NUMERICAL ANALYSIS OF BRAZILIAN DISC TEST WITH A CENTRAL NOTCH: CONSTRAIN LEVEL FOR VARIOUS NOTCH INCLINATION ANGLES

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Abstract. *The Brazilian disc test is widely used to determinate indirect tensile strength or fracture mechanical parameters of rocks, concrete and polymers. The fracture mechanical parameters evaluated from such a test provides information of mode I, mode II and mixed mode I/II fracture initiation. While the geometry functions for calculation of stress intensity factors are known, the T-stress values are lacking. Especially, the lack of T-stress can be noticed, when the T-stress equals to zero. This contribution aims to find crack inclination angle for which the T-stress values are close to zero and evaluate stress intensity factor for such an angle. For this, a numerical study was performed.*

Keywords

Brazilian disc, constraint, Mixed Mode I/II, Stress Intensity Factor, T-stress.

1. Introduction

The prior renovation durability and sustainability of structures made from cementitious materials are often investigated in order to prevent accidents, unnecessary expenses and to get a basic understanding of the material used. To obtain a material sample from renovated structure, a core-drill is used to remove a cylindrical sample from the structure. The concrete samples are submitted for laboratory tests to identify the material's characteristics. The common material characteristics determined through testing the cylindrical specimen are: bulk density, the Young's modulus, the compressive strength, the flexural strength, etc. see [1][2]. The design and structural behaviour of the abovementioned structures are, in most cases, advanced and complex. These structures are not only subject to uniaxial load, but very often to mixed mode I/II (biaxial) load conditions. To perform a modern advanced analysis of structural behaviour, knowledge of the material's fracture mechanical parameters is essential.

Due to the structural geometry, the loading conditions or the construction technology, concrete structures and their structural elements are typically subjected to a combination of bending and shear load. The fracture process in such structural elements can be divided into actions from tensile loads (mode I), shear loads (mode II) or any combination of tensile and shear loads (mixed mode I/II).

To avoid an expensive reshaping of the core drill sample into a rectangular shape, it is very efficient to perform a fracture mechanical test on the specimen made directly from the core-drill sample. The Brazilian disc test with a central notch (BDCN) [3-6] suggest such application and provides information about fracture behaviour under modes I, II and mixed-mode I/II loading. The test performed on the BDCN specimen (circle cut from the core-drill cylinder) is carried out under relatively simple experimental conditions, using only the testing press with sufficient load capacity. The evaluation of the fracture parameters for modes I, II and mixed mode I/II is done by inclining the notch by angle α against the load position (see Fig. 1).

The aim of this contribution is to evaluate and compare the values of the stress intensity factors (SIF) under the mixed mode load (combination of mode I and mode II) and find the value of T-stress close to zero. This was done by a numerical model of Brazilian disc with various boundary conditions in which the constraint effect was studied and the geometry functions were calculated.

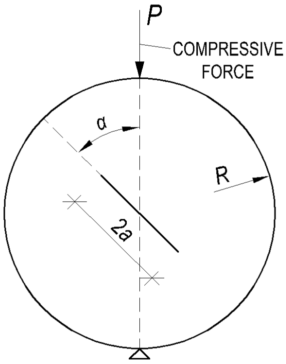

Fig. 1: Brazilian disc test with central notch and loading conditions.

2. Theoretical Background

This contribution is based on a linear elastic fracture mechanics. The linear elastic fracture mechanics concept uses the stress field in the close vicinity of the crack tip described by Williams' expansion [7]. This expansion is an infinite power series originally derived for a homogenous elastic isotropic cracked body, which can be described by a following equation:

$$\sigma_{i,j} = \frac{K_I}{\sqrt{2\pi r}} f_{i,j}^{I}(\theta) + \frac{K_{II}}{\sqrt{2\pi r}} f_{i,j}^{II}(\theta) + T + O_{i,j}(r,\theta), \quad (1)$$

where σ_{ij} represents the stress tensor components, K_I, K_{II} are the SIFs for mode I and respectively mode II, $f_{i,j}^{I}(\theta), f_{i,j}^{II}(\theta)$, are known shape functions for mode I and mode II usually written as Y_I and Y_{II}, T (or T-stress) represents the second independent term on r, O_{ij} represents higher order terms and r, θ are the polar coordinates (with origin at the crack tip; crack faces lie along the x-axis).

The values of the SIFs for a finite specimen and the polar angle $\theta = 0°$ can be expressed in the following form [8-9]:

$$K_I = \frac{P\sqrt{a}}{RB\sqrt{\pi}} \frac{1}{\sqrt{1-\frac{a}{R}}} Y_I(a/R, \alpha), \quad (2)$$

$$K_{II} = \frac{P\sqrt{a}}{RB\sqrt{\pi}} \frac{1}{\sqrt{1-\frac{a}{R}}} Y_{II}(a/R, \alpha), \quad (3)$$

where P is the compressive load, a is a crack length, R is the radius of the disc ($D/2$), B is disc thickness, α is the inclination angle and $Y_I(a/R,\alpha)$, $Y_{II}(a/R, \alpha)$ are the dimensionless shape functions for mode I and mode II, respectively. Geometry functions Y_I and Y_{II} are derived in subsection 3.1.

3. Numerical Model

A numerical simulation was performed in the finite element (FE) software Ansys 17.2 [10] to derive dimensionless geometrical functions for mixed mode of SIFs and to calculate T-stress. A two-dimensional (2D) numerical model with a plane strain boundary condition was used to calculate SIFs (K_I and K_{II}). The numerical model was meshed with an 8-node quadratic element (type PLANE183) taken from ANSYS elements library and the meshing command KSCON was used to consider the crack tip singularity. Fig. 2(a) indicates the mesh pattern of the BDCN specimen while the refined mesh pattern around the specimen crack tip is shown in Fig 2(b).

Input material properties of concrete used in FE analysis were following the Young's modulus and Poisson's ratio, E = 40 GPa and ν = 0.2, respectively. Simulated BDCN had radius R = 75 mm and the relative crack length a/R was 0.4. The numerical model was loaded with constant force P = 100 N in all studied cases.

The numerical study was performed with a loading applied at the top edge of the BDCN, while the bottom edge was considered a rigid support. Adequate boundary conditions were added to prevent rigid body translations (See Figure 2(a)).

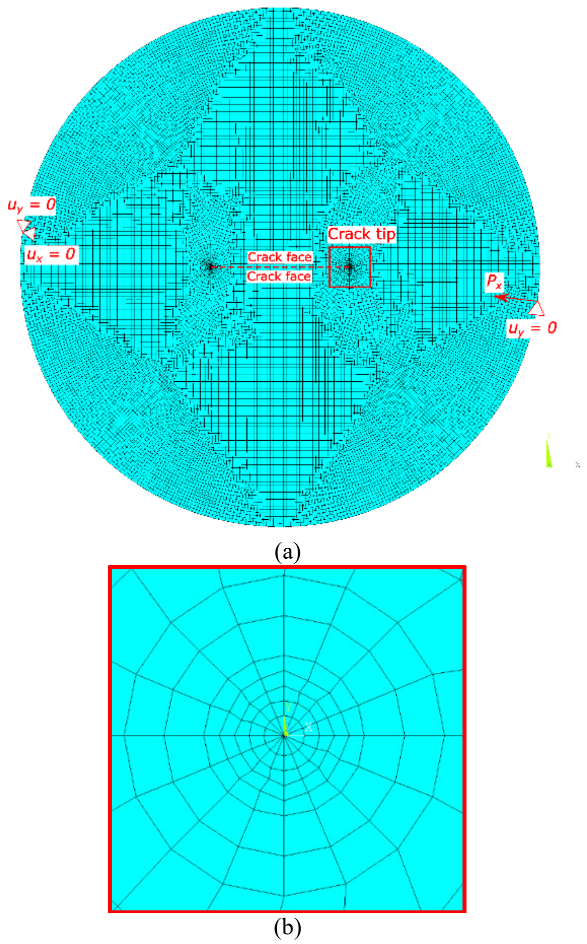

Fig. 2: Numerical model with boundary conditions (a) and detail on crack tip (b).

3.1. Geometry functions

The FE software ANSYS uses following equations for calculation of SIFs – K_I, K_{II} for $\theta = \pm 180°$ (KCALC command).

$$K_I = \sqrt{2\pi}\frac{2G}{1+\kappa}\frac{|v|}{\sqrt{r}} \quad (4)$$

$$K_{II} = \sqrt{2\pi}\frac{2G}{1+\kappa}\frac{|u|}{\sqrt{r}} \quad (5)$$

where u, v are nodal displacements, G is the shear modulus, κ is the Kolosov's constant for plane strain or plane stress boundary conditions and r is the coordinate in the cylindrical coordinate system. However, the KCALC command was replaced by new command CINT, which uses and interaction integral. The interaction integral is defined as:

$$I = \frac{2}{E^*}(K_1 K_1^{aux} + K_2 K_2^{aux}) + \frac{1}{\mu} K_3 K_3^{aux}, \quad (6)$$

where K_i is the stress intensity factor for mode I, II and III, K_1^{aux} is the auxiliary stress intensity factor for mode I, II and III, E^* is the Young's modulus for plane strain $E/(1-\nu^2)$, ν is the Poisson's ratio and μ is the shear modulus. For a 2D problem, the SIF for mode II K_3 is 0. The interaction integral is implemented in command CINT. The used SIFs and T-stress values are calculated as average value of five contours defined in CINT command.

The basic idea of two-parameter fracture mechanics consists of the fact that the crack behaviour under different conditions is the same, if both parameters, the SIF K and the T-stress, are identical. Hence, the value of T-stress is introduced. To calculate T-stress, a direct extrapolation method [11], [12] is used, for angle $\theta = 0°$ the following equation is used:

$$T = \lim_{r\to 0}(\sigma_{xx} - \sigma_{yy}), \quad (7)$$

where σ is stress in given direction x or y.

Using the FE model of BDCN mentioned above a dimensionless geometrical function can be derived by using Eq. (20) for mode I and Eq. (21) for mode II [6] and [8-9]. The calculation of geometry functions Y_I and Y_{II} considers various notch angle α and a/R ratio:

$$Y_I(a/R,\alpha) = \frac{K_I R B\sqrt{\pi}}{P\sqrt{a}}\sqrt{1-\frac{a}{R}}, \quad (8)$$

$$Y_{II}(a/R,\alpha) = \frac{K_{II} R B\sqrt{\pi}}{P\sqrt{a}}\sqrt{1-\frac{a}{R}}. \quad (9)$$

4. Numerical results

The numerical results performed by using CINT command in this study have been compared with the values of T-stress found in the literature [6] and [8]. The values of T-stress are presented in Tab. 1 and it shows good agreement with values found in the literature with an error limited to 2%.

From Tab. 1 it can be noticed, that the inclination angle α, where T-stress reaches the values close to zero is $\alpha = 35°$. The whole curve of T-stress is presented in Fig. 3.

Tab.1: A comparison of the T-stress values with the values found in the literature.

α [°]	**T-stress [MPa] by [6]**	**T-stress [MPa] by [8]**	**T-stress [MPa] present**
0	-3.091	-3.038	-3.059
10	-2.685	-2.596	-2.641
20	-1.685	-1.581	-1.626
25	-1.133	-1.131	-1.028
35	-	-	0.0138

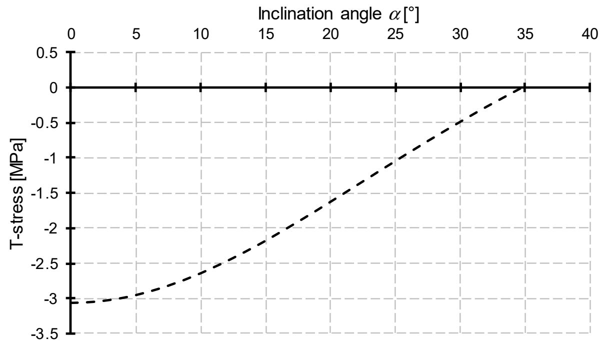

Fig. 3: The development of the T-stress values over the various notch inclination angles α.

Subsequently, the values of geometry function Y_I and Y_{II} has been evaluated in range of α = <0°-35°> to provide information about the SIFs for the case where the T-stress equals to zero. Both curves for Y_I and Y_{II} are presented in Fig. 4.

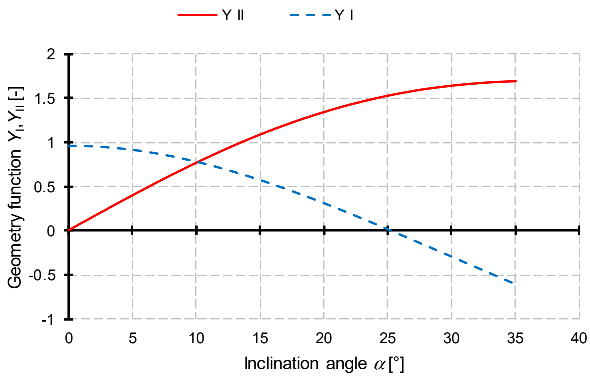

Fig. 4: Geometry function Y_I and Y_{II} over the various notch inclination angle α.

From Fig. 4 it can be noticed, that the geometry function for mode I after the value of α = 25° reaches the negative values. This should be taken in to account in during the experimental measurement, where the T-stress = 0 due the fact, that the negative value of K_I will produce the crack face closing instead of its opening. This can have a negative influence on the experimental results.

5. Conclusion

In this contribution, a numerical study of the Brazilian disc test with a central notch was performed with focus set on finding the notch inclination angle α, where the T-stress reaches value of zero. It was shown, that such a case is for $\alpha = 35°$. The presented T-stress values were compared to literature and show good agreement.

Consequently, a value of SIFs has been calculated, from which a geometry functions Y_{I} and Y_{II} were evaluated in the range of $\alpha = <0°; 35°>$. It was shown, that for the case where the T-stress reaches value of zero, the K_{I} starts to have negative value, which produces crack closing instead of opening. This fact should be taken into account in the experimental measurement.

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