WEDGE SPLITTING TEST: DISPLACEMENT FIELD ANALYSIS BY MULTI-PARAMETER FRACTURE MECHANICS

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Abstract. *Multi-parameter fracture mechanics is nowadays quite extensively applied when cracked structures/specimens are investigated. The reason was that it has been shown that it can be helpful and bring results that are more accurate when for describing of fracture processes a larger region around the crack tip is used. This can be typical for material like concrete or other materials with quasi-brittle behaviour. Various relative crack length configurations were chosen in order to investigate the importance of the higher-order terms of the Williams expansion (WE) on the crack-tip stress field distribution in Wedge splitting test specimen. The higher-order terms were calculated by means of the over-deterministic method from displacements of nodes around the crack tip obtained by a finite element analysis in different radial distances from the crack tip. The effect of the constraint level (second member of WE) was investigated. Although the third and higher terms of the Williams series are very often neglected, their influence on the opening stress values was investigated and discussed.*

Keywords

WST, ODM, Williams series, FEM, displacement, fracture mechanics.

1. Introduction

The wedge splitting test (WST), introduced by Linsbauer and Tschegg [1], [2] and later developed by Brühwiler and Wittmann [3], is an interesting experimental method since it does not require sophisticated testing equipment to achieve a quasi-static (stable) fracture process. An ordinary electromechanical testing machine with a constant actuator displacement rate can be used and no sophisticated test stability control apparatus (i.e. closed loop control unit with e.g. crack tip opening displacement as a feedback signal) is necessary. The test is nowadays widely used by researchers see e.g. Zimmerman and Lehký [4] – for two classes of cement based concrete C 40/50 and C 50/60, Seitl et al [5] – for foam concrete, Merta et al [6]- concrete with natural fibers, Walter [7] – for concrete-steel interface, Korte et al for self-compacting concrete [8][9][10] and for vibrated concrete [9][11], ASTM 1221-96 [12] - for steel, etc.

The test specimens can be prepared either from the cube Merta [13] or cylindrical specimens by Seitl [14] casted into standardized moulds, or as prismatic or cylindrical specimens taken from existing structures by sawing or core-drilling, respectively. The paper by Sobek [15] is focused on the stress field analysis (especially the shape functions) in WST for approximation of the stress and displacement fields in cracked specimens in two variants of plane problem – the plane stress and the plane strain condition.

The aim of this contribution is to introduce displacement field in specimen for WST. The various relative crack lengths (from small $a/W = 0.1$ up to long $a/W = 0.8$) and quantify influence of boundary conditions on the displacement field in the vicinity of the crack tip for the constant value of the first member of Williams expansion series (stress intensity factor values equals 1 MPa m$^{1/2}$). For simulation of stress/displacements fields in WST, the finite element method (FEM) was used.

2. Theoretical Background

In the following subchapters description of the Williams [16] stress field near the crack tip and an over deterministic method is introduced in next subchapter.

The crack-tip stress/displacement field in this work is approximated via the Williams expansion (WE), particular relations can be written in the form:

$$\sigma_{i,j} = \sum_{n=1}^{\infty} \frac{n}{2} r^{\frac{n}{2}} A_n f_{\sigma_{i,j}}(\theta, n), \text{ where } i,j \in \{x,y\} \qquad (1)$$

More important is expression for displacement equations:

$$u_i = \sum_{n=1}^{\infty} r^{\frac{n}{2}} A_n f_i^u(\theta, n, E, \eta), \text{ where } i \in \{x,y\} \qquad (2)$$

Displacement field is obtain from analysis done in software ANSYS, materials input properties (E, ν) are given for each material from measurement.

Over-deterministic method

The over-deterministic method (ODM) was proposed by Ayatollahi and Nejahi [17] and provides accurate results of the WE coefficients, it has been chosen with regard to the previous experience of the authors. Its largest advantage is that it requires only the knowledge of the nodal displacement solution around the crack tip. Thus, the displacements together with the polar coordinates of a selected set of nodes defined at a particular radial distance around the crack tip serve as an input for Eq. 2 and the only variables A_n are calculated. At least the one condition that has to be fulfilled should be mentioned: a minimum of $N/2 + 1$ nodes are needed to be able to determine N coefficients see e.g. in [17] and [18]. The influence of rounding is mentioned by Ruzicka et al [19].

3.   Modelling in FEM Software

3.1.   Geometry of WST

Geometry of investigated WST specimen was width $(S) \times$ height (W), particularly 75 mm × 150 mm [13], notch was modelled as a crack (i.e. with zero width). The material for upper part can be made from various kinds of marble [20] or steel [1]. To provide accurate results the upper part was modelled as a steel plate [13] and steel part for forces application and a bottom part (mentioned as steel support) were modelled with dimensions 60 mm × 20 mm, 10 mm × 10 mm, 5 mm × 10 mm, respectively. The Model's geometry and boundary conditions are shown in Fig. 1. Sketch of symmetrical half is rotated by -90° against the experimental performance due to crack tip with origin at global Cartesian coordination origin.
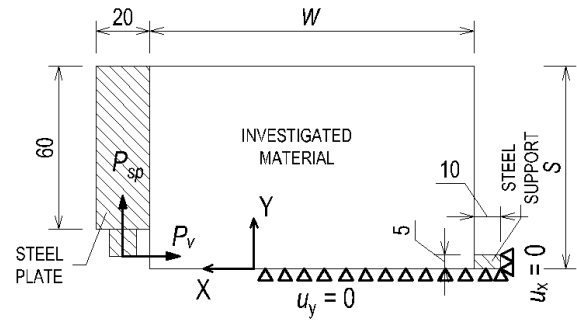

Fig. 1: Dimensions of studied geometry, symmetric half rotated by -90°.

The numerical model was loaded with two forces simulating the induced displacement of the wedge into the specimen P_v (vertical direction) and P_{sp} (horizontal/splitting force). The vertical force P_v can be expressed as follows:

$$P_v = \frac{1}{2} P_{sp} 2\tan(\alpha_w), \qquad (3)$$

where $\alpha_w = 19.2°$ represents wedge angle [13].

3.2.   Numerical Model

The numerical model was created in a finite element software (FEM) Ansys 17.2 [21] as a plane symmetric 2D with the plane strain boundary conditions. Studied model was meshed with element type PLANE183 and command KSCON was applied to take into account the crack singularity. A linear elastic material model was used with input parameters: Young's modulus and Poisson's ratio. For the investigated material like concrete) material's constants were selected $E = 30$ GPa and $\nu = 0.2$, whereas $E = 210$ GPa and $\nu = 0.3$ for the steel frame.

Half model's boundary conditions are shown in Fig. 2, note that the numerical model is rotated by 90° in comparison to the testing reality, due to more accurate results of nodal displacements when the crack tip is coincident with the origin of the global Cartesian coordinate system.
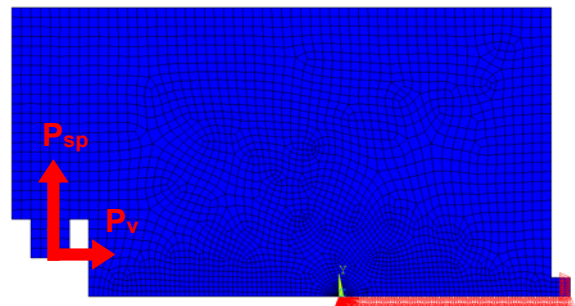

Fig. 2: Example of the symmetric half of WST specimen as numerical model rotated by -90° with used boundary conditions (arrows is applied force and triangles are boundary conditions).

In Fig. 3, detail in the vicinity of crack tip and selected nodes considered for displacements evaluation in radial distance $r = 1; 2; 3; 4$ and 5 mm are shown. For ODM

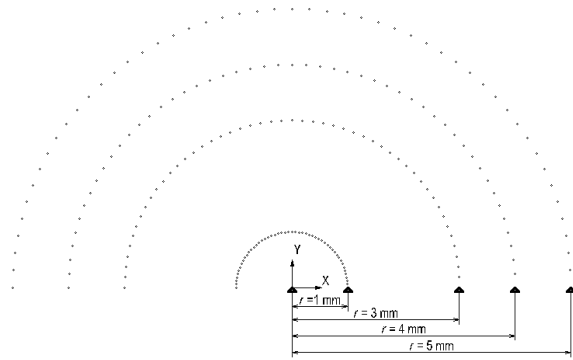analysis the values from nodes in distance $r = 1, 3, 5$ mm were used.

Fig. 3: Detail in the vicinity of crack tip and selected nodes considered for displacements evaluation in radial distance 1; 3; 4 and 5 mm.

The values of forces used in numerical simulation were chosen for all relative crack lengths α to produce the same value of the stress intensity factor (SIF). The SIF values were calculated by means of KCALC command in software ANSYS. The numerical results were compared to results in literature [22] and [23]. For this study, K_I was chosen as 1 MPam$^{1/2}$ to provide better comparison of changes in the displacement field near the crack tip. The applied values of forces for selected relative crack lengths are mentioned in Tab. 1.

Tab. 1: Overview of applied forces for given relative crack length used for numerical analysis.

α [-]	0.1	0.2	0.3	0.4
P_v [N]	30.42	23.65	18.65	14.22
α [-]	0.5	0.6	0.7	0.8
P_v [N]	10.20	6.63	3.69	1.53

These forces generate values of T-stress (second term related to A_2) as presented in Tab. 2.

Tab. 2: T-stress values and equivalent values of A_2 for given relative crack length used for numerical analysis generated by forces mentioned in Tab. 1.

α [-]	0.1	0.2	0.3	0.4
T [MPa]	4.24	5.22	5.60	5.75
A_2 [ ]	1.06	1.30	1.40	1.44
α [-]	0.5	0.6	0.7	0.8
T [MPa]	5.74	5.65	5.64	5.91
A_2 [ ]	1.43	1.41	1.41	1.48

4.   Results and Discussion

4.1.   Results from FEM

The results of nodal displacement from the numerical analysis were used as an input parameter for the ODM method. The nodal displacements were generated for $\alpha = <0.1 \div 0.8>$ and for the radial distance from the crack tip $r = \{1; 3; 5\}$ mm. The example of nodal displacements generated by the FE model are presented in Fig. 4 and Fig. 5.
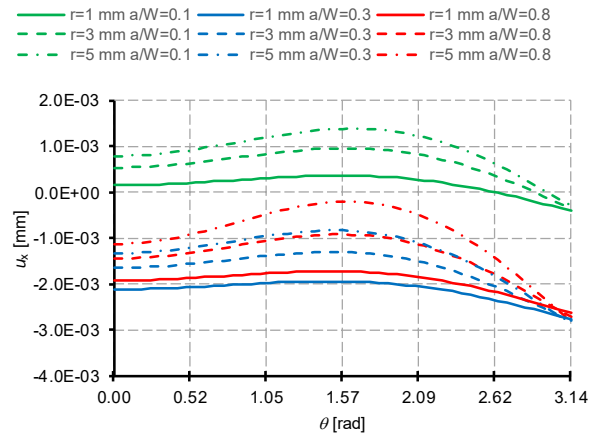

Fig. 4: Nodal displacements u_x generated via FE software for the relative crack length $\alpha = 0.1, 0.3$ and 0.8 in radial distance $r = 1, 3$ and 5 mm.

The nodal displacements u_x as presented in Fig. 4. The plot for different a/W=0.1 - 0.8 has increasing values for long crack and it is caused by reduced stiffness of the specimen as the relative notch length and radial distance increases.
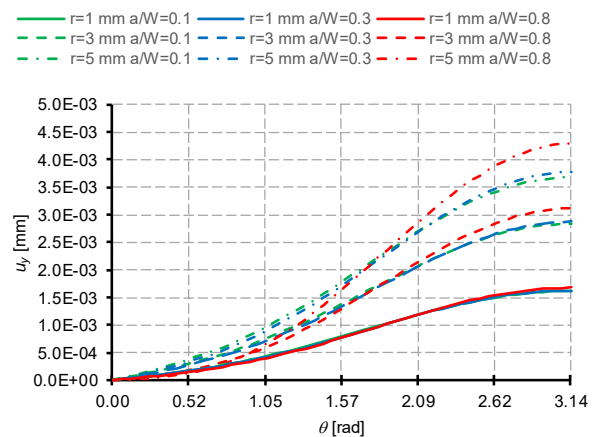

Fig. 5: Nodal displacements u_y generated via FE software for the relative crack length $\alpha = 0.1, 0.3$ and 0.8 in radial distance $r = 1, 3$ and 5 mm.

The nodal displacement u_y is shown in Fig. 5. The values are identical for all studied relative crack lengths in radial distance $r = 1$ mm, where the u_y opens the specimen in mode I deformation very near of crack tip and stress is mathematically going to infinity. This agrees with the assumption of loading the specimen with forces producing $K_I = 1$ MPam$^{1/2}$. The variation of the displacement in radial distance $r = 3$ and 5 mm is caused by the various boundary conditions.

4.2. Results obtained by ODM

The above-mentioned source of nodal displacement field was generated by FEM (using the ANSYS solution tools). For every α ratio and every half-ring distance the set of coefficients of WE were calculated. In total 51 points from FEM allows to calculate 50 coefficients of A_n. For faster calculation with sufficient precision, the calculation was limited up to the first 20 coefficients. The first five WE coefficients(see, eq. (1)) for $\alpha = 0.1$, $\alpha = 0.3$ and $\alpha = 0.8$ are mentioned in Tab. 3 for various radial distance $r = 1$; 3; 5 mm. Note that, stress field in the vicinity of the crack tip is described analytically usually by SIF ($K_I = \sqrt{2\pi} \times A_1$) and T –stress ($T = 4 \times A_2$).

Tab. 3: Initial first five WE coefficients calculated via ODM for selected relative crack length and studied distance from crack tip at $r = 1$ mm.

n [-]	$r = 1$ mm		
	$a/W = 0.1$	$a/W = 0.3$	$a/W = 0.8$
1	12.61445	12.61461	12.61505
2	-0.19515	0.181144	0.467172
3	-0.05358	-0.11717	-0.50842
4	0.005094	0.001388	0.020191
5	-0.00295	-0.00074	-0.00886

Tab. 4: Initial first five WE coefficients calculated via ODM for selected relative crack length and studied distance from crack tip at $r = 3$ mm.

n [-]	$r = 3$ mm		
	$a/W = 0.1$	$a/W = 0.3$	$a/W = 0.8$
1	12.61418	12.61443	12.61524
2	-0.19538	0.180936	0.467012
3	-0.05416	-0.11773	-0.50901
4	0.005112	0.001342	0.020133
5	-0.00219	0.000061	-0.00798

Tab. 5: Initial first five WE coefficients calculated via ODM for selected relative crack length and studied distance from crack tip at $r = 5$ mm.

n [-]	$r = 5$ mm		
	$a/W = 0.1$	$a/W = 0.3$	$a/W = 0.8$
1	12.61421	12.61453	12.6157
2	-0.19544	0.180887	0.466981
3	-0.05428	-0.11785	-0.50912
4	0.00511	0.001335	0.020123
5	-0.00213	0.000126	-0.00791

As it is seen in Tabs 3 – 5, results obtained from ODM are in good agreement for fist WE coefficient A_1, which is related to K_I for all studied α ratios and radial distances r. The difference in all cases in 0.1%. However, the higher WE coefficients calculated by ODM shows great

mismatch with the demanded results. This fact is visible for values of A_2 coefficient calculated for $\alpha = 0.1$ for all radial distances, where the A_2 should have positive values as presented in Tab. 2.

(a) – $r = 1$ mm, $\alpha = 0.1$

(b) – $r = 3$ mm, $\alpha = 0.1$
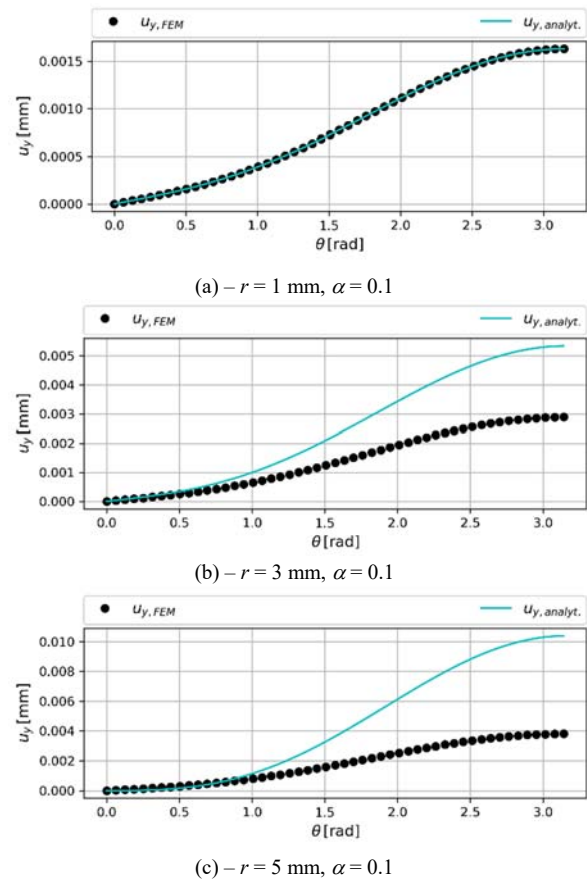

(c) – $r = 5$ mm, $\alpha = 0.1$

Fig. 6: Comparison of displacements obtained from analytical for first member and numerical solution (infinite number of terms) in radial distance from crack tip $r = 1, 3, 5$ mm and for $\alpha = 0.1$.

Presented comparison of nodal and calculated displacements u_y for various radial distances are presented in Fig. 6. From Fig. 6(a) in can be noted that the differences in u_y are limited to tenth of percent. However, the difference in u_y calculated by FEM and ODM increases with increasing radial distance from the crack tip. This effect is greatly visible in Fig. 6(c) for $r = 5$ mm. These disagreements are caused by lack of calculated coefficients of WE and authors recommend calculating more than 20 WE coefficients. This fact should be considered since the u_y displacement opens the crack in mode I deformation.

5. Conclusion

A calculation of higher order coefficients of Williams expansion by over-deterministic method was presented based on the input of nodal displacement calculated by finite element software. In the study, the calculated WE coefficients shows good agreement only for first member, which is directly related to K_I. The disagreement in higher

order terms is caused but choosing insufficient number of WE coefficients.

Determination of a particular number of terms that allows a sufficiently correct approximation of a displacement field in a real situation should therefore be supported by an analysis similar to that presented here with a possible additional convergence analysis

The results of this study may have significant implications for transfer of material properties from fracture and fatigue analyses of civil engineering materials applied in structures.

Acknowledgements

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