COMPARISON OF S-N CURVES FOR CONCRETE IN EN 1992-2 AND *FIB* MODEL CODE 2010

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Abstract. *The paper is focused on the comparison of Wöhler's curves for concrete in accordance with fib Model Code 2010 and EN 1992-2. It discusses the great difference for bridge fatigue assessment between Model Code 2010 and EN 1992 observed in previous studies. Respective S-N curves were compared with the experimental data for concrete C20. Evaluation is performed for concrete exposed to compression only. Wöhler's curve in fib Model Code 2010 is more complex and can better fit experimental data than the Eurocode model.*

Keywords

Fatigue, fib Model Code 2010, EN 1992, Wöhler's curve for concrete.

1. Introduction

The study is based on the practical problem where the great differences in bridge fatigue life predicted by probabilistic analysis in accordance with EN 1992-2 [1] (EC) and *fib* Model Code 2010 [2] (MC) have been observed in [3] and in related unpublished studies of the authors. Estimation of service life of bridges is significantly affected by fatigue induced by crossings of heavy vehicles and significantly depends on a type of S-N curve adopted in the assessment. While in common cases fatigue resistance of concrete is assessed indirectly through the check of stresses due to service loads, a more advanced assessment using S-N curves is often needed for slender structures, existing structures failing to comply with the stress limit, or some members of prestressed structures.

Augustin proposed in 1850 a theory of the fatigue failure of a material, describing the relationship between stress amplitudes and a number of loading cycles until failure. This dependence is called Wöhler's fatigue curve (S-N curve), which is the most widely used tool for the assessment of fatigue life. Wöhler's curves can be used for various construction materials in the building industry (steel, concrete, aluminium, etc.). Shape of curves should be based on the measured data, but often differs amongst codes of practice.

All calculations in this study are performed for theoretical minimum and maximum compression in concrete. Mean values obtained by the MC and EC models are critically compared with limited experimental data. It is foreseen that the achieved experience will be utilised in refinements of fatigue resistance models for concrete structures. The presented approach could be also utilised to validate fatigue resistance models for UHPC where fatigue often dominates structural reliability.

2. EC Models

2.1. Design Values

Wöhler's curve in EC is determined by the following equations:

$$N = 10^{14\left(1-\frac{E_{cd,\max}}{\sqrt{1-R}}\right)} \quad (1)$$

$$R = \frac{E_{cd,\min}}{E_{cd,\max}} \quad (2)$$

$$E_{cd,\min} = \frac{\sigma_{cd,\min}}{f_{cd,fat}} \quad (3)$$

$$E_{cd,\max} = \frac{\sigma_{cd,\max}}{f_{cd,fat}} \quad (4)$$

$$f_{cd,fat} = k_1 \beta_{cc}(t_0) f_{cd}\left(1-\frac{f_{ck}}{250}\right) \quad (5)$$

$$\beta_{cc} = e^{s\left(1-\left(\frac{28}{t_0}\right)^{0.5}\right)} \quad (6)$$

$$f_{cd} = \alpha_{cc} \frac{f_{ck}}{\gamma_c} \tag{7}$$

where N denotes the ultimate number of constant amplitude cycles, $E_{cd,min}$ is the minimum compressive stress level, $E_{cd,max}$ the maximum compressive stress level, R the stress ratio, $\sigma_{cd,min}$ the lower stress in a cycle in MPa, $\sigma_{cd,max}$ the upper stress in a cycle in MPa, $f_{cd,fat}$ the design value of concrete fatigue strength in MPa, k_1 the factor equal to 0.85, $\beta_{cc}(t_0)$ the coefficient for concrete strength at the first load application in accordance with EN 1992-1-1 [4], f_{cd} design value of concrete compressive strength [4] in MPa, f_{ck} characteristic compressive cylinder strength of concrete at 28 days [4] in MPa, s the coefficient which depends on the strength class of cement in accordance with EN 1992-1-1 [4] (see Tab. 1), t_0 the time of the start of the cyclic loading on concrete in days, α_{cc} the coefficient taking account of long term effects on the compressive strength and of unfavourable effects depending on the way of load application (equal to one in accordance with [4]), and γ_c = 1.5 is the partial factor for concrete [4].

Tab.1: Coefficient s in accordance with EN 1992-1-1 [4].

Strength class of cement	32.5 N	32.5 R 42.5 N	42.5 R 52.5 N 52.5 R
s	0.38	0.25	0.20

2.2. Mean Values

The mean concrete strength as an input for Wöhler's curves is obtained as [1] (in MPa):

$$f_{cm} = f_{ck} + 8 \tag{8}$$

$$f_{cm,fat} = k_1 \beta_{cc}(t_0) f_{cm} \left(1 - \frac{f_{cm}}{250}\right) \tag{9}$$

where f_{cm} is the mean value of concrete cylinder compressive strength in accordance with EN 1992-1-1 [4] and $f_{cm,fat}$ is the mean value of concrete fatigue strength.

The change of characteristic and design value of concrete compressive strength from Eq. (5) to the mean value of compressive strength in Eq. (9) might seem to be inappropriate. However, this is fully consistent with the MC model; see Eq. (15) below. Note that γ_c is not considered when estimating the mean values.

3. MC Models

3.1. Design Values

Wöhler's curve in the MC is slightly more complex than the EC curve:

$$\log N_1 = \frac{8}{Y-1}\left(S_{cd,\max} - 1\right) \tag{10}$$

$$\log N_2 = 8 + \frac{8\ln(10)}{Y-1}\left(Y - S_{cd,\min}\right)\log\left(\frac{S_{cd,\max} - S_{cd,\min}}{Y - S_{cd,\min}}\right) \tag{11}$$

if $\log N_1 \le 8$, then $\log N = \log N_1$

if $\log N_1 > 8$, then $\log N = \log N_2$

$$Y = \frac{0.45 + 1.8 S_{cd,\min}}{1 + 1.8 S_{cd,\min} - 0.3 S_{cd,\min}^2} \tag{12}$$

$$S_{cd,\max} = \frac{\gamma_{Ed}\sigma_{c,\max}\eta_c}{f_{cd,fat}} \tag{13}$$

$$S_{cd,\min} = \frac{\gamma_{Ed}\sigma_{c,\min}\eta_c}{f_{cd,fat}} \tag{14}$$

$$f_{cd,fat} = \frac{0.85\beta_{cc}(t) f_{ck}\left(1 - \frac{f_{ck}}{25 f_{ck0}}\right)}{\gamma_{c,fat}} \tag{15}$$

where $S_{cd,min}$ denotes the minimum compressive stress level, $S_{cd,max}$ is the maximum compressive stress level, γ_{Ed} the partial factor for loading (1.0 assumed here as the compression is here associated with no particular loading case), $\sigma_{c,min}$ the lower stress in a cycle in MPa (equal to $\sigma_{cd,min}$ in EC), $\sigma_{c,max}$ the upper stress in a cycle in MPa (equal to $\sigma_{cd,max}$ in EC), η_c the averaging factor of concrete stresses in the compression zone considering the stress gradient ($2/3 \le \eta_c \le 1$), $\gamma_{c,fat}$ the partial factor for concrete fatigue resistance (1.5 according to MC), and f_{ck0} = 10 MPa is the reference strength. For $S_{cd,min} > 0.8$, the S-N relationship for $S_{cd,min} = 0.8$ applies.

3.2. Mean Values

The mean fatigue strength for the MC model is estimated in a similar way as for the EC model, using Eq. (8):

$$f_{cm,fat} = 0.85\beta_{cc}(t) f_{cm}\left(1 - \frac{f_{cm}}{25 f_{ck0}}\right) \tag{16}$$

As can be seen, the mean value of fatigue strength according to MC is same as for EC, but it is obtained in a clearer way as it is derived from f_{ck} only and the partial factor is not included in Eq. (15).

3.3. Influence of η_c

The stress gradient for concrete in the compression zone of a cracked section may be taken into account by multiplying the maximum stress in the compression zone by a factor η_c:

$$\eta_c = \frac{1}{1.5 - 0.5\frac{|\sigma_{c1}|}{|\sigma_{c2}|}} \quad (17)$$

where σ_{c1} is the lower absolute value of the compressive stress within a distance of 300 mm from the surface under the relevant load combination of actions and σ_{c2} is the larger absolute value of the compressive stress within a distance of 300 mm from the surface under the same load combination considered to determine σ_{c1}.

4. Critical Comparison of Methods

EC and MC Wöhler's curves for the mean values of the basic variables are displayed in Fig. 1, 2 and 3. The figures show the dependence of maximum compressive stress level $E_{c,max}$ and $S_{c,max}$ on ultimate number of constant amplitude cycles N. The influence of stress gradient η_c is also shown in Fig. 2. Fig. 3 includes the experimental data for C20 [5].

Figure 1 shows that for $\eta_c = 1$ the EC model seems to be more conservative for minimum compressive stress level $S_{c,min} = E_{c,min} \leq 0.6$. For higher minimum compressive stress levels $S_{c,min} = E_{c,min} > 0.6$, the EC and MC models could lead to similar results.

The effect of η_c is shown in Fig. 2. Considering $0.667 \leq \eta_c \leq 1$, it follows from the figure:

- No significant effect of η_c is observed for very low $S_{c,min}$.
- For medium values of $S_{c,min}$, the MC model provides bounds around the EC model (models could lead to similar results) and
- For high values of $S_{c,min}$, the MC model tends to become more conservative compared to the EC model.

Tab.2: Basic variables used for comparison.

Eurocode 1992-2		Model Code 2010	
f_{ck} [MPa]	20	f_{ck} [MPa]	20
k_1 [-]	0.85	f_{ck0} [MPa]	10
s [-]	0.38	s [-]	0.38
t_0 [days]	90	t_0 [days]	90
$\sigma_{cm,min}$ [MPa]	1.25	$\sigma_{cm,min}$ [MPa]	1.25
$\sigma_{cm,max}$ [MPa]	1.25 - 25	$\sigma_{cm,max}$ [MPa]	1.25 - 25
		η_c [-]	1

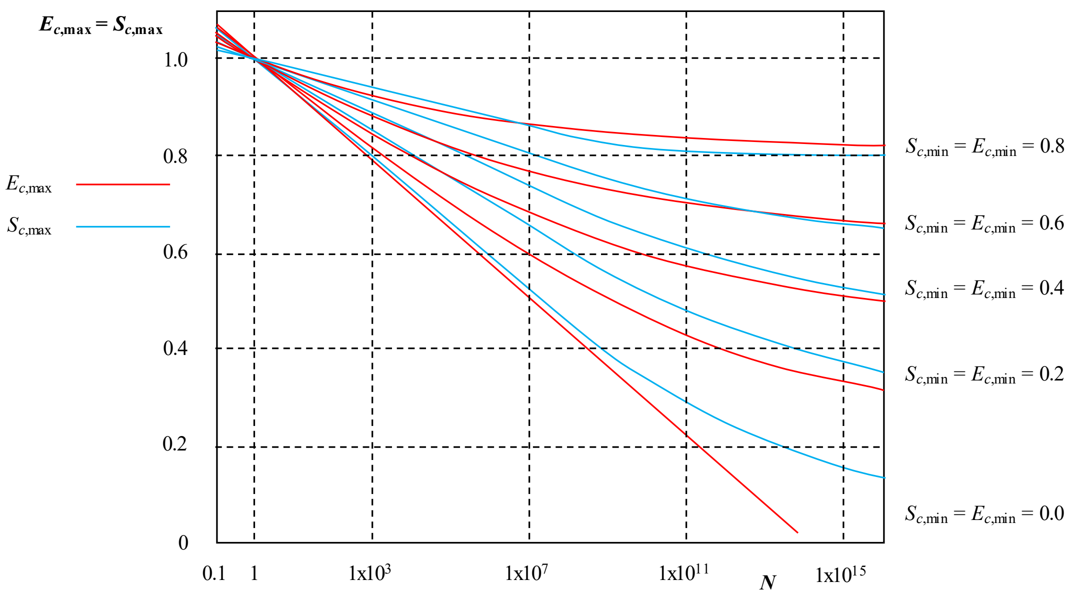

Fig. 1: Comparison S-N curves in EC and MC for mean values with different minimum compressive stress level for $\eta_c = 1$.

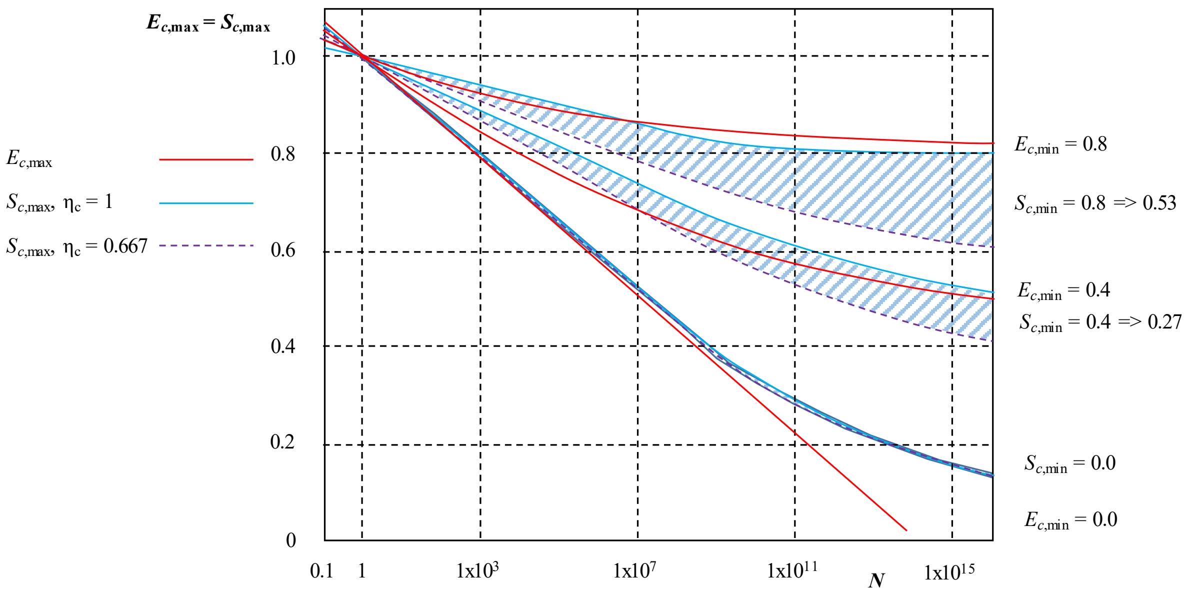

Fig. 2: Influence of η_c on S-N curves.

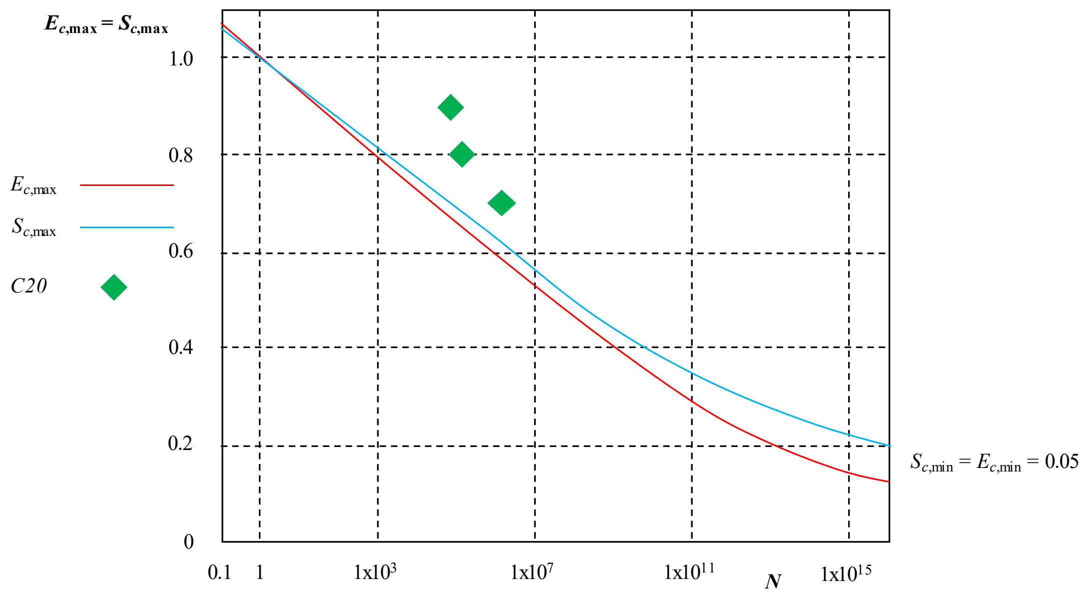

Fig. 3: Comparison S-N curves in EC and MC with the data published in [5].

Considering the values of the basic variables in Tab. 2, EC and MC Wöhler's curves are compared with the experimental data in Fig. 3. Both the models seem to be slightly conservative (the MC model fits the data slightly better), but further research is needed to justify this statement (more experimental data are required).

5. Discussion

Achieved experience will be utilised in refinements of fatigue resistance models for normal and high-strength concrete structures. In particular, for the latter, fatigue is foreseen to frequently dominate structural reliability. The methodology utilised in this study can be readily applied to various construction materials, e.g. to steel reinforcement, prestressed steel reinforcement or to structural steel.

6. Conclusion

Wöhler's S-N fatigue curves for concrete in fib Model Code 2010 and EN 1992-2 are critically compared in the contribution. Both models seem to be slightly conservative in comparison with the data and the MC curve provides slightly better fit to the data. The MC stress factor decreases the ultimate number of constant amplitude cycles for maximum compressive stress level. This effect becomes more important with an increasing minimum compressive stress level.

It is foreseen that the achieved experience will be utilised in refinements of fatigue resistance models for normal and high-strength concrete structures.

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