Legion Bridge in Prague – Assessment of Stone Arches

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Abstract. *The article deals with available engineering assessment methods for the stone vault bridges according to relevant standards. The article summarizes variable methodologies used for design and assessment of masonry vaults in last period. All methods were applied for assessment of the Legion Bridge over Vltava River in Prague and results were compared. The models were validated by results of static load test.*

Keywords

Masonry arch bridge, Vault, Bridge, Assessment, Load carrying capacity, Thrust line

1. Introduction

When assessing existing bridges, the requirement for sufficient mechanical resistance and stability is usually given by the maximum load that the structure is able to carry safely - the maximum weight of the road vehicle which can pass the bridge under the specified conditions, i.e. the load carrying capacity.

It is necessary to know the actual material characteristics of the carrying elements of the structure and the geometry of the structure as a basis for determining the load-carrying capacity. Degradation of the structure in the course of time changes both the material and geometric parameters of the carrying and non-carrying elements of the bridge. Therefore, finding out the degree of degradation is critical to determine the load-carrying capacity. Determining actual defects of the bridge, the degree of degradation and the real bridge geometry are objective of diagnostics.

The load carrying capacity is influenced by the fact that the bridge structures were designed to be loaded according to the standards valid at the time of bridge construction. The requirements of load to be transferred by the bridges are increasing. The Legion Bridge was designed to be loaded by a herd of cattle, and a load test of a cattle herd was also carried out on the bridge of Emperor Francis, which is name of the Legion Bridge at the time of completion. Therefore, a number of historic structures do not have the load carrying capacity that the administrator of bridge wants (even in state without defects and degradation).

Determination of the load bearing capacity of the existing bridge today is governed by the same principles as the design of new structure (see applicable technical standards and regulations EN, DIN, and MVL).

In case of masonry vault structures this task is complicated by the structural behaviour and typical property of the material - a negligible tensile strength, see [1] and [9]. Due to these facts, the procedures for determination of the load carrying capacity are non-linear and must involve a large number of parameters with significant variability. Therefore, it is very difficult to set up a simple analytical model and special programs developed directly for vault structures are usually used.

In case of road bridges three kinds of load carrying capacity (according to [6]) can be calculated :

- Vn - normal load carrying capacity of the bridge – represents the maximum weight of one typical lorry, which may pass the bridge without any restrictions (position limited by safety barriers only).

- Vr - exclusive load carrying capacity – represents the maximum weight of one truck, which can pass the bridge as single vehicle in any position or lane respectively (no other traffic loads except pedestrians is permitted). It can move anywhere on the bridge.

- Ve - exceptional load carrying capacity - which represents the maximum weight of special vehicle, which can pass the bridge under special conditions (specified velocity, specified path and eccentricity).

Determination of each load carrying capacity presents separate calculation in both limit states (SLS and ULS) considering all other relevant loads (temperature, wind, flood etc.).

The calculation of load carrying capacity was carried out using the results of diagnostics according to chapter 3. The conditions of reliability used to verify the structure see in chapter 4. The methods of determining the internal forces and stresses see in chapter 5.

2. Brief history and description of the bridge

The Legion bridge connects the Old town with the Lesser town of Prague through the Střelecký Island. The foregoer bridge of Legion Bridge was Bridge of Emperor František and it was second bridge in Prague finished in 1841. The Legion Bridge was constructed at the position of Bridge of Emperor František. The construction of new bridge ran from 1898 to 1901. In contrast to the original bridge, the superstructure is made of a massive stone vault structure. The construction of the bridge consists of nine flat vaults of different spans: $26.6+34.3+38.5+42.0+27.8+27.8+31.9+28.7+25.6\ m$. Two vaults above the Střelecký Island are vaults of circular segments, the other vaults are elliptical. The construction of the bridge is made of granite blocks with a gap of $12-15\ mm$ filled with cement mortar. The stone facade of front walls of light sandstone and red granite symbolizing national colors. The bridge has wide footways and motorway lanes, electric tracks were also built on the bridge, trams run along an existing bridge from June 17, 1901.

The Legion Bridge is an immovable cultural monument and is therefore protected according to the provisions of law On State Monument Care, as amended. Due to the fact that it is a building located in the territory of the Prague Historical Reserve (PPR), the provisions of the Government Decree On the Historical Heritage in the Capital City of Prague. The historical monument in Prague, representing the historical center of Prague, was included in the UNESCO World Heritage List in 1992.

3. Diagnostics of the bridge

In order to find the geometric and material properties of the bridge, following procedures were carried out. For detailed results see [11]:

1. detailed mapping of all visible damages,
2. geodetic survey,
3. geotechnical boreholes,
4. core boreholes from the superstructure (getting E, f_b, f_m),
5. georadar survey of thickness of arches,
6. continuous measurement of temperatures,
7. diving survey of substructure,
8. static and dynamic load test – see chapter 7.5.

3.1. Detailed mapping

As an example of detailed mapping of visible damages, which was done together with the acoustic test of degradation of stones of arches, following figure is shown:

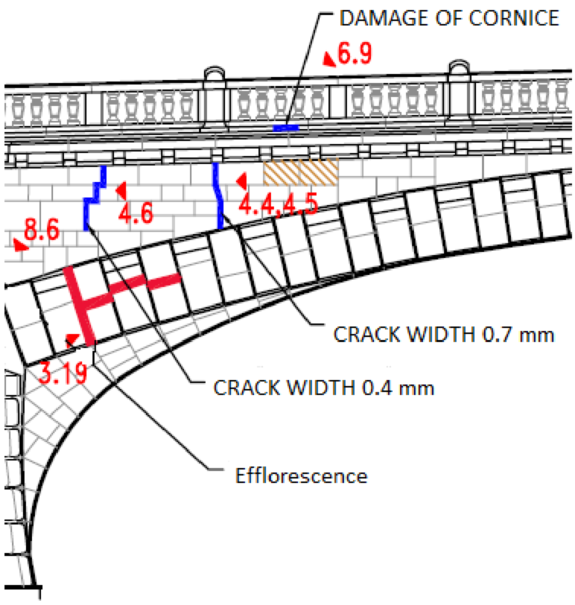

Fig. 1: Mapping of visible damages in the spandrel wall near pier No. 2

Because of the cracks in the spandrel walls, the wall is considered just as a load in the models, not as a carrying element.

3.2. Geotechnical boreholes

Piers number 2, 3, 4, 6, 8 were drilled from the bridge deck to the foundations. From that, we get the information about thickness and material properties of asphalt layers, backfill, masonry of piers and foundation ground. It was found out, that masonry of pier has much lower strength, than the superstructure, see chapter 3.3.

3.3. Core boreholes

Compressive strength was tested on 34 granite specimens from superstructure. The measured average strength was $f_b = 132\ MPa$. Mortar of superstructure has average strength 25.8 MPa, tested on 114 specimens.

Compressive strength was tested on 131 granite and sedimentary rock specimens from substructure. The measured average strength was $f_m = 86\ MPa$. Mortar of superstructure has average strength 25.2 MPa, tested on 169 specimens.

The characteristic compressive strength of masonry

$$f_k = K f_b^{\alpha} f_m^{\beta}, \tag{1}$$

was calculated 28.9 MPa for superstructure and 22.8 MPa for substructure (which are very high values),

where:

K is coefficient depending on type of masonry and masonry elements,

α is coefficient depending on type joints and mortar,

β is coefficient depending on type of mortar.

The Young modulus of granite of superstructure was found 41.3 GPa on 17 specimens, Young modulus of granite of substructure was found 31.1 GPa on 29 specimens.

3.4. Georadar survey of thickness of arches

The Georadar method (GPR) is based on the principle of transmitting high-frequency electromagnetic waves into the examined environment and then registering the wave image of reflected waves. The wave image is affected by local inhomogeneities, especially with different conductivity and other electromagnetic properties. Inhomogeneities can have both planar character (discontinuity surfaces, structural interfaces) and local character (eg, cavities, etc.). Inhomogeneities are manifested by amplification of the registered reflected signal amplitude. In Figure 2 there are blue lines representing the arch shape taken from geodetic survey. The yellow area and white lines are taken from the Georadar method.

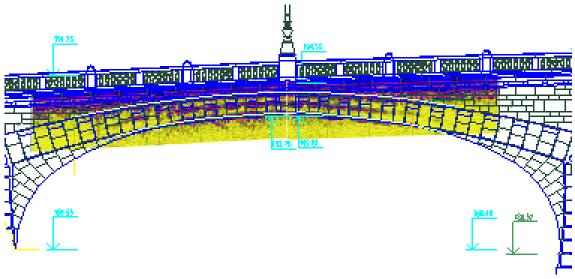

Fig. 2: Georadar method in span No. 9

Measuring in longitudinal direction of the bridge showed good agreement with the archive documentation. On the vault of span 7 was found $0.15 - 0.2\ m$ concrete layer, which was not expected. Measuring in transverse direction showed constant thickness, which was expected.

4. Limit states

4.1. Ultimate limit state

In the ultimate limit state (ULS) the behaviour of the structure just before the collapse is investigated. For the bearing capacity determination load factors according to appropriate EN are considered (1.35 for dead load and 1.35 for live load). Generally, it is assumed that plastic hinges are fully developed through the structure. For details on masonry arch bridges behaviour see in [1] and [2]. Resistances for axial and shear forces at the ULS can be written according to [3] as:

$$N_{Rd} = f_d b \left(h - 2e_u\right), \tag{2}$$

$$V_{Rd} = \left(f_{vk0} + 0.4\sigma_d\right) b \left(h - 2e_u\right) / \gamma_M, \tag{3}$$

where: f_d is the design strength of masonry in compression, f_{vk0} is the characteristic value of initial shear strength at normal stress equal to 0, b, h is width or height - respectively, e_u is the eccentricity of the resultant axial force in cross-section at the ultimate limit state, σ_d is the design compressive stress in the compressed area at the ultimate limit state (the stress is uniformly distributed, see Figure 3), 0.4 is the coefficient of friction in the masonry joint, γ_M is the factor of the material.

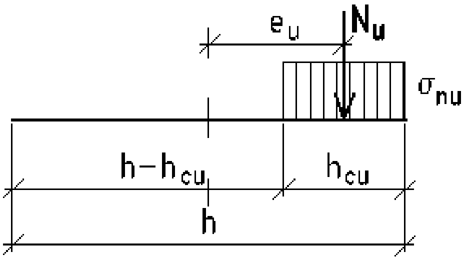

Fig. 3: Stress distribution of masonry cross-section at the ultimate limit state.

4.2. Serviceability limit state

The serviceability limit state (SLS) describes the behaviour of the structure under ordinary operating conditions. Fulfilling the conditions of serviceability limit state provides the required properties and behaviour of the structure throughout its lifetime. In terms of serviceability limit state, crack width and structural stress under operating load are verified (load factor equals to 1.0). In terms of verification of vault structures, it is necessary to verify the maximum axial stress in the cross-section and the height of the compressed area at the cross-section (see [4] and [3]). Elastic behaviour of the structure is considered with a linear distribution of axial stress in the compressed area of the cross section. Tensioned part of the section is excluded for stress determination (see Figure 4):

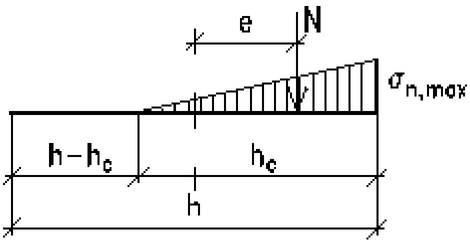

Fig. 4: Stress distribution of masonry cross-section at the serviceability limit state.

$$\sigma_{n,max} = \frac{N_{Ek}}{3b\,(h-2e)} \leq 0.45 f_k, \quad (4)$$

$$h_c \geq \frac{h}{2} \longrightarrow e \leq \frac{h}{3}, \quad (5)$$

$$e = \frac{M_{Ek}}{N_{Ek}}, \quad (6)$$

where: M_{Ek}, is characteristic moment caused by load, N_{Ek} is characteristic normal force caused by load.

5. Methods of calculation - arch

1. Graphical method (controls all the requirements)
2. Linear calculation (controls all the requirements)
    (a) Beams – 2D or 3D
    (b) Plane-stress elements – 2D
    (c) 3D solid elements
3. Non-linear calculation (controls SLS requirements)
    (a) 2D – plane
    (b) 3D – solid elements – not used in this article
4. Equilibrium method — LimitState:Ring (controls only collapse of the structure)

5.1. Graphical methods

Various graphical methods had been used until computer aided design came to engineering practice. It provides very simple and quick design approach independent on arch bridge shape. Graphical methods are based on the thrust line determination. Thrust force at the cross section can be found as centroid of the axial stress diagram. When the thrust line is known, stress at the cross-section can be calculated as well. The method of finding the thrust line runs in following order (symmetric arch according to [5]):

1. Divide the arch in 2 symmetric parts, find the weight of one half and from the geometry of arch we find force H (horizontal force in the top of arch)
2. Divide one half of arch into several partitions (vertical lines can be used to divide)
3. Draw graphical representation of all parts – Fi - size of vector in chosen scale, acts in its centroid
4. Force H we locate for example in the upper bound of cross section core and reaction in the lower bound of cross section
5. For getting the resulting force R1 in first partition, we graphically add the force Fi to H, for getting next resulting forces in each partition, we graphically add the forces Fi to previous resulting force

Basic principle of calculation is shown in Fig.5 and [8].

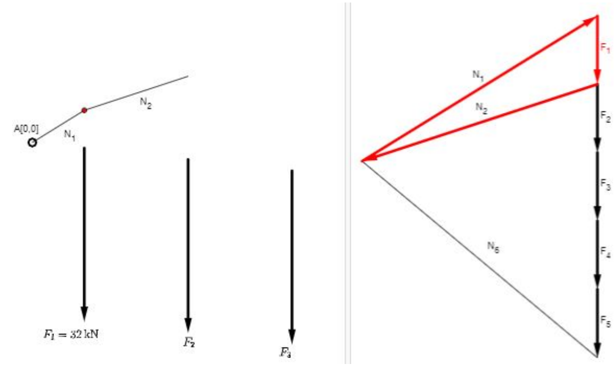

Fig. 5: Basic principle of the graphical method.

5.2. Linear calculation

Structural analysis using linear calculation method was done in two different options. In the first one the arch was represented by sequence of beams in its center line. The backfill was represented by vertical beams provided in appropriate longitudinal distance, see Figure 6. While modelling using linear calculation, one should not forget, that this calculation method doesn't take into account material non-linearity (and geometry changes due to excluding the tensioned part of cross section). For the second option the arch and backfill were modelled by the 3D solid elements with various mechanical properties – see Figure 11.

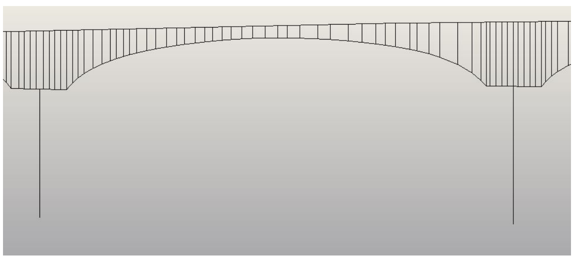

Fig. 6: Linear 2D beam model.

5.3. Non-linear calculation

For performing an non-linear analysis and assessment of the structure, the material – mortar and masonry elements - is homogenized to preserve its properties in relation to the real behaviour of the structure or its part. It is assumed that the dimensions (thick) of the masonry elements and joints between them do not significantly affect the distribution of stress in the masonry element. The real stress-strain diagram of the masonry shows non-linear behaviour (see [1]) particularly due to negligible tensile strength. In this article, it is considered that the material acts only in compression and when the tensile stress occurs, cracks open up, see Figure 7). If, subsequently, (e.g. in another load combination) the tensile stresses in the cross section disappear, the cracks close and the cross-section acts again as full.

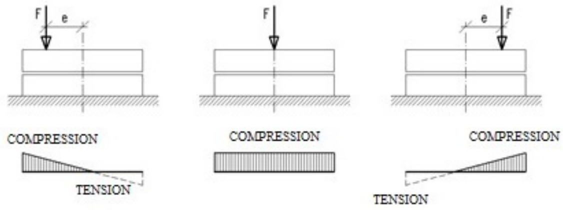

Fig. 7: Non-linear behaviour of masonry.

The structural model in program Midas was prepared using plane-stress elements, modelling the joints between the granite blocks as set of elastic links with the property "Compression only", see Figure 8.

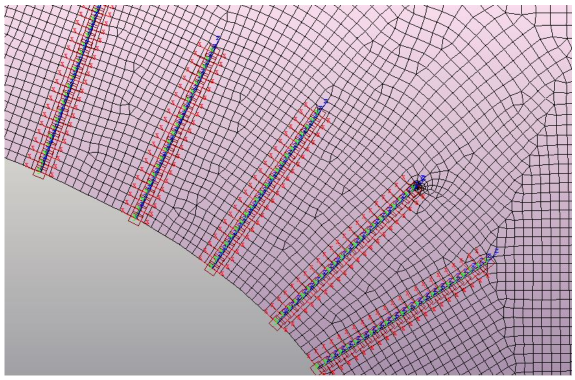

Fig. 8: Elastic links between nodes in joints of masonry.

5.4. Equilibrium method on rigid blocks

LimitState:RING is a special analysis software for checking the load bearing capacity of the vault in the plane of the longitudinal section of the bridge structure, including the load distribution by the backfill. It uses equilibrium equations on the parts of vault act as a rigid bodies, the structure is divided into rigid bodies depending on forming of the plastic hinges in locations with the lowest height of compressed area. Load distribution is considered according to Bousinesq, see example in Figure 9. For more details see [7].

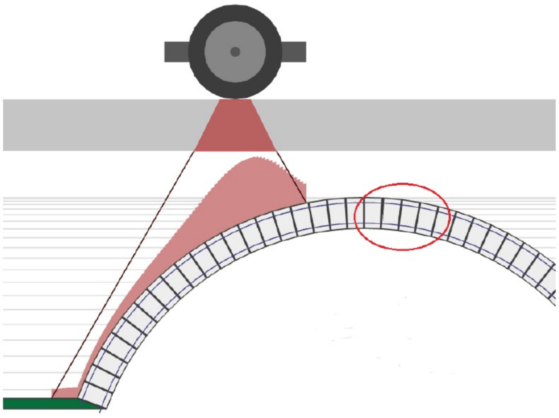

Fig. 9: Vertical traffic load distribution (dispersion) to the vault considered for arch modelling in LimitState:RING software.

6. Methods of calculation - transverse direction

6.1. Linear calculation

1) Beam elements

Modelling using this method is done by representing the arch by beams in its middle line and dividing the backfill in chosen interval to represent it by beams as well, see Figure 6. If the model shown in Figure 6 is copied several times in the transverse direction the analysis model representing the arch as a body can be arranged. The stiffness of transverse beams is chosen as a stiffness of arch. Such 3D linear model can be used to study the effect of eccentricity of live load on the bending moments distribution in the transverse direction – see Figure 10.

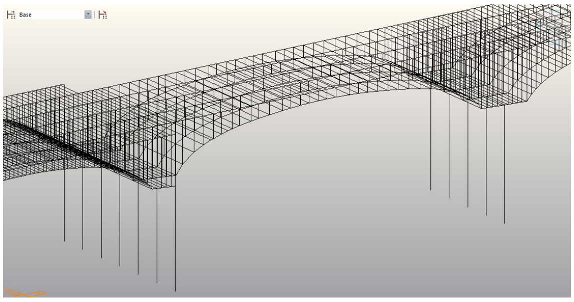

Fig. 10: Linear 3D beam model.

2) Solid elements

Another option for linear analysis is to model the body of the structure by the 3D solid elements – see Figure 11. Both longitudinal and transverse direction can be modelled by this way.

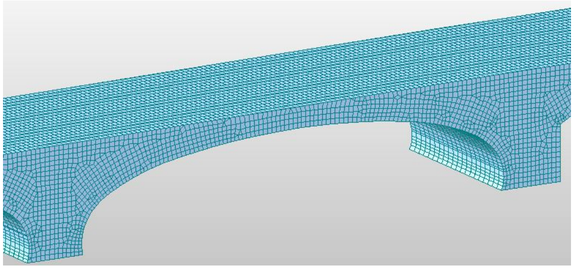

Fig. 11: 3D solid model.

6.2. Effective width

Principles of "modelling" of the bridge span 4 in transverse direction using effective width can be seen from Figure 12. This way of modelling is used in most codes. It considers conservative idea, that non-loaded lane of arch doesn't carry any load. So the shear and bending stiffness between loaded and non-loaded elements is considered equal zero.

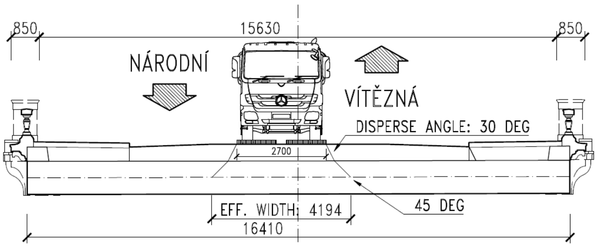

Fig. 12: Calculation of effective width.

7. Results

7.1. Graphical method

Result of graphical method for span 4 are shown in Figure 13.

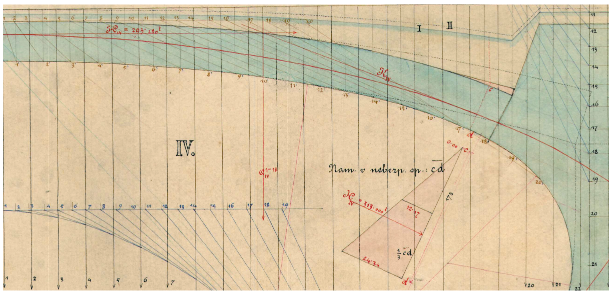

Fig. 13: Graphical solution of the span 4 from the archive documentation.

7.2. Linear calculation

The bending moments on the beams are shown in Figure 14.

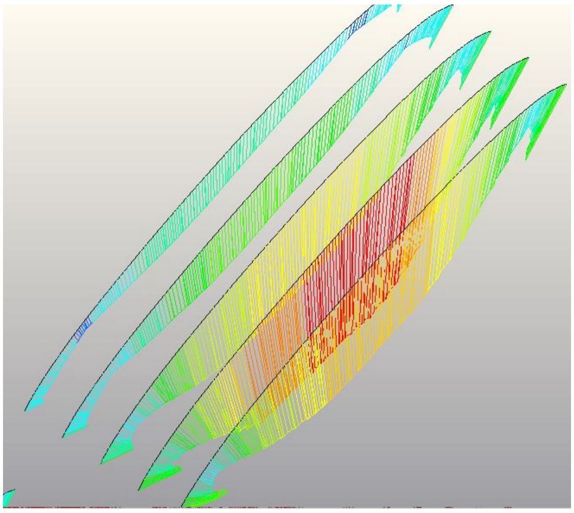

Fig. 14: Bending moment on the beams representing arch in longitudinal direction.

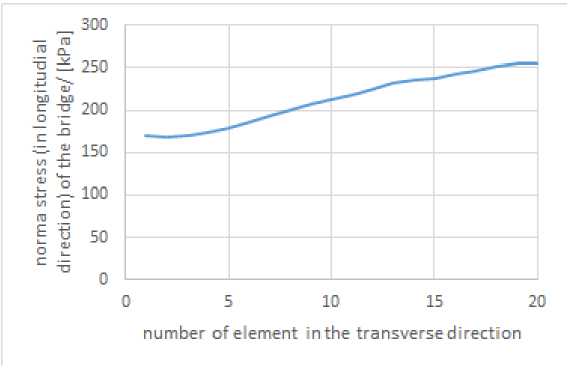

Fig. 15: Resulting stress from the load of exclusive load model.

7.3. Non-linear calculation

Principal stress in the arch of span 4 can be seen in Figure 16.

The legend of curves in the Figure 17 is following:

- *sw* means self weight,
- N means non-linear combination,
- *ohr* means heat-up,
- *och* means cooling,
- $4VnT!$ means load by live load.

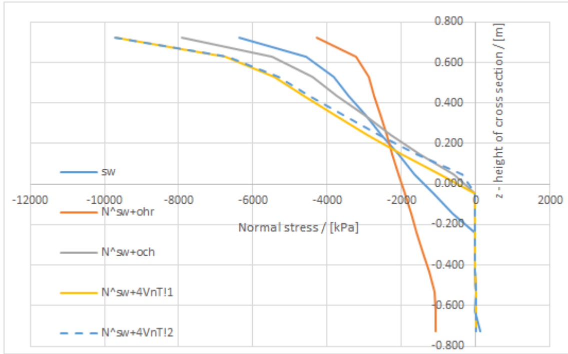

Fig. 17: Normal stress distribution in the middle of span 4 versus the cross section height.

7.4. Shear between the blocks

As a demonstrations of failure mode, picture of span 4 from program LimitSte:RING is attached:

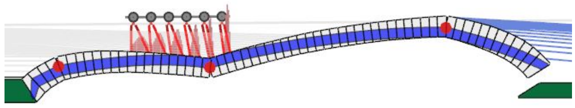

Fig. 18: Failure mode of span 4

The resulting shear force acting on the cross section depends on the geometry of the arch. In beam model, it depends on chosen geometry of axis of the beams. In solid model, it depends on choice of material model – linear or non-linear. In Fig. 20 can be seen, that changes in geometry according to material non-linearity affects resulting shear force highly. Figure shows resulting shear force from the linear beam model with changed axis near the bottom of arch.

7.5. Static load test

Six vehicles of weight $31.6, 31.2, 31.7, 31.5, 31.7, 31.5\ t$ were used for the static load test. Test was carried out according to [10] in spans 3, 4, 5, 6. In this article, only results of span 4 are presented. The measured displacements are quite small in comparison to preciseness of measurement, which is equal $0.25\ mm$. Four functions of transverse vertical displacement are compared in the Figure 21:

1. linear beam model,
2. linear solid model,
3. "non-linear beam model",
4. "non-linear solid model".

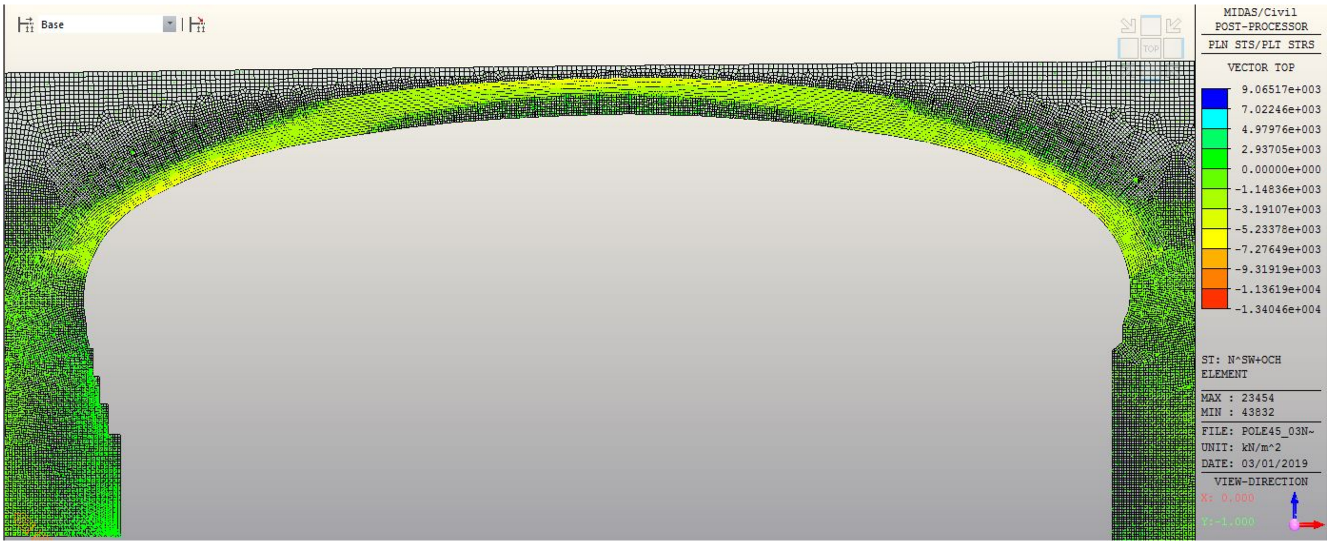

Fig. 16: Principal stress from the non-linear model.

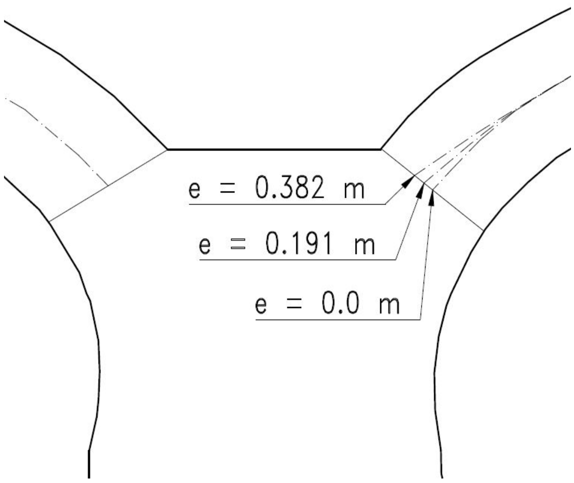

Fig. 19: Changes of geometry of beam axis depending on eccentricity of load

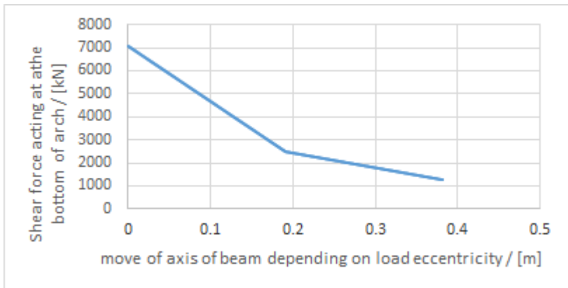

Fig. 20: Resulting shear force depending on eccentricity of load

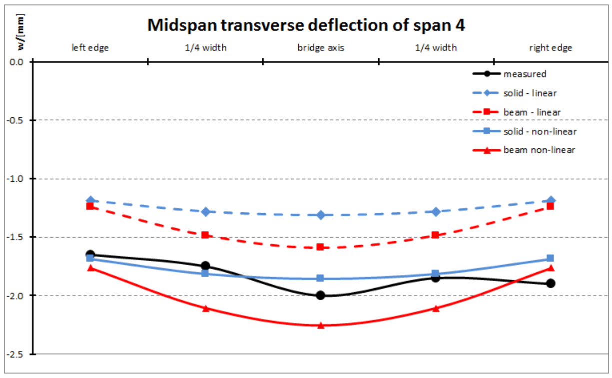

Fig. 21: Displacement comparison

Results from the effective width calculation is not presented in the plot, because it is evident, that it is very conservative – resulting displacement on the left and right edges would be zero, which is not true. The function "non-linear beam model" and "non-linear solid model" was created by multiplying the linear result by k_{non}, which was calculated as $k_{non} = w_{non-linear}/w_{linear}$, where $w_{non-linear}$ is displacement from the 2D non-linear model loaded by self weight and w_{linear} is displacement from the linear solid model loaded by self weight.

The resulting "non-linear" calculated displacements are in the range of measured displacements +- preciseness (which is equal to 0.25 mm). The results approve the non-linear behaviour of the bridge.

8. Comparison of methods and discussion

8.1. Self weight – the main load

In the Figure 22 we can see the eccentricity from the linear, non-linear model and from graphical solution from the archive documentation. It is evident, that eccentricity from the non-linear model is higher, as expected, because tension, which is allowed in the linear model pushes the resultant thrust line to the centroid of the cross section. The non-linear analysis is more time consuming (on the effort of the engineer as well as the effort of the computer), but the real behaviour of the structure is better described by the non-linear model (see the behaviour of masonry in Figure 7 and 21). The results from the non-linear model are more dangerous and are closer to the limit states.

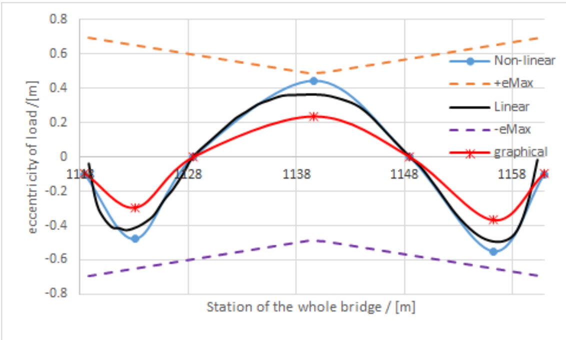

Fig. 22: Comparison of linear, non-linear and graphical method for span 4.

8.2. Traffic load and its distribution in transverse direction

Load distribution in the transverse direction was compared on three models – 3D solid, 3D beam model and effective width model. Result can be found in Figures 14, 15 and 12. 3D linear solid model assumes linear behaviour in all directions, therefore gives the most non-conservative results. The most conservative model is effective width model, because it entirely excludes part of cross section. The real behaviour is somewhere between. The beam model gives results between the two mentioned method, in opinion author is therefore the most real. The beam model is much simpler and the stiffness in transverse direction can be easily changed. Real 3D solid model (which considers non-linear behaviour) is complicated with the fact, that we don't know many crucial characteristics of masonry – such as bending and shear stiffness of mortar between the blocks – neither in longitudinal nor the transverse direction. That is the reason the 3D solid model is recommended just for special structures and if we know the parameters.

8.3. Final results of load carrying capacity – normal stress

The final results of load carrying capacity reflects both the results of modelling the arch itself and modelling of transverse direction.

Tab. 1: Resulting load carrying capacity – normal stress. (m. means model)

Arch m.	Transverse m.	Vn	Vr	Ve
Linear	3D beam	41	122	230
Non-linear	effective width	32	83	185
RING	effective width	46	105	182

It is in general known, that LimitState:RING gives non-conservative results. Non-linear model gives conservative results, first because of the method of assessing the load distribution in transverse direction, second because of modelling of the arch itself. The linear model results are between two mentioned method. For Ve and Vr, the beam model gives the most non-conservative results. The load is concentrated to small strip of the arch in effective width model, but in 3D beam model all the beams carry part of the load.

8.4. Final results modelling – shear stress

When calculating the resistance of masonry blocks to the shear, crucial is friction coefficient. In LimitState:RING, default value 0.6 is considered (according to laboratory tests, see [12]). For comparison, values from 0.597 to 0.705 were obtained according to [13]. According to [3], coefficient 0.4 should be used. For the comparison, the coefficient 0.4 is used for all models.

Tab. 2: Resulting load carrying capacity – shear stress. (m. means model)

Arch m.	Transverse m.	Vn	Vr	Ve
Linear	3D beam	0	0	0
Non-linear	effective width	42	111	240
RING	effective width	35	79	161

As can be seen in Figure 20 and 19, geometry changes are crucial and model, that doesn't consider that, gives unreal shear forces. This is the case of linear beam models of vaults of low sagitta.

9. Conclusion

Several models of the Legion bridge were carried out. Results were validated by the static load test. Modelling of arches showed, that the most real behaviour of arches describes the non-linear model, because it considers the non-linearity, which impacts the calculation the most – negligible strength in tension. The results from non-linear modelling are non-conservative in comparison to other methods. Modelling of load distribution in transverse direction showed, that 3D solid model gives upper bound (non-conservative), effective width gives the lower bound (conservative results) and 3D beam model is somewhere between, which is the the most real behaviour. However, using effective width is precise enough for small spans (spans of 90% of stone arch bridges are lower than 10 m), but leads to very non-conservative results for such a bridge with very large span (span of Legion bridge is the largest in Czech Republic). For modelling of such a large spans therefore other 3D models should be made.

Verification of shear resistance showed also importance of considering the material non-linearity. The shear forces resulting from linear beam model, which doesn't consider the non-linearity showed to be unreal, the structure collapses even loaded by self-weight. Non-linear model and LimitState:RING model are in good agreement, considering, that weight of models of traffic loads (Vn, Vr, Ve) is quite small in comparison to self-weight.

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