THE PURE BENDING TASK IN CASE OF COMPOSITE ROD BASED ON FOUR-ELEMENT MODEL

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Abstract. In the presented paper is stated distribution of deformations and stresses development at external constant force moment in the rod with taking into account geometric characteristic as well as the time factor based on the four-element model. The values of the curvature and rotation angle are estimated. The values of deflection of rod with dependence on its coordinate and time are determined. The behavior and distribution of arisen in the cross-section of rod stresses at constant deformation (the case of relaxation) depended on the time is also determined.

Keywords

Bending, stress, deformation, relaxation.

1. Introduction

Is stated the rather general description of composites hardening on the polymer binding. So briefly, description of hardening process is necessary for illustration of problem under study and understanding of its mechanics are stated the examples of non-symmetrical layered composite's curved plates. The theory that gives the possibility to analyze orthogonally reinforced non-symmetric layered composites deformations at cooling at temperature below of hardening temperature are stated. Are described the numerical examples that reflects the impact of temperature, properties of material of layer and geometry of plate. When it is possible the results of calculation are compared with data of carried out by classic author's experiments. Composites with fiber matrix and ebony (organic) filler are characterized with complex nature of deformation. This is also expressed even in the case of constant loading by developed over time deformations, also occurs the creeping phenomenon. They are also characterized by relaxation (at constant deformation arisen in the cross-section stresses are decreasing over time). The theoretical theory is often used to describe the properties of such materials that represent certain combinations of pure elastic and viscous elements. The simplest cases of models are presented by two-dimensional Maxwell and Kyle-Voigt models [1, 2], although they cannot even qualitative describe the characteristics of these composites. Much better results are achieved by consideration of generalized models in that the quantity of simple elements is more than two.

2. Basic part

The raised in the presented case task is considered as four elements (two elastic and two viscous) generalized models (Fig. 1). Its main characteristic is the existence of a single binding element [3, 4, 7]. Conditionally it represents a combination of the connecting in parallel of Maxwell and Kelvin-Voigt models.

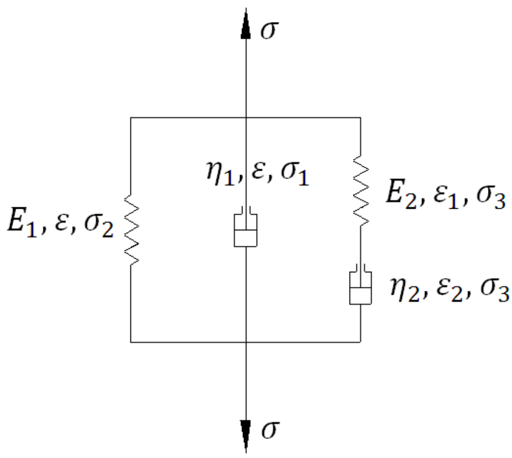

Fig. 1. Four element model with elastic binder element

The linear task of such model deformation is reduced to differential dependence, generation of the rheological equation (1) that connects the total stress and deformation.

$$\frac{\mathrm{d}^2\varepsilon}{\mathrm{d}t^2} = \frac{1}{\eta_1}\frac{\mathrm{d}\sigma}{\mathrm{d}t} - \frac{(E_1+E_2)\eta_2 + E_2\eta_1}{\eta_1\eta_2}\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} + \\ + \frac{E_2}{\eta_1\eta_2}\sigma - \frac{E_1E_2}{\eta_1\eta_2}\varepsilon. \quad (1)$$

At consideration of (1) slow and fast deformation is possible to neglect the corresponding derivatives and free members. In such case, simple dependencies are obtained: $\sigma = E\varepsilon$ at slow deformation and $\dot{\sigma} = H\dot{\varepsilon}$ at fast deformation [1, 2]. With taking into account (1), we will obtain for the long-term and instantaneous modules of elasticity:

$$E = E_1 \text{ and } H = \frac{(E_1+E_2)\eta_2+E_2\eta_1}{\eta_2}, \quad (2)$$

with additional designations:

$$n = \frac{\eta_2}{E_2} \text{ and } \eta = \eta_1. \quad (3)$$

Due consideration of (2) and (3) expression (1) will be as following:

$$\frac{\mathrm{d}^2\varepsilon}{\mathrm{d}t^2} = \frac{1}{\eta}\frac{\mathrm{d}\sigma}{\mathrm{d}t} - \frac{H}{\eta}\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} + \frac{1}{\eta n}\sigma - \frac{E}{\eta n}\varepsilon. \quad (4)$$

The equation (4) would be solved at a particular case of deformation or stress, for example: $\sigma = \sigma_c = \text{const}$ is corresponding to the case of creeping and $\varepsilon = \varepsilon_c = \text{const}$ – case of relaxation [1, 2]. At this time, are obtained specific analytical solutions that more or less well are describing the actual characteristics of such composites.

Using the four-element generalized model, the task of the pure bending of composite rod will be reduced to the task of linear deformation for each layer. On the Fig. 2 is presented bending scheme of having rectangular cross-section rod, where *mn* is a neutral layer, and *kp* is displaced from it with certain z distance layer.

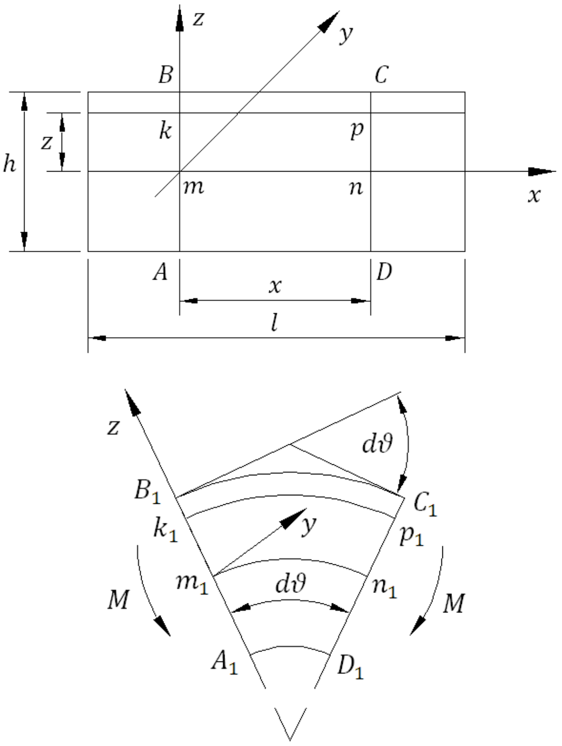

Fig. 2. Composite rod pure bending scheme

In the case of pure bending, the shear forces in the cross-section of rod are equal to 0, there are existing only bending moments M. We have the following dependencies between the geometric characteristics of the rod and deformations and stresses in the cross-section points:

$$\varepsilon = \varkappa z, \\ \varkappa = \frac{M}{HI_y} = \frac{M}{HI}, \quad (5) \\ \sigma = H\varepsilon = H\varkappa z = H\frac{M}{HI}z = \frac{M}{I}z,$$

where $\varkappa = \frac{1}{\rho}$ is a curvature, and ρ – is a radius of curvature. $I_y = I$ is the moment of inertia of cross-section with respect of the neutral axis *y* [1, 2].

If we consider that bending moment $M = M_c = \text{const}$ for the specific fixed *z* value, and introduce the defined by expression (5) deformation and stress values in the expression (4), we will obtain the following equation:

$$\ddot{\varkappa} + \frac{H}{\eta}\dot{\varkappa} + \frac{E}{\eta n}\varkappa = \frac{M_c}{\eta n I}. \quad (6)$$

The solution of equation (6) with the following initial conditions: if $t = 0$, then $\varkappa(0) = \frac{M_c}{HI}$ and $\dot{\varkappa}(0) = \nu_0$, are given as:

$$\varkappa(t) = C_1 e^{k_1 t} + C_2 e^{k_2 t} + \frac{M_c}{EI}, \quad (7)$$

where $k_1 = \frac{-H+\sqrt{H^2-4E\eta/n}}{2\eta} < 0$ and $k_2 = \frac{-H-\sqrt{H^2-4E\eta/n}}{2\eta} < 0$ are the solutions of the characteristic equation of the equation (6), and coefficients C_1 and C_2 accordingly of the initial conditions will be as following:

$$C_1 = \frac{\nu_0 I + M_c k_2\left(\frac{1}{E} - \frac{1}{H}\right)}{I(k_1 - k_2)}; \\ C_2 = -\frac{\nu_0 I + M_c k_1\left(\frac{1}{E} - \frac{1}{H}\right)}{I(k_1 - k_2)}. \quad (8)$$

Accordingly to dependency (7) the typical graphical representation is given in Fig. 3.

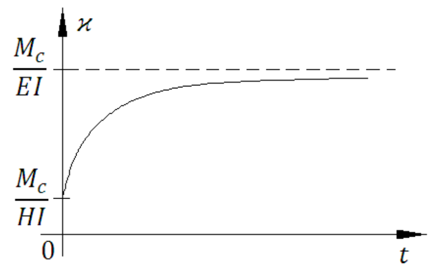

Fig. 3. Typical curve of dependency at curvature $\varkappa(t)$

Now let's consider the relaxation process that provides estimation of the time dependence on stress values in the rod under the conditions of fixed deformation. Let's assume that deformation $\varepsilon = \varepsilon_c = \varkappa_0 z = \text{const}$ at fixed z. If we introduce this expression in equation (4), we will obtain:

$$\dot{\sigma} + \frac{1}{n}\sigma = \frac{Ez}{n}\varkappa_0. \qquad (9)$$

The general solution of the equation (9) will be as:

$$\sigma(t) = Ce^{-\frac{t}{n}} + E\varkappa_0 z. \qquad (10)$$

If we take into consideration that relaxation process starts after a long time as the deformation starts, accordingly by taking into account expression (7), curvature $\varkappa_0 = \varkappa(t \to +\infty) = \frac{M_c}{EI}$. From the expression (5) the relaxation process starts when $t = 0, \sigma_0 = \sigma(0) = \frac{M_c z}{I}$. Taking into account these conditions we will assume that in the expression (10) $C = 0$ and we finally will obtain:

$$\sigma(t) = E\varkappa_0 z = \frac{M_c}{I}z. \qquad (11)$$

As it is clear from the expression (11), at relaxation process arisen in the points of the rod's cross-section stress is not dependent on time, it is depending only on their location, in particular, the stress is proportional to the distance from neutral axis, and directly in neutral layer it is equal to 0.

In the case of composite body, the bending moment M, as well as deflection y is the x function, depending on the time t and location of cross-section on the length of rod. I.e. $M = M(x,t)$ and $y = y(x,t)$. There is a well-known dependence of how is expressed curvature $\varkappa$ of rod by the first and second order derivatives on deflection y [1, 2, 6]:

$$\varkappa = \frac{y"}{\sqrt{(1 + y'^2)^3}}. \qquad (12)$$

If we consider that for an arbitrary magnitude of time $\varkappa$ is fixed value, then it is possible to solve the differential equation (12) and deflection is the function of x coordinate depending on the $\varkappa$ parameter. It will be as:

$$y = -\frac{1}{\varkappa}\sqrt{1 - \varkappa^2(x + A_1)^2} + A_2. \qquad (13)$$

In the case of pure bending, deflection of the ends of the attached rod with l length is equal to 0, and due the symmetry axis, in the middle of rod the angle of rotation is also equal to 0. In the expression (13) to find constants A_1 and A_2 let's apply the following conditions:

when $x = 0, y(0) = 0$ and when $x = \frac{l}{2}, y'\left(\frac{l}{2}\right) = 0$. (14)

By taking into account the conditions (13) let's accept for the constants: $A_1 = -\frac{l}{2}$ and $A_2 = \frac{1}{\varkappa}\sqrt{1 - \frac{\varkappa^2 l^2}{4}}$.

Considering that the $\varkappa$ parameter depends only on time and has a form of (7), then expression (13) finally will be as:

$$y(x,t) = -\frac{1}{\varkappa(t)}\sqrt{1 - \varkappa^2(t)\left(x - \frac{l}{2}\right)^2} + \\ + \frac{1}{\varkappa(t)}\sqrt{1 - \frac{\varkappa^2(t) l^2}{4}}. \qquad (15)$$

The angle of rotation of the rod would be easily find from the expression (15):

$$\vartheta(x,t) = \frac{\partial y(x,t)}{\partial x} = \frac{\varkappa(t)\left(x - \frac{l}{2}\right)}{\sqrt{1 - \varkappa^2(t)\left(x - \frac{l}{2}\right)^2}}. \qquad (16)$$

3. Conclusion

The considered task reflects mode of deformation corresponding to the pure bending of made from composite material rectangular rod. Despite of the simplicity of task, in contrary to the metals, will be obtained the complex image, deformation and stresses in the points of cross-section with coordinates will depend also on time. The application of general models and in this case the four-element model gives the possibility for the most cases to write down the solutions in analytical form that is very important for further researches.

This stated task will be reduced on the case of linear deformation that is described in detail for various cases in the literature [1, 2, 3, 4, 5, 6, 7]. The considered model belongs to a category that has a binding elastic element, and as a result at origination of the creeping process, deformation will reach the certain constant value. This factor limits the class of composite materials and would only be used for those who have similar properties.

The expressing curvature dependency (6) includes combination of two exponential summands and the number of independent parameters is large that gives the possibility to better compatibility of theoretical and experimental data. The relaxation process is relatively simple to describe. It is also possible to write down the values of the rod deflections and the rotation angle in exact analytical form as depending on the coordinate and time functions. This circumstance represents prerequisite for further studies in contrary of other forms of data entry (table or graphic form). It is substantially better.

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BIDZINA ABESADZE was born in Kutaisi, Georgia. He received his Major in Engineering Sciences in 2007. His research interests include Physics, Mathematics and Technology.