A Comparison of Crack Paths Growth in Brazilian Disc with a Central Notch Using Various Criteria for Crack Initiation

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Abstract. *A numerical parametric study of the crack propagation under various level of mixed-mode conditions is evaluated. By focusing of the Linear Elastic Fracture Mechanics, the model with concrete input data of Brazilian Disc Test with a central notch is considered and its analysis with defined radius R, force P and material properties is performed. The results are obtained by varying the length an of the central notch and its angle of inclination α with respect to the horizontal axis. The crack growth direction is estimated through three different criteria: the maximum tangential stress criterion, the strain energy density criterion and the crack tip displacement criterion. This study involves the classical single-parameter fracture mechanics concept, in which the stress distribution in a cracked specimen is described by the stress intensity factors. The Williams solution of the crack-tip stress and displacement fields are taken into account. As a comparison, the solution is also provided with J-Integral. The model is created with the finite element method software ANSYS. Lastly, the suggested crack paths obtained by criteria are compared with experimental data.*

Keywords

Crack propagation direction, Crack-tip field, Mixed-mode I/II, Single-parameter MTS/SED/CTD criteria, J-Integral.

1. Introduction

In general, structures are usually loaded by various types of loading (gravity load, temperature loads etc.), which leads into complex structural behaviour. This means that instead of a uniaxial load, structures are subjected to mixed mode I/II loading conditions. As a consequence, detailed and precise knowledge of the material and of its properties is fundamental, in order to achieve an advanced structural analysis.

The tests generally performed in order to obtain the fracture mechanical parameters of cementitious material are done on prismatic beams or cubes, characterized by a rectangular cross-section specimen, usually with the notch. Examples of such tests are the three-point bending test (3PBT) [1] and the four-point bending test (4PBT) [1], the eccentric asymmetric four-point bending specimen for mixed load (EA4PBT) [3], the wedge splitting test (WST) [4]-[6] and WST wit biaxial loading [7], the combination WST/3PBT [8], and a modified compact tension test (MCT) with circular [8], [9] or rectangular [10] cross section.

On the contrary, in this research a Brazilian disc specimen with a central notch (BDCN) used for fracture mechanical test has been analysed [11]-[13]. This is due to the fact that the BDCN specimen is made directly from the core-drill sample obtained from the investigation structure prior to renovation. That demonstrates its efficiency as a test and that the expensive process of the reshaping of the specimen is avoided.

The BDCN specimen is used in a fracture mechanical test that delivers parameters related to fracture mechanical behaviour of the cementitious material under uniaxial modes I, II and mixed-mode I/II loading. The experimental conditions, in which the test is performed are quite simple. It requires only a testing press with sufficient load capacity and some disposal container for cracked specimens.

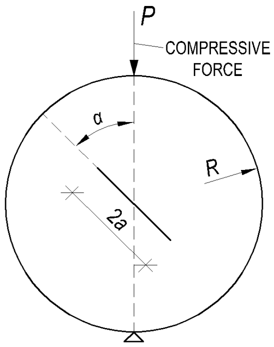

Fig. 1: Sketch of Brazilian disc test with a central notch and boundary/loading conditions.

The aim of this contribution is analysed crack initiation and propagation direction by using various well-known criteria [14] (see next section) in Brazilian disk with initiation notch. The initiation notch is under various angles of inclination α with respect to the horizontal axis, see Fig. 1. The theoretical path calculated by various criteria is compared with the experimental one [15],[16].

2. Theoretical Background

This contribution is based on a linear elastic fracture mechanics. The linear elastic fracture mechanics concept uses the stress field in the close vicinity of the crack tip described by Williams expansion [17]. This expansion is an infinite power series originally derived for a homogenous elastic isotropic cracked body, which can be described by a following equation:

$$\sigma_{i,j} = \frac{K_I}{\sqrt{2\pi r}} f_{i,j}^{I}(\theta) + \frac{K_{II}}{\sqrt{2\pi r}} f_{i,j}^{II}(\theta) + T + O_{i,j}(r,\theta), \quad (1)$$

where σ_{ij} represents the stress tensor components, K_I, K_{II} are the stress intensity factors (SIF) for mode I and respectively mode II, $f_{i,j}^{I}(\theta), f_{i,j}^{II}(\theta)$, are known shape functions for mode I and mode II usually written as Y_I and Y_{II}, T (or T-stress) represents the second independent term on r, O_{ij} represents higher order terms and r, θ are the polar coordinates (with origin at the crack tip; crack faces lie along the x-axis)[18].

There are several criteria, which predicts onset of fracture under mixed mode I/II. In this contribution, a single parameter fracture criterions are used to calculate crack initiation angle θ_0. For this, only $K-$ factors are used in angle θ_0 prediction. In total, three selected criteria are compared [14].

The first one is the maximum tangential stress (MTS) criterion proposed by Erdogan and Sih [19], which can be expressed in the following form:

$$\frac{\partial \sigma_{\theta\theta}}{\partial \theta}\Big|_{\theta=\theta_0} = 0, \quad \frac{\partial^2 \sigma_{\theta\theta}}{\partial^2 \theta} < 0. \quad (2)$$

Conventional MTS criterion uses only first terms (singular terms K_I, K_{II} are considered) in the series for $\sigma_{\theta\theta}$.

According to the first hypothesis of the MTS criterion, the angle of maximum tangential stress θ_0 is determined from Eq. (3) in following form:

$$[K_I sin\theta_0 + K_{II}(3cos\theta_0 - 1)] = 0 \quad (3)$$

that leads to:

$$\theta_0 = 2\tan^{-1}\frac{-2K_{II}}{K_I + \sqrt{K_I^2 + 8K_{II}^2}}. \quad (4)$$

The second criterion to predict onset of fracture was strain energy criterion (SED), which assumes the onset on fracture in the direction of the minimum strain energy density criterion, as postulated by Sih [20]. This condition can be written as shown in Eq. 5.

$$\frac{\partial S}{\partial \theta} = 0, \quad \frac{\partial^2 S}{\partial \theta^2} > 0, \quad (5)$$

where S is strain density function and it is defined as follows

$$S = \frac{1}{2\mu}\left[\frac{k+1}{8}(\sigma_{rr} + \sigma_{\theta\theta})^2 - \sigma_{rr}\sigma_{\theta\theta} + \sigma_{r\theta}^2\right], \quad (6)$$

where σ_{ij} are the stress tensor components in polar coordinates system, μ is the shear modulus and κ is constant postulated by Kolosov. After operations on the equation (5), the crack initiation angle for SED criterion can be expressed as:

$$\theta_0 = -\tan^{-1}\frac{2K_I K_{II}}{K_I^2 + K_{II}^2}. \quad (7)$$

In his research, Li defined [21], the vector crack tip displacement (CTD) criterion, which relies on the concept of the vector crack tip displacement. Crack tip displacement vector is the driving force for the crack growth of fatigue phenomenon. Considering the crack tip opening displacement vector δ_I, produced by Mode I loading, and the crack tip shear displacement vector δ_{II} produced by Mode II loading, the vector summation constitutes the CTD vector. Therefore, the crack propagates in the direction of the abovementioned vector, which can be calculated as illustrated in the Eq. 8.

$$\theta_0 = \tan^{-1}\frac{\delta_{II}}{\delta_I}. \quad (8)$$

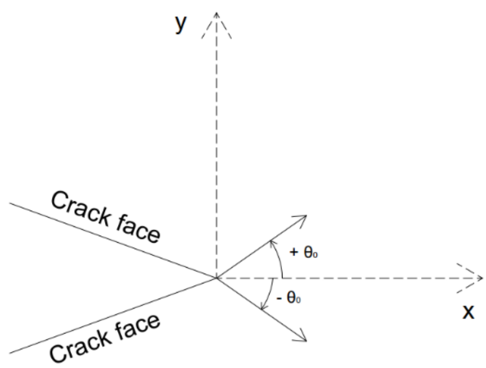

Fig. 2: An illustration of the calculated crack initiation angle θ_0 orientation.

3. Numerical Model

A numerical simulation was performed in the finite element (FE) software Ansys 17.2 [22] to derive dimensionless geometrical functions for mixed mode of SIFs and to calculate *T*-stress. A two-dimensional (2D) numerical model with a plane strain boundary condition was used to calculate SIFs (K_I and K_{II}). The numerical model was meshed with an 8-node quadratic element (type PLANE183) taken from ANSYS elements library and the meshing command KSCON was used to consider the crack tip singularity. Fig. 3(a) indicates the mesh pattern of the BDCN specimen while the refined mesh pattern around the specimen crack tip is shown in Fig 3(b).

Input material properties of used in FE analysis were following the Young's modulus and Poisson's ratio, $E = 40$ GPa and $\nu = 0.2$, respectively. Model of BDCN had radius R = 75 mm and the relative crack length a/R was 0.4. The numerical model was loaded with constant overall force P = 100 N in all cases.

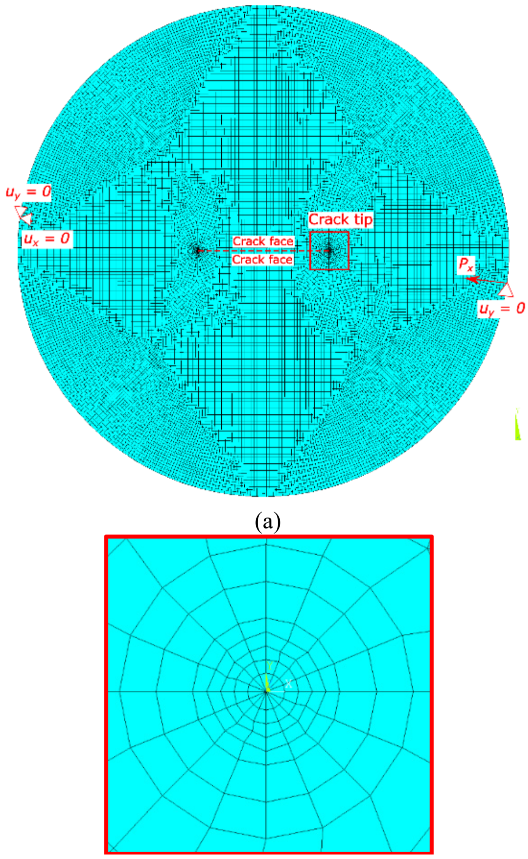

Fig. 3: Numerical model with boundary conditions (the 1st node from crack tip is in distance of ¼ element length due to application of KSCON command) (a) and detail on crack tip (b).

The numerical study was performed with a loading applied at the top edge of the BDCN, while the bottom edge was considered a rigid support. Adequate boundary conditions were added to prevent rigid body translations i.e. horizontal displacement $u_y = 0$ (See Fig. 3(a)).

The FE software ANSYS uses an interaction integral to calculate SIFs, which is defined as:

$$I = \frac{2}{E^*}(K_1 K_1^{aux} + K_2 K_2^{aux}) + \frac{1}{\mu} K_3 K_3^{aux}, \qquad (9)$$

where K_i is the stress intensity factor for mode I, II and III, K_1^{aux} is the auxiliary stress intensity factor for mode I, II and III, E^* is the Young's modulus for plane strain $E/(1-\nu^2)$, ν is the Poisson's ratio and μ is the shear modulus. For a 2D problem the SIF for mode II K_3 is 0. The interaction integral is implemented in command CINT (contour integral). The used SIFs and T-stress values are calculated as an average value of five contours defined in CINT command.

4. Numerical results

To produce a numerical crack pattern a loop was used, in which a SIFs were calculated for each step and crack initiation angle θ_0 was calculated. Then the geometry and mesh deletion in each step of the loop is used, i.e. the crack initiation angle θ_0 is calculated in step i for specific geometry and then the new geometry in step i+1 is created. The crack growth uses a constant increment of r = 1 mm. The loop is terminated, when the crack reaches the edge of the Brazilian disc. The crack increment follows conditions as it is illustrated in Fig. 2.

The generated numerical crack shape shows relatively good agreement with experimentally obtained crack in BDCN specimen published in [23]. The examples of crack patterns for the α = 15° and a/R = 0.4 are presented in Fig. 4 for CTD criterion, in Fig. 5 for MTS criterion and in Fig. 6 for SED criterion, while the actual crack pattern from the experiment mentioned in [23] is presented in Fig. 7.

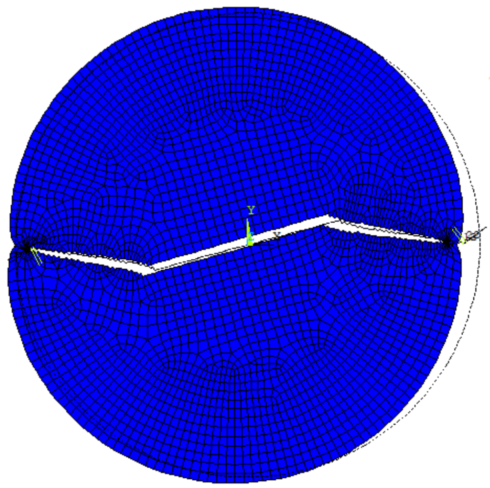

Fig. 4: BDCN specimen: CTD criterion crack pattern for α = 15°.

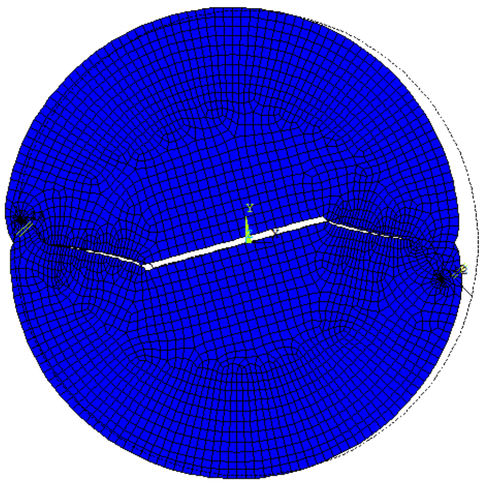

Fig. 5: BDCN specimen: MTS criterion crack pattern for $\alpha = 15°$.

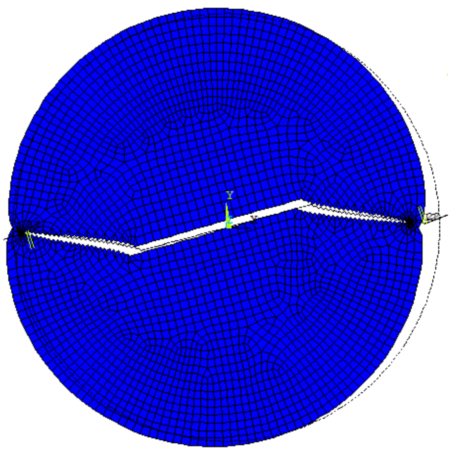

Fig. 6: BDCN specimen: SED criterion crack pattern for $\alpha = 15°$.

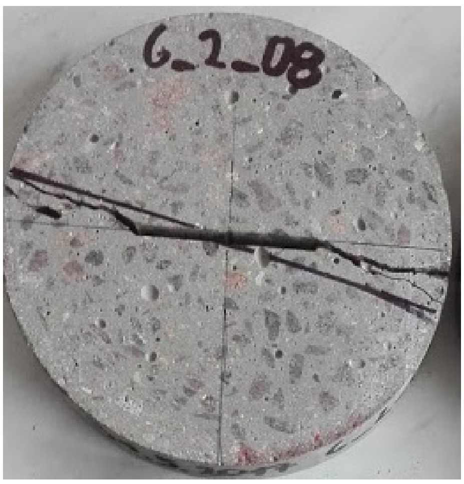

Fig. 7: Actual experimental BDCN test pattern for $\alpha = 15°$, see [23].

From numerical results presented in Figs. 4-6 it can be noticed, that the criteria CTD and SED predicts the crack path in the BDCN specimen in good agreement compared to actual crack pattern propagated in disc's radius. The MTS criterion predicts the crack pattern in the disc's radius up to crack length $a/R = 0.6$, then the crack starts to kink. This is caused by the sensitivity of MTS criterion for high-stress concentrations in front of the crack tip, from which the SIF are calculated.

5. Conclusion

In this contribution, three various fracture criteria for prediction of a crack propagation path were applied for crack path in the BDCN specimen. The predicted paths were compared with the experimental one. The following conclusion can be drawn:

- The SED and CTD criteria predicted the crack paths in relatively good agreement with experiment one.
- The MTS criterion provides relatively good results only for short crack lenghts.
- This should be taken into account in an assessment of such criteria in different geometries.

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References

[1] KARIHALOO B. L. *Fracture Mechanics and Structural Concrete* (Concrete Design and Construction Series), Longman Pub Group, 1995, ISBN-13: 978-0582215825

[2] SHAH, S.P. and A. CARPINTERI. Fracture mechanics test methods for concrete: report of Technical Committee 89-FMT Fracture Mechanics of Concrete, Test Methods, RILEM, *Chapman and Hall*, London; New York, 1991 ISBN 978-0442313838

[3] MALÍKOVÁ, L., V. VESELÝ, and S. SEITL. Crack propagation direction in a mixed mode geometry estimated via multi-parameter fracture criteria.

International Journal of Fatigue, 2016, vol. 89 pp. 99-107. ISSN 0142-1123 DOI: 10.1016/j.ijfatigue.2016.01.010

[4] LINSBAUER, H. and E. TSCHEGG. Fracture energy determination of concrete with cube-shaped specimens. *Zement und Beton*. 1986 vol. 31, pp. 38-40

[5] SEITL, S., S., KORTE, W., DE CORTE, V., BOEL, J., SOBEK and V. VESELÝ. Selecting a suitable specimen shape with low constraint value for determination of fracture parameters of cementitious composites. *Key Engineering Materials*. 2014, vol. 577-578, pp. 481-484. ISSN 1013-9826 DOI: 10.4028/www.scientific.net/KEM.577-578.481

[6] SEITL, S., P.MIARKA, V. RŮŽIČKA, L. MALÍKOVÁ and I. MERTA, Wedge Splitting Test: Displacement Field Analysis by Multi-parameter Fracture Mechanics. Transactions of the VŠB – *Technical University of Ostrava, Civil Engineering Series*, 2018, Vol. 18(2), pp. 50-55. ISSN: 1804-4824

[7] SEITL, S. P. MIARKA, I. MERTA and Z. KERŠNER, Numerical Stress Analysis of the Biaxial Tension-Compression Wedge-Splitting Test in Vicinity of the Crack Tip. *Key Engineering Materials* Trans Tech Publications, Switzerland, 2018. pp. 85-90. ISBN: 978-3-0357-1242-1. ISSN: 1662-9795

[8] CIFUENTES, H. M. LOZANO, T. HOLUŠOVÁ, F. MEDINA, S. SEITL and A. FERNÁNDEZ-CANTELI, Modified Disk- Shaped Compact Tension Test for Measuring Concrete, Fracture Properties. *International Journal of Concrete Structures and Materials*, 2017, Vol. 11(2) pp. 215-228. ISSN: 2234-1315

[9] FERNÁNDEZ-CANTELI, A. L. CASTAÑÓN, B. NIETO, B. M. LOZANO, T. HOLUŠOVÁ, and S. SEITL, Determining fracture energy parameters of concrete from the modified compact tension test. *Frattura ed Integrita Strutturale*, 2014, Vol. 8(30), pp. 383-393. ISSN: 1971-8993.

[10] SEITL, S., J.D., RÍOS, H., CIFUENTES, and V. VESELÝ. Effect of the load eccentricity on fracture behaviour of cementitious materials subjected to the modified compact tension test. *Solid State Phenomena*. 2017, vol. 258, pp. 518-521. ISSN 1662-9779 DOI: 10.4028/www.scientific.net/SSP.258.518

[11] LI, D. and L.N.Y. WONG. The Brazilian disc test for rock mechanics applications: Review and new insights, *Rock Mechanics and Rock Engineering*. 2013, vol. 42, iss. 2, pp. 269-287. ISSN 0723-2632 DOI: 10.1007/s00603-012-0257-7

[12] HOU, CH., Z., WANG, W., LIANG and J. LI. Determination of fracture parameters in center cracked circular discs of concrete under diametral loading: A numerical analysis and experimental results. *Theoretical and Applied Fracture Mechanics*. 2016 vol. 85 pp. 355-366. ISSN 0167-8442 DOI: 10.1016/j.tafmec.2016.04.006

[13] MARTÍNEZ-LÓPEZ, M., G., MARTÍNEZ-BARRERA, L.C.S., NUNES, J.M.L., REIS and H.S., DA COSTA MATTOS. Mixed mode fracture analysis in a polymer mortar using the Brazilian disk test. *Engineering Fracture Mechanics*, 2016, vol. 154, pp. 140-151, ISSN 0013-7944 DOI: 10.1016/j.engfracmech.2016.01.007

[14] QIAN, J. and A. FATEMI. Mixed mode fatigue crack growth: a literature survey. *Engineering fracture mechanics*. 1996, vol. 55, pp. 969–990. ISSN 0013-7944 DOI: 10.1016/S0013-7944(96)00071-9

[15] MALÍKOVÁ, L. Multi-parameter fracture criteria for the estimation of crack propagation direction applied to a mixed-mode geometry. *Engineering Fracture Mechanics*. 2015, vol. 143, pp. 32–46. ISSN 0013-7944 DOI: 10.1016/j.engfracmech.2015.06.029

[16] MALÍKOVÁ, L. and V. VESELÝ, Application of multi-parameter fracture mechanics to study of crack propagation angle in selected mixed-mode geometry, *Key Engineering Materials*. 2014, vol. 592-593, pp. 209–212, ISSN 1013-9826 DOI: 10.4028/www.scientific.net/KEM.592-593.209

[17] WILLIAMS, M.L. On the Stress Distribution at the Base of a Stationary Crack. *Journal of Applied Mechanics*. 1956, Vol. 24. ISSN 0021-8936

[18] ANDERSON, T.L. *Fracture mechanics: fundamentals and applications*, CRC press, 2017. ISBN 978-0849316562

[19] ERDOGAN, F. and G.C. SIH. On the Crack Extension in Plates Under Plane Loading and Transverse Shear. *Journal of Basic Engineering*. 1963, vol. 85, pp. 519-525. ISSN 0098-2202 DOI: 10.1115/1.3656897

[20] SIH. G.C. Strain-energy-density factor applied to mixed mode crack problems. *International Journal of Fracture*. 1974, vol. 10, pp. 305-321. ISSN 0376-9429 DOI: 10.1007/BF00035493

[21] LI. C. Vector CTD criterion applied to mixed mode fatigue crack growth. *Fatigue Fracture of Engineering Materials and Structures*. 1989, vol. 12, pp. 59-65. ISSN 1460-2695 DOI: 10.1111/j.1460-2695.1989.tb00508.x

[22] ANSYS®, Academic research, Crack analysis guide Mechanical APDL Documentation guide, Release 19.1., 2018

[23] SEITL, S., P. MIARKA and V. BÍLEK. The Mixed-Mode Fracture Resistance of C 50/60 and its Suitability for Use in Precast Elements as Determined by the Brazilian Disc Test and Three-Point Bending Specimens. *Theoretical and Applied Fracture Mechanics*. 2018 vol. 97 pp. 108–119. ISSN 0167-8442 DOI: 10.1016/j.tafmec.2018.08.003

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