FULLY STOCHASTIC NONLINEAR ANALYSIS OF SLENDER REINFORCED CONCRETE COLUMN

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Abstract. *The following article presents nonlinear resistance assessment of a slender reinforced concrete column using commercial FEM software ATENA. Furthermore, three different approaches are used to determine design value of resistance. Firstly, the most commonly used method of partial safety factors is described. Secondly, method ECoV (estimate of coefficient of variation) is presented, which is one of the possible options used in fib Model Code to asses design resistance of structures using nonlinear analysis. Eventually, a fully probabilistic analysis is performed using commercial software package SARA.*

Keywords

Non-linear analysis, safety formats, reliability.

1. Introduction

Non-linear structural analysis plays an important role while assessing reliability of existing, or even newly designed concrete (concrete-steel) structures. Although the simplified linear elastic analysis is a powerful tool and allows us relatively quickly design dimensions of a desired structure, if we want to get a deeper insight into the real structural behaviour under certain load conditions, the linear elastic analysis quickly becomes insufficient because it doesn't account for geometrical (equilibrium on deformed structure) and most importantly material (concrete cracking and crushing, reinforcement yielding, etc.) nonlinearities. It also worth mentioning that elastic distribution of internal forces in statically indeterminate structures is usually close to reality only under very low load levels, which is in contrary with design procedure described e.g. in Eurocode 2 (EC2) [1], where the elastic distribution of internal forces is used to design the cross-sectional dimensions (and reinforcement) under an assumption of plastic material behaviour. Nevertheless, although this design procedure doesn't properly reflect real behaviour of the structure, it has been proved by many years of experience that it provides conservative designs. On the other hand, if one wants to simulate the real behaviour of given reinforced concrete structure, it is necessary to account for nonlinearities. One of the possible solutions is to use commercial FEM software ATENA [2], which can model discrete reinforcement and uses fracture-plastic material model to simulate the real concrete behaviour. Principles of the model can be characterized by following features:

- Smeared cracks within finite elements;
- Crack band control of strain localization including effects of crack orientation and element shape function;
- Fixed crack model;
- Two fracture failure modes are considered on the crack face, Mode I due to normal stress action (exponential law of crack opening controlled by fracture) and Mode II due to shear stress (shear deformation controlled by a shear factor and shear strength depending on aggregate interlock);
- Reduction of compressive strength due to damage by cracks;
- Menétrey-Willam [3] plasticity formulation for concrete in compression with non-associated flow rule.

More detailed information about the material model can be found in [4].

The purpose of this paper is to compare three different methods of global design resistance evaluation, method of partial safety factors, method ECoV (estimate of coefficient of variation) and full probabilistic analysis. Results will be presented on the relatively simple reinforced concrete structure – slender column.

2. Global Safety Format

In standard design procedure, the following condition has to be fulfilled

$$F_d < R_d \,, \tag{1}$$

where F_d stands for design action and R_d stands for design resistance. Most commonly used design method described e.g. in Eurocode 2 is based on fulfilling condition (1) in all the critical cross-sections of the given structure. F_d is than design internal force (normal force, bending moment, etc.) calculated by means of linear elastic analysis on the given structure loaded by prescribed load combinations margined by partial safety factors (e.g. γ_G for permanent load, γ_Q for imposed load, γ_P for prestressing load, etc.) and R_d is resistance of the selected cross-section of the structure to the acting internal force, which is usually determined under plastic material conditions using material safety factors γ_M. In this procedure we assume failure probabilities of separate materials in the critical cross-sections, but the failure probability of the entire structure remains unknown. On the other hand, in non-linear analysis, R_d is design global resistance, i.e. set of forces representing an imposed load (load combination) which lead to failure of some part of the structure. Unlike in sectional design, the global resistance accounts for interaction of the whole structure and does not only asses specific cross-sections. It can be expressed as [5]

$$R_d = \frac{R_m}{\gamma_R}, \tag{2}$$

where R_m is the mean resistance and γ_R is the global safety factor, which includes all uncertainties and under an assumption of log normal distribution used in Eurocode 2 it can be expressed by means of coefficient of variation of resistance V_R as

$$\gamma_R = \exp\left(\alpha_R \beta V_R\right), \tag{3}$$

where α_R ($\alpha_R = 0.8$ for 0.001 probability of failure) is the sensitivity factor for resistance and β ($\beta = 3.8$ for a reference period of 50 years) is the reliability index. It is important to note that coefficient of variation V_R includes uncertainties of various origins and can be expressed as [6]

$$V_R = \sqrt{V_G^2 + V_m^2 + V_{Rd}^2}, \tag{4}$$

where V_G, V_m, and V_{Rd} are coefficients of variation of associated random variables to account for geometry, material and numerical model uncertainties.

There are different approaches of assessment of design structure reliability which differ in the level of approximation. Three of them are briefly described below.

2.1. Full Probabilistic Analysis

The full probabilistic analysis is the most rational way of assessing the structural reliability. Its principal lies in running the non-linear simulations on many samples of the investigated structure, while chosen parameters of the numerical model (material properties, dimensions, boundary conditions, etc.) are systematically varied within the samples according to certain probability distribution functions (PDF). There are different algorithms for generating the samples, which are described in detail e.g. in [7] and won't be further discussed in this paper. The randomization of chosen probabilistic quantities in numerical model can be carried out in two different ways: (a) **random variables**, where the quantity remains constant within the sample (structure), but differs between samples and (b) **random fields**, where the quantity varies randomly in space and of course between the samples. It is important to note that certain parameters are actually correlated (e.g. in case of concrete, the higher the compressive strength, the higher the tensile strength) and therefore this correlation should be taken into account to properly reflect the reality. After running the numerical simulations we end up with an array of resistance values which can be fit by chosen PDF of resistance (e.g. in EC2 it is log normal PDF). The only remaining step is to choose the probability of failure we want to obtain the resistance value for. In our case, we are interested in design value of resistance, which usually corresponds to the 0.001 probability of failure (excluding the uncertainty of action force). Although this method is the most rational and robust way of determining the structural design resistance, it is computationally very demanding (large number of simulations has to be performed). Furthermore, if one wants to obtain resistance values for very small (or very large) failure probabilities (which is the case of design resistance), the results strongly depend on choice of PDF, because there is usually a lack of samples on the edges of PDF and the resistance value is therefore extrapolated.

2.2. ECoV Method – Estimate of Coefficient of Variation

ECoV is a simplified probabilistic method proposed in [9]. It is based on the idea of determining the coefficient of variation from two samples only and assumes lognormal distribution of resistance. The first sample is calculated using the mean material parameters (which supposedly corresponds to the median) and the second one uses the characteristic parameters (that are supposed to yield 5% quantile). After running the non-linear analysis on both samples, coefficient of variation of structural resistance can be calculated as

$$V_R = \frac{1}{1.65}\ln\left(\frac{R_m}{R_k}\right), \tag{5}$$

where R_k is characteristic resistance.

The resulting design resistance is then obtained using formulas (2) and (3). This method is general and much less computationally demanding than fully probabilistic approach. However, it is arguable if it properly reflects all types of failure. In [6] authors propose similar method which requires more samples than ECoV, but is still much less computationally demanding than fully probabilistic approach.

2.3. Partial Safety Factors

The method of partial safety factors described in most of the design codes can be also applied on global analysis to obtain design resistance of the structure. The design values of material parameters f_d are calculated as $f_d = f_k / \gamma_M$, where f_k is characteristic value and γ_M is material safety factor, which can be calculated as [6]

$$\gamma_M = \exp(\alpha_R \beta V_R - 1.64 V_m) \tag{6}$$

for reinforcing steel and as

$$\gamma_M = 1.15 \exp(\alpha_R \beta V_R - 1.64 V_m) \tag{7}$$

for concrete. An additional factor 1.15 in (7) has been introduced to account for the lower concrete strength in real structures than in carefully cured experimental cylinders.

Specific values of coefficients of variation which lead to well-known values $\gamma_C = 1.5$ for concrete and $\gamma_S = 1.15$ after using (4) in (6) and (7) are shown in Tab. 1.

Tab. 1: Statistical representation which leads to the partial safety factors in Eurocode 2.

Type of uncertainty	Reinforcing steel	Concrete
V_{Rd} (model)	2.5%	5%
V_G (geometry)	5%	5%
V_m (material)	4%	15%

Considering that design values calculated by means of partial safety factors represent extremely low values of material properties, this method could lead to distorted failure modes (meaning that failure mode of the structure can be unrealistic due to the extremely low values of material properties which are unlikely to occur). On the other hand, years of experience proved, that this method gives mostly conservative and therefore safe results.

3. Case Study

The three above mentioned methods of assessing the global structural reliability will be presented on the case study of axially loaded reinforced concrete slender column. The loading scheme and basic dimensions of the column are shown in Fig. 1. The detailed drawing of reinforcement can be further seen in Fig. 2.

3.1. Numerical Model

3D numerical model of presented case study with discrete reinforcement was created using commercial FEM software ATENA and its details are described in the following subchapters.

1) Materials

To investigate the influence of concrete strength, the structural behavior is simulated with two considerably different classes of concrete, C50/60 and C16/20. The reinforcement steel is B500B. The following material properties are used (Tab. 2, Tab. 3 and Tab. 4).

Tab. 2: Material properties for concrete class C50/60 from EC2.

C50/60	Mean	Characteristic	Design
E_c [MPa]	37277	35650	32102
ν	0.2	0.2	0.2
f_c [MPa]	-58	-50	-35.24
f_t [MPa]	4.1	2.9	2.04
G_f [N/m]	102	72.5	51.1
ε_{cp}	-0.000897	-0.00111	-0.00156

Tab. 3: Material properties for concrete class C16/20 from EC2.

C16/20	Mean	Characteristic	Design
E_c [MPa]	28608	25331	22803
ν	0.2	0.2	0.2
f_c [MPa]	-24	-16	11.27
f_t [MPa]	1.9	1.3	0.916
G_f [N/m]	47.5	32.5	22.9
ε_{cp}	-0.00104	-0.00132	-0.001507

E_c is modulus of elasticity, ν is Poisson's ratio, f_c is compressive strength, f_t is tensile strength, G_f is fracture energy and ε_{cp} is plastic strain at compressive strength level. It is important to note, that the mean modulus of elasticity E_{cm} can be according to EC2 calculated as $E_{cm} = 22000(f_{cm}/10)^{0.3}$, but EC2 doesn't say anything about the characteristic value of E modulus. Actually, it proposes that the mean value should be always used. However, in case of the analysis that can strongly depend on the value of E modulus (e.g. because of buckling phenomena), it is necessary to use E_k and E_d different from E_m. The simplest interpretation of EC2 is to apply analogy to the formula used to determine the mean value of E and calculate characteristic value of modulus of elasticity as $E_{ck} = 22000(f_{ck}/10)^{0.3}$ and design value as $E_{cd} = 22000(f_{cd}/10)^{0.3}$, which is done in Tab. 2 and Tab. 3. Furthermore, to compare results obtained from full probabilistic analysis and from ECoV with method of partial safety factors, we only need to account for material uncertainty in material safety factors. Using formulas (4), (6), and substituting $V_G = 0$ and $V_{Rd} = 0$ gives us

$$\gamma_S = \exp(\alpha_R \beta V_R - 1.64 V_m) =$$
$$= \exp(0.8 \times 3.8 \times 0.04 - 1.64 \times 0.04) = 1.058$$

for steel reinforcement safety factor and using (7) analogically gives us

$$\gamma_C = 1.15 \times \exp\left(\alpha_R \beta V_R - 1.64 V_m\right) =$$
$$= \exp\left(0.8 \times 3.8 \times 0.15 - 1.64 \times 0.15\right) = 1.419$$

for concrete safety factor.

Tab. 4: Material properties for reinforcement steel B500B from EC2.

B500B	Mean	Characteristic	Design
E_s [MPa]	200000	200000	200000
f_y [MPa]	550	500	472.8
ε_{lim}	0.05	0.05	0.05

where f_y is the yield strength and ε_{lim} is the limit strain.

2) Loading

ATENA uses incremental method of applying loads by means of so called intervals. Loading of our specific model is divided into two intervals:

- Interval 1: deadload (important due to the horizontal orientation of column, concrete density 2300 kg/m^3); 1 step.
- Interval 2: axial displacement load; 0.1 mm steps until failure.

3) Mesh sensitivity analysis

To ensure that sufficiently accurate results are obtained from the analysis, two different mesh grids were used. Both use linear solid hexahedral elements. The coarser mesh uses finite elements with dimensions 50×50×23 mm (*x*, *y*, *z*) and the finer mesh uses finite elements with dimensions 30×50×12.5 mm. The mesh grid with coordinate system orientation can be seen in Fig. 3. Mesh sensitivity analysis was performed only for concrete class C50/60 and the resulting load-displacement (*L-D*) diagrams are shown in Fig. 4. It is clear that for this case study coarser mesh can be used because the results differ only slightly and the computational time decreases considerably.

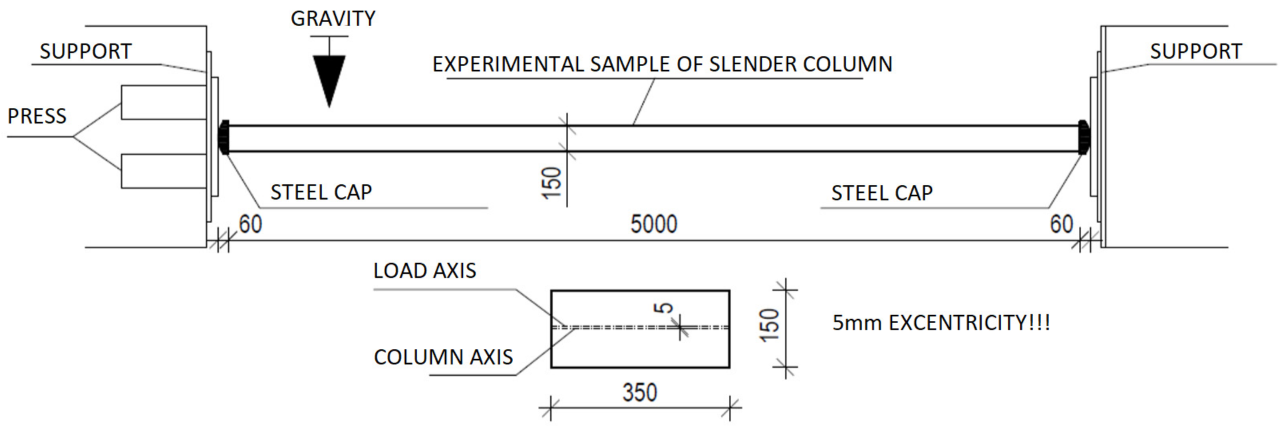

Fig. 1: Schematic drawing of slender column.

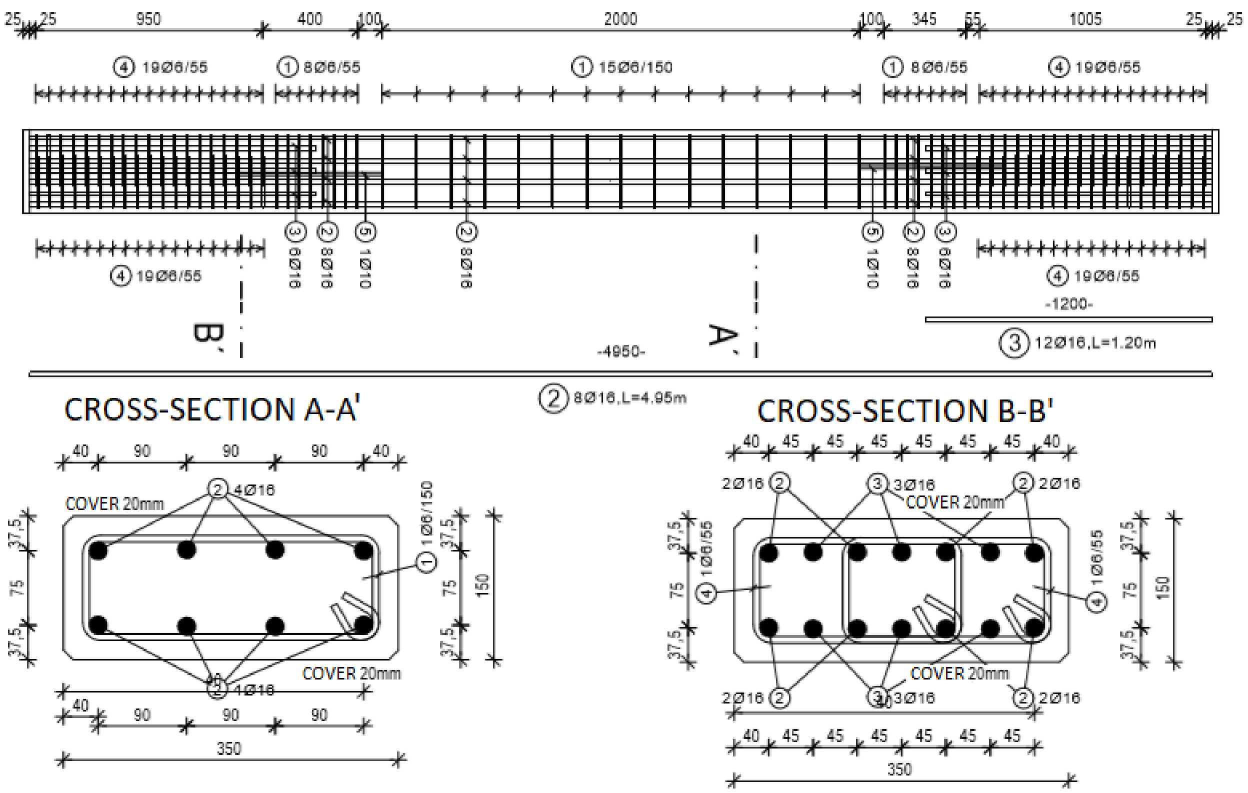

Fig. 2: Reinforcement of slender column.

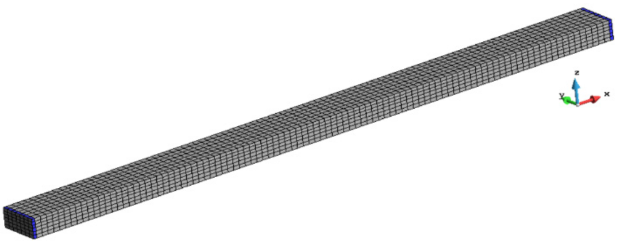

Fig. 3: Finite element mesh grid (coarse).

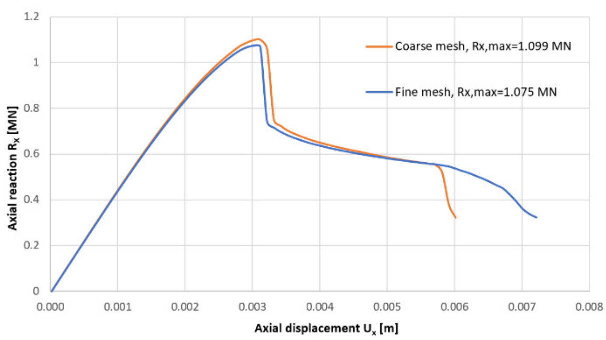

Fig. 4: Comparison of *L-D* diagrams for coarse and fine mesh.

3.2. Full Probabilistic Analysis

Full probabilistic analysis of the case study is performed using commercial software SARA [7, 8], which is used to randomize material parameters and further cooperates with ATENA, where the simulations are run. The randomized material parameters are modulus of elasticity of concrete E_c, concrete tensile strength f_t, concrete compressive strength f_c, concrete fracture energy G_f and concrete plastic strain when the compressive strength is reached, ε_{cp}. All of the randomized parameters are assumed to have normal distribution with given mean values and standard deviations calculated such that 5% quantile of the resulting probability distribution function represents the characteristic value (Tab. 2 and Tab. 3). This approach is reasonable for compressive and tensile strength of concrete and questionable for rest of the parameters, but different approach would require more experimental data. Furthermore, some of the parameters are correlated in reality, which is taken into account by setting up the statistical correlation matrix (Tab. 5).

Tab. 5: Statistical correlation matrix (symmetrical).

	E_c	f_t	f_c	G_f	ε_{cp}
E_c	1	0	-0.7	0	0
f_t		1	-0.5	0.8	0
f_c			1	0	-1
G_f				1	0
ε_{cp}					1

Correlation measures the degree of statistical association between two variables. In our case correlation measures the degree of linearity of the relationship. The higher the value of statistical correlation is, the more correlated the quantities are. Negative correlation means that the higher one quantity is, the lower is the second one (and vice versa). The simulation is performed on 60 samples (see Fig. 5, analogical for all randomized parameters for C50/60 and C16/20).

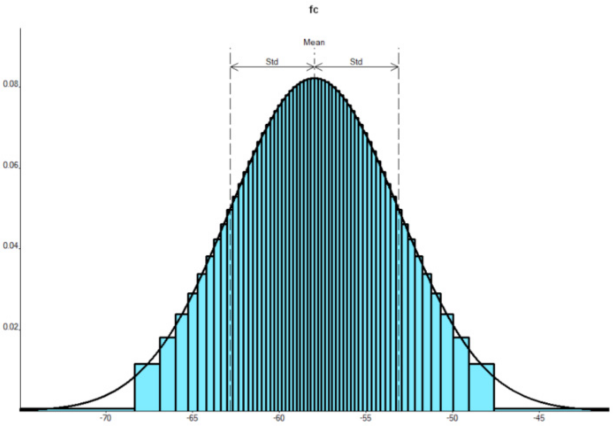

Fig. 5: Normal distribution of compressive concrete strength for 60 samples, C50/60 [10].

After running ATENA non-linear analysis for all the generated samples (for both concrete classes) we get following *L-D* diagrams (Fig. 6 and Fig. 7) and reliability histograms (Fig. 8 and Fig. 9). Finally, we can obtain the global resistance value for selected probability of failure, which in our case corresponds to 0.001 (design resistance) and is equal to $R_{x,prob,c50} = 0.973$ MN for concrete class C50/60 and $R_{x,prob,c16} = 0.347$ MN for concrete class C16/20.

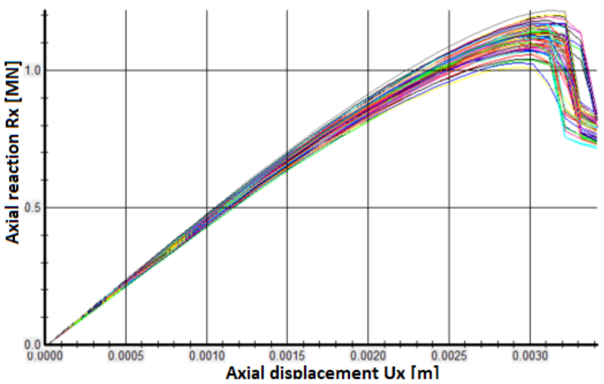

Fig. 6: Set of *L-D* diagrams for concrete class C50/60.

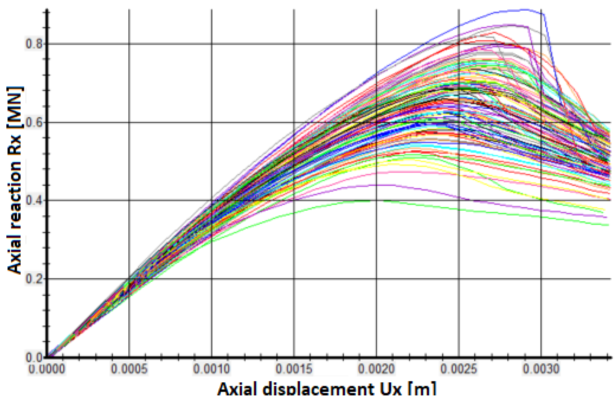

Fig. 7: Set of *L-D* diagrams for concrete class C16/20.

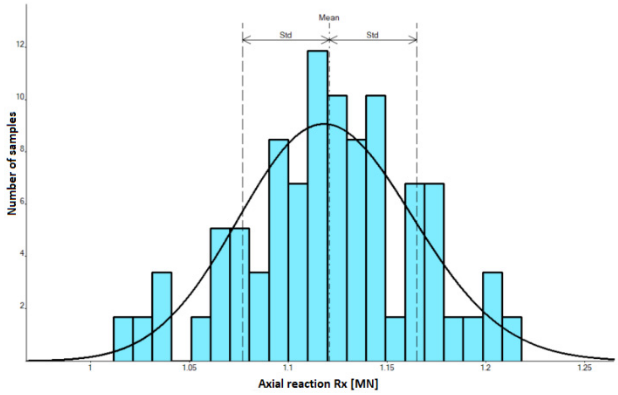

Fig. 8: Reliability histogram, lognormal PDF, C50/60 [10].

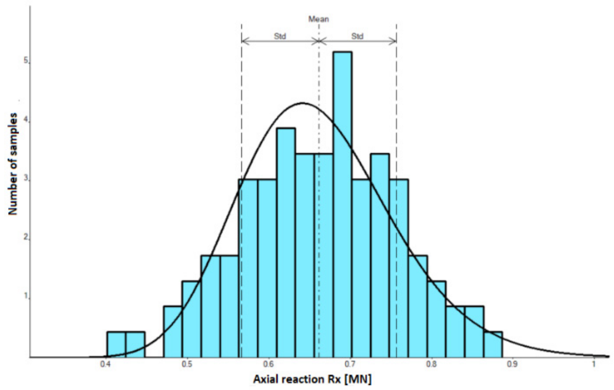

Fig. 9: Reliability histogram, lognormal PDF, C16/20 [10].

3.3. ECoV Method

To determine design resistance using ECOV method, only two samples are required for each concrete class. Using the mean and characteristic material properties (Tab. 2 and Tab. 3) in our numerical model lead to the following resistance values. For concrete C50/60 $R_{x,m,c50} = 1.099$ MN and $R_{x,k,c50} = 0.987$ MN, for concrete C16/20 $R_{x,m,c16} = 0.648$ MN and $R_{x,k,c16} = 0.466$ MN. To determine design safety factor, one has to apply formula (2) to determine coefficient of variation:

$$V_{R,c50} = \frac{1}{1.65} \ln\left(\frac{1.099}{0.987}\right) = 0.0651,$$

$$V_{R,c16} = \frac{1}{1.65} \ln\left(\frac{0.648}{0.466}\right) = 0.199,$$

then (3) to determine design safety factor:

$$\gamma_{R,c50} = \exp(0.8 \times 3.8 \times 0.0651) = 1.219,$$

$$\gamma_{R,c16} = \exp(0.8 \times 3.8 \times 0.199) = 1.831,$$

and finally (5) to evaluate the resulting design resistance:

$$R_{d,ECOV,c50} = \frac{1.099}{1.219} = 0.901 \text{ MN},$$

$$R_{d,ECOV,c16} = \frac{0.648}{1.831} = 0.354 \text{ MN}.$$

3.4. Partial Safety Factors Method

This method applies safety factors on material parameters before the non-linear analysis (Tab. 2, Tab. 3 and Tab. 4) and considers the resulting resistance value as design one. The following results were obtained for each concrete class, $R_{d,part,c50} = 0.758$ MN for C50/60 and $R_{d,part,c16} = 0.349$ MN for C16/20.

3.5. Results and Discussion

The final summary of results obtained by applying above described methods is shown in Fig. 10 for C50/60 and Fig. 11 for C16/20.

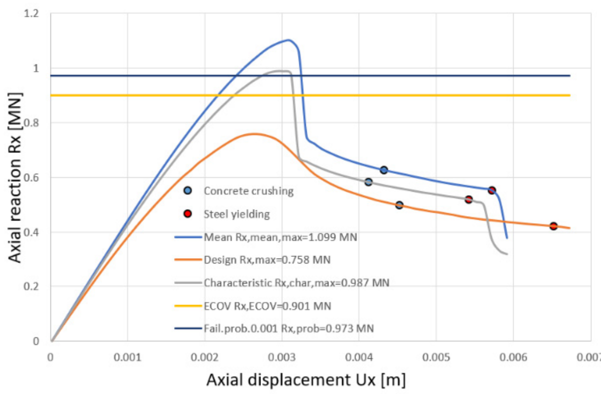

Fig. 10: Comparison of three design resistance assessment methods, C50/60.

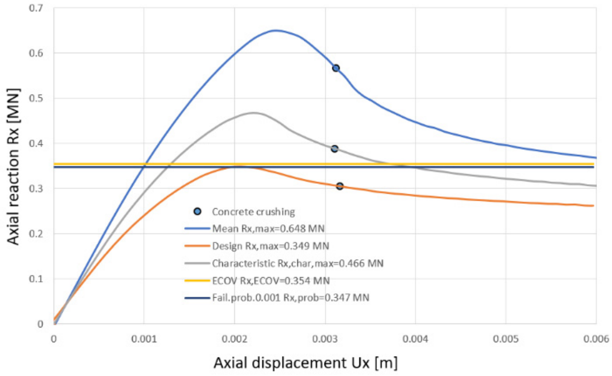

Fig. 11: Comparison of three design resistance assessment methods, C16/20.

As is clear from the diagrams, in case of higher strength concrete (C50/60), the resulting values of design resistance obtained by various methods differ considerably, while for low strength concrete, the resulting design resistances match perfectly. This result could lead to conclusion, that in case of higher strength concrete, the partial safety factors method is unnecessarily conservative and use of ECoV, or possibly full probabilistic analysis would result in more economical designs. On the other hand, it still remains unclear why in case of higher strength concrete the design resistance obtained by method of partial safety factors differs that much from the values obtained by other two methods. One of the possible explanations can be that EC2 defines mean compressive strength value as $f_{cm} = f_{ck} + 8$ MPa, which for low strength concrete classes leads to considerable higher relative difference between the mean and characteristic values and therefore the full probabilistic analysis and method ECoV give smaller values of design resistance i.e. closer to the partial safety factors method.

4. Conclusions

Three methods of global design resistance assessment of reinforced concrete structures were presented whereas each one of these methods uses different level of approximation. Non-linear structural analysis were performed in commercial FEM software ATENA using discrete reinforcement and 3D fracture-plastic material for concrete. Firstly, the most robust, but computationally demanding full probabilistic analysis was performed on 60 samples for two concrete classes by means of commercial tool for structural reliability assessment, SARA, and the design resistance was determined for 0.001 probability of failure. Secondly, simplified probabilistic method ECoV, which requires only two samples (mean and characteristic), was applied to calculate the design resistance. At last, most commonly used method of partial safety factors was applied to reduce material parameters to design values, which input the non-linear analysis. Eventually, all the results were compared in diagrams and the differences were discussed.

Acknowledgements

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