maximum steel stress in a crack in the crack formation stage, obtained for pure tension as:

$$\sigma_{sr} = (f_{ct}\ A_{c,ef} / A_{sl})\ (1 + \alpha_e\ A_{sl} / A_{c,ef}) \quad (3)$$

where:

$A_{c,ef}$ – $b \times \min[2.5(h - d), (h - x)/3, h/2]$ is the effective area of concrete in tension and x is the distance from neutral axis to compression edge.

Tab. 2 provides notation of variables, values of which are estimated from the values of the basic variables given in Tab. 1. These estimates, particularly that one of tensile strength, introduce some additional uncertainty. However, its effect is considered small and this uncertainty is disregarded in the following analysis.

Tab.2: Basic variables derived from measurements.

Symbol	Description	Value
$E_c = 22000\ (0.1\ f_c)^{0.3}$	modulus of elasticity	29.6 GPa
$f_y \approx 1.1 f_{yk}$	yield strength of longitudinal reinforcement	550 MPa

5 MODEL UNCERTAINTY

For each specimen, the crack width is estimated from Equation (2) and a model uncertainty value is obtained from Equation (1). Sample characteristics of the model uncertainty (mean μ_θ and coefficient of variation V_θ) for the whole database are given in Tab. 3 for the reference value of crack width of 0.4 mm.

Tab.3: Sample characteristics of the model uncertainty.

Description of the sample	μ_θ	V_θ
Whole database, $n = 12$	2.24	0.36
Reinforced beams ($\rho_l \leq 0.5$ %), $n = 6$	1.50	0.21
Moderately reinforced beams ($\rho_l \approx 1.5$ %), $n = 6$	2.99	0.06
Beams without stirrups, $n = 4$	2.16	0.36
Beams with stirrups, $n = 8$	2.28	0.34
Beams with concrete cover 20 mm, $n = 6$	2.25	0.40
Beams with concrete cover 70 mm, $n = 6$	2.24	0.36

It follows from Tab. 1 that the MC2010 predictions tend to excessively underestimate crack widths – with bias between two and three – and significant dispersion of model results is observed – coefficients of variation up to 0.4. The values provided in Tab. 3 suggest that model uncertainty will likely dominate reliability of reinforced concrete beams with respect to crack width as its coefficient of variation exceeds typical values of other basic variables.

The study is based on a limited test database and the conclusions with general validity can thus hardly be drawn. It seems that longitudinal reinforcement ratio or, similarly, strain of reinforcement (Fig. 1) is found to have significant effect on characteristics of model uncertainty while the influence of other variables such as shear reinforcement ratio, concrete cover and stirrup spacing is marginal. Obviously, these findings need to be verified by further studies based on larger databases.

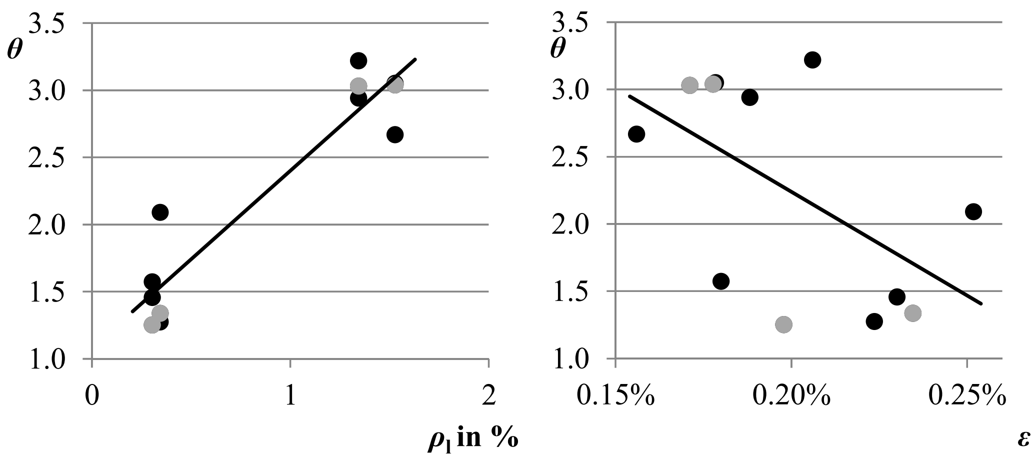

Fig.1: Variation of model uncertainty values with longitudinal reinforcement ratio (left) and strain of reinforcement (right) for MC2010 model (grey points - beams without stirrups).

Additional numerical investigations, results of which are not discussed here in detail, reveal that:

- Characteristics of model uncertainty change insignificantly when the results of theoretical crack width predictions are compared with test results for crack width around 0.2 or 1 mm; somewhat higher coefficients of variation, up to 0.5, are observed for the 0.2 mm level.

- Considering the EC2 model for maximum crack width [3], the bias in model predictions is somewhat reduced (around two); the coefficient of variation is similar to that one observed for the MC2010 model.

The bias in the EC2 model predictions for mean crack width is slightly less than two and the coefficient of variation reduces to 0.25 for crack width levels of 0.4 and 1 mm. The recent study [6] shows that crack widths can by well predicted by FEM. However, uncertainty in maximum crack width estimates seems to be in all cases much larger than that related to the models for Ultimate Limit State verifications [7].

6 CONCLUSIONS

A wide consensus on appropriate theoretical models for crack width prediction seems to be missing at present. The submitted study is based on a limited test database and the conclusions with general validity can thus hardly be drawn. It seems that:

- The predictions based on the *fib* MC2010 model tend to excessively underestimate crack widths with bias between two and three and the coefficient of variation up to 0.4.
- Longitudinal reinforcement ratio is found to have significant effect on characteristics of model uncertainty while the influence of other variables such as shear reinforcement ratio, concrete cover and stirrup spacing is marginal.
- The characteristics of model uncertainty change insignificantly when model predictions are compared with test results for the crack width levels of 0.2, 0.4 or 1 mm.

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LITERATURE

[1] CALDENTEY, P. A., PEIRETTI, C. H., IRIBARREN, P. J. & SOTO, G. A., 2013. Cracking of RC members revisited: influence of cover, ϕ/ρs, ef and stirrup spacing - an experimental and theoretical study. *Structural Concrete* 14(1): 69-78.

[2] *fib Model Code for Concrete Structures 2010*. Lausanne: *fib*, 2013.

[3] MARKOVA, J. & SYKORA, M. Uncertainties in Crack Width Verification of Reinforced Concrete Structures In: *Risk, Reliability and Safety. Innovating Theory and Practice*. London: Taylor & Francis Group, 2016, pp. 2433-2438. ISBN 978-1-138-02997-2.

[4] HOLICKY, M., RETIEF, J. V. & SYKORA, M. Assessment of model uncertainties for structural resistance; In: *Probabilistic Engineering Mechanics* 45: 188 – 197, 2016, ISSN: 0266-8920, DOI: 10.1016/j.probengmech.2015.09.008.

[5] JCSS 2015. *JCSS Probabilistic Model Code* (periodically up-dated, online publication). Joint Committee on Structural Safety. <www.jcss.byg.dtu.dk>.

[6] CERVENKA, V., MARKOVA, J., MLCOCH, J., CALDENTEY, P. A., SAJDLOVA & T., SYKORA, M. Uncertainties of Crack Width Models (accepted for publication), Maastricht *fib* Symposium, 2017

[7] SYKORA, M., HOLICKY, M., PRIETO, M. & TANNER, P. Uncertainties in resistance models for sound and corrosion-damaged RC structures according to EN 1992-1-1; In: *Materials and Structures* 48(10): 3415-3430, 2015, Print ISSN 1359-5997, Online ISSN 1871-6873, DOI 10.1617/s11527-014-0409-1.