10.2478/v10160-010-0001-8

Transactions of the VŠB – Technical University of Ostrava

No.1, 2010, Vol.X, Civil Engineering Series

paper #1

Radim ČAJKA[1], Pavlína MATEČKOVÁ[2]

CARRYING CAPACITY AND SERVICEABILITY PARAMETRIC ANALYSIS OF PRE-STRESSED CONCRETE ROOF PURLINE

Abstract

This paper deals with different computational methods of pre-stressed components according to formerly valid CSN and Eurocode system 1992-1-1 (next only EC2). The different computational methods are applied on a particular component of a pre-stressed prefabricated concrete roof purlin with a span of 16 m. The limit state of serviceability has been analyzed in relation to relative humidity and the dead and live load ratio.

Keywords

Prefabricated concrete, pre-stressed concrete, roof purlin, limit strain, ultimate deflection.

Abstrakt

Příspěvek se zabývá výpočetními metodami předpjatých prvků podle dříve platných norem ČSN a podle soustavy Eurokódů. Různé výpočetní metody jsou aplikovány na předpjaté střešní vaznici o rozpětí 16 m. Dále se analyzuje mezní stav použitelnosti s ohledem na různou vlhkost prostředí a rozdílný poměr stálého a nahodilého zatížení.

Klíčová slova

Prefabrikované dílce, předpjatý beton, střešní vaznice, mezní přetvoření, mezní průhyb.

1 INTRODUCTION

In the paper, a prefabricated pre-stressed concrete purlin with a span of 16 m is analysed; the cross-section is trapezoidal with a height of 700 mm and width of 180-260 mm. This purlin is pre-stressed with 6 strands in three ranks and is moreover reinforced with reinforcing steel. After calculating all losses, the prestress works out to 1,000 MPa.

2 LIMIT STATE OF THE CARRYING CAPACITY

2.1 Material properties, CSN 73 12 01 versus EC2

Roof purlin is made of concrete B55 [1]. This grade corresponds to concrete C45/55 according to EC2 [2]. The design strength values and limit strains are sequenced in Table 1. (Design strength values are not always identical).

[1] Prof. Ing. Radim Čajka, CSc., Department of Building Structures, Faculty of Civil Engineering, VSB – TU Ostrava, Ludvíka Podéště 1875/17, 708 33 Ostrava - Poruba, tel.: (+420) 597 32 1344, e-mail: radim.cajka@vsb.cz.

[2] Ing. Pavlína Matečková, Ph.D., Department of Building Structures, Faculty of Civil Engineering, VSB – TU Ostrava, Ludvíka Podéště 1875/17, 708 33 Ostrava - Poruba, tel.: (+420) 597 32 1394, e-mail: pavlina.mateckova@vsb.cz.

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Fig.1: Pre-stressed roof purlin in real structure

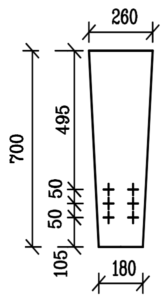

Fig.2: Purlin cross-section

The design strength of pre-stressed reinforcement is derived according to CSN from a contractual yield strength of 0.2% [1]; according to EC 2 it is derived from a contractual yield strength of 0.1% [2]. CSN in Appendix 2 brings forward characteristic and design strength values for different types of pre-stressing reinforcement; EC 2 refers to EN 10138, which has not been instituted in the Czech Republic yet, while some data is given in [3]. If more accurate values are not available, the recommended value of the ratio between characteristic strength f_{pk} and proof stress 0.1% $f_{p0,1k}$ is $f_{p,0,1,k}/f_{pk}$=0,9. Design strength values and limit strains are sequenced in Table 1.

Tab. 1: Summary – material characteristics

		ČSN 731201 [1]		EC 2 [2]		
concrete	Concrete grade	B 55		C45/55		
	Characteristic strength	R_{bn}	39,5	f_{ck}	45	MPa
	Design strength	R_{bd}	30	f_{cd}	30	MPa
	Ultimate strain	e_{bu}	0.0025	e_{cd}	0.0035	-
Pre-stressing steel	Characteristic strength	R_{pn}	1800	f_{pk}	1800	MPa
	contractual strength of 0.1%			$f_{p0,1k}$	1620	MPa
	Design strength	R_{pd}	1440	f_{pd}	1409	MPa
	Ultimate strength	e_{pd}	0.015	e_{ud}	0.02	-

The partial safety factor (coefficient) for prestressed reinforcement according to CSN 73 12 01 is $\gamma = 1{,}25$ [1]. The partial safety factor for prestressed reinforcement for a permanent design situation according to EC [2] is $\gamma = 1{,}15$, but the characteristic value is assumed to be lower. The resulting design strength values are comparable.

$$R_{pd} = \frac{R_{pn}}{\gamma_p} = \frac{R_{pn}}{1{,}25} \tag{1}$$

$$f_{pd} = \frac{f_{p0,1k}}{\gamma_p} \cong 0{,}9.\frac{f_{pk}}{1{,}15} = \frac{f_{pk}}{1{,}28} \tag{2}$$

where:

R_{pd} – design strength of pre-stressing steel [MPa],

R_{pn} – characteristic strength of pre-stressing steel [MPa],

γ_p – partial safety factor for pre-stressing steel [-],

f_{pd} – design strength of pre-stressing steel [MPa],

f_{pk} – characteristic strength of pre-stressing steel [MPa],

$f_{p,0,1k}$ – contractual yield strength of 0.1% [MPa].

In Fig. 3 there is an idealized and design stress-strain diagram for prestressed reinforcement. As given in paragraph 3.3.6 [2] with regards to design, both an inclined branch of the diagram, with the strain limit ε_{ud}), or a horizontal top branch without any strain limit, may be used.

CSN 731201 [1] defines stress-strain diagrams similarly, but the diagram with an inclined branch can be used only for designing on a second limit state.

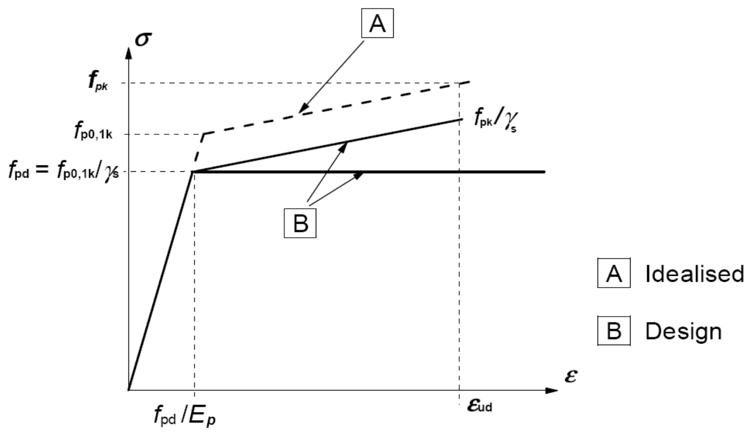

Fig.3: Idealized and design stress-strain diagram for prestressed reinforcement [2]

2.2 Ultimate strain

When evaluating the ultimate bending moment of the carrying capacity, the ultimate strain both in pre-stressed reinforcement and in pressed concrete has to be checked. Achieving the ultimate strain in prestressed reinforcement is bound by condition (3), while achieving the ultimate strain in stressed concrete is bound by condition (6), Fig. 4.

$$x \geq x_1 \tag{3}$$

$$x_1 = \frac{|\varepsilon_{bu}|.h_1}{|\varepsilon_{bu}| + \Delta\varepsilon_p}\text{, obdobně } x_1 = \frac{|\varepsilon_{cu}|.d_1}{|\varepsilon_{cu}| + \Delta\varepsilon_p} \tag{4}$$

$$\Delta\varepsilon_p = \varepsilon_{pd} - \varepsilon_{p\infty}\text{, obdobně } \Delta\varepsilon_p = \varepsilon_{ud} - \varepsilon_{p\infty} \tag{5}$$

$$x \leq x_3 \tag{6}$$

$$x_3 = \frac{|\varepsilon_{bu}|.h_3}{|\varepsilon_{bu}| + \Delta\varepsilon_{pe}}\text{, obdobně } x_3 = \frac{|\varepsilon_{cu}|.d_3}{|\varepsilon_{cu}| + \Delta\varepsilon_{pe}} \tag{7}$$

$$\Delta\varepsilon_{pe} = \frac{\Delta\sigma_p}{E_p} \tag{8}$$

where particular symbols are perceptible from the Fig.4.

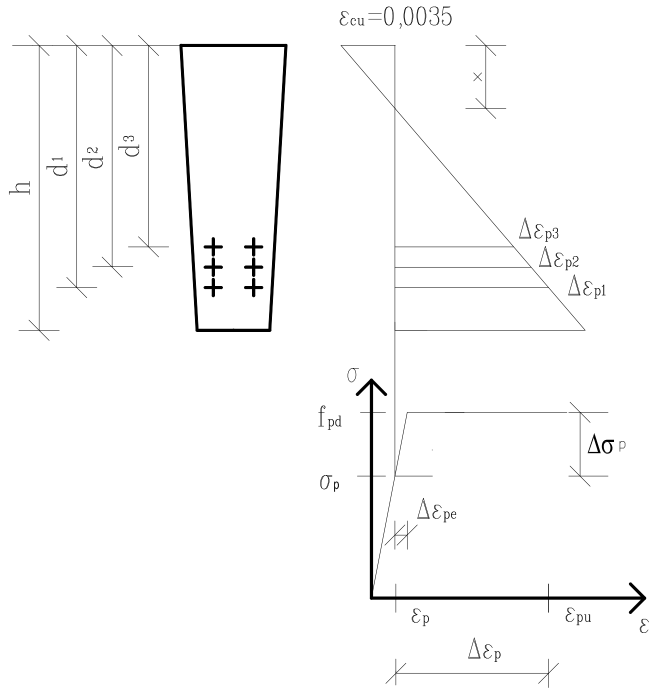

Fig.4: Checking ultimate limit strain

The bending moment of the carrying capacity has to be calculated using the limit strain method according to CSN 731201. Only for reinforced concrete and prestressed concrete in some cases is it possible to use the limit equilibrium method, paragraph 5.2.6 [1].

EC 2 provides no computation method for calculating the bending moment of the carrying capacity, but the assumption of the stress-strain diagram with an inclined branch requires a limit strain method and this method is also generally recommended [3].

If the conditions of (3) and (6) are met, it is possible to reckon with the ultimate strength in all prestressed strands, and the carrying capacity could be solved using the simple limit equilibrium method. As is clear in Table 2, the range between x_1 and x_2 according to EC2 is wider than according to CSN 731201 and the possibility of using the simple limit equilibrium method seems to be broader.

Tab. 2: Summary – ultimate limit strain

	ČSN 731201 [1]	EC 2 [2]	
$\Delta\varepsilon_p$	0,0095	0,0145	-
$\Delta\sigma_p$	440	409	Mpa
E_p	200	200	GPa
$\Delta\varepsilon_{pe}$	0,0022	0,0021	-
x_1	0,124	0,116	m
x_3	0,238	0,309	m

2.1 Bending moment of the carrying capacity

In Table 3 the bending moment of the carrying capacity of a roof purlin cross-section is compared according to CSN 731201 and EC 2 using the limit equilibrium method. The relevant carrying capacities are comparable, the difference being $\pm$ 2 %.

In Table 4 the bending moment of the carrying capacity of a roof purlin cross-section is calculated according to EC 2 using the limit strain method. The calculation is made on the assumption of a horizontal top branch and an inclined branch. The use of strain hardening in the stress-strain diagram shows an insignificant increase in the bending moment of the carrying capacity, the difference being $\pm$ 2,5 %.

Tab. 3: Bending moment of the carrying capacity – limit equilibrium method

ČSN [1]		EC [2]		
h_1	0,595	d_1	0,595	m
h_2	0,545	d_2	0,545	m
h_3	0,495	d_3	0,495	m
x	0,203	x	0,199	m
P_1	283,1	P_1	283,1	kN
P_2	283,1	P_2	283,1	kN
P_3	283,1	P_3	283,1	kN
ΔP_1	125,0	ΔP_1	115,8	kN
ΔP_2	125,0	ΔP_2	115,8	kN
ΔP_3	125,0	ΔP_3	115,8	kN
P_c	1224,34	P_c	-1196,64	kN
M_U	567,29	M_{Rd}	557,09	kNm

Tab. 4: Bending moment of the carrying capacity – limit strain method

	Horizontal branch		Increasing branch	
EC [2]	step 1	step 2	step 1	step 2
ε_{cu}	0,0035	0,0035	0,0035	0,0035
$\Delta\varepsilon_{p1}$	0,02	0,00698	0,02	0,00664
$\Delta\varepsilon_{p2}$	0,0180	0,0061	0,0180	0,0058
$\Delta\varepsilon_{p3}$	0,0161	0,0052	0,0161	0,0049
d_1	0,595	0,595	0,595	0,595
d_2	0,545	0,545	0,545	0,545
d_3	0,495	0,495	0,495	0,495
x	0,089	0,199	0,0886	0,205
P_1	283,1	283,1	283,1	283,1
P_2	283,1	283,1	283,1	283,1
P_3	283,1	283,1	283,1	283,1
ΔP_1	115,8	115,8	115,8	131,12
ΔP_2	115,8	115,8	115,8	128,59
ΔP_3	115,8	115,8	115,8	126,06
P_c	-544,35	-1196,6	-544,35	-1235,2
ΣP	652,35	0,06	652,35	-0,19
	M_{Rd} [kNm]	557,09	M_{Rd} [kNm]	571,89

3 SERVICEABILITY LIMIT STATE

3.1 General

The serviceability limit state of roof purlin is analyzed according to EC2 [2]. In EC 2 the following limit states are taken into consideration: stress limitation, crack control, deflection control. In the paper, crack control and deflection control are analyzed for different states of relative humidity of the environment and a different dead and live load ratio.

3.2 Cracking moment

The cracking moment is based on the condition that the strain in the bottom part of a prestressed cross-section is smaller than the average tensile strength of the concrete (9). The cracking moment is then M_{cr} =344 kNm.

$$\sigma_d + \frac{M_{cr}.z_{Id}}{I_I} \leq f_{ctm} \tag{9}$$

$$\sigma_d = -\frac{P_1 + P_2 + P_3}{A_I} + \frac{P_1(d_1 - z_{Ih}).z_{Id}}{I_I} + \frac{P_2.(d_2 - z_{Ih}).z_{Id}}{I_I} + \frac{P_3.(d_3 - z_{Ih}).z_{Id}}{I_I} \tag{10}$$

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where:

M_{cr} – cracking moment [kNm],

f_{ctm} – average tensile strength of the concrete [kPa],

σ_d – strain in the bottom part of a cross-section from prestressing [kPa],

z_{Id} – distance between gravitational centre and bottom part of cross-section [m],

z_{Ih} – distance between gravitational centre and upper part of cross-section [m],

A_I – area of cross-section [m^2],

I_I – second moment of area [m^4].

$$\left(g_d + q_d\right) \leq \frac{8.M_{Rd}}{L^2} = \frac{8.557}{16^2} = 14{,}41\ \text{kNm}^{-1} \tag{11}$$

$$M_{Ek} = \frac{1}{8}\frac{\left(g_d + q_d\right)}{\gamma_g}.L^2 = \frac{1}{8}.\frac{14{,}41}{1{,}35}.16^2 = 342\ \text{kNm} \tag{12}$$

$$M_{Ek} = 342\ \text{kNm} \leq M_{cr} = 344\ \text{kNm} \tag{13}$$

3.3 Creep coefficient

The deflection of a prestressed component is affected by common factors (the amount of load, the stiffness of the cross-section), but also by stiffness degradation, when the cracking moment is exceeded, and also by the time-dependent characteristics of the concrete (shrinkage, creep). It is supposed that the cracks would not occur and that the shrinkage is significant only for non-symmetrically reinforced cross-sections. In the deflection analysis of pre-stressed purlin, only creep is taken into consideration, which is characterized by the creep coefficient.

The creep coefficient is determined according to Appendix B of EC 2. The initiation of prestress is given in time t_0=1 day. The basic creep coefficient is especially influenced by the relative humidity of the environment. According to CSN 731201, the usual environment is defined as having a relative humidity in the range of 30% to 80%. For the parametric analysis, the humidity is considered using the average value RH_1 = 50% and lowest value RH_2 = 30%.

In the paper [5] the shrinkage and creep determined according to EC 2 and measured on the actual structure are compared. The time-dependent strain had been measured for 9 years and the strain had significantly risen the whole time. One of the reasons is probably due to the lower relative humidity in air-conditioned buildings. Taking into consideration that purlin is used especially in the air-conditioned buildings of shopping centres, the value of the relative humidity for the parametric analysis is given the lowest value.

The basic values of the creep coefficient are:

- $\varphi_{0,RH=50} = 2.46$ for the relative humidity of environment RH_1 = 50%
- $\varphi_{0,RH=30} = 2.85$ for the relative humidity of environment RH_2 = 30%

It is also possible to determine the creep coefficient as a function of time. The coefficient β_c describes the development of creep over time after loading and takes into account the time of the active load and the relative humidity of the environment. The creep coefficient as a function of time for a relative humidity of 50% and 30% is in Fig. 5 and Fig. 6.

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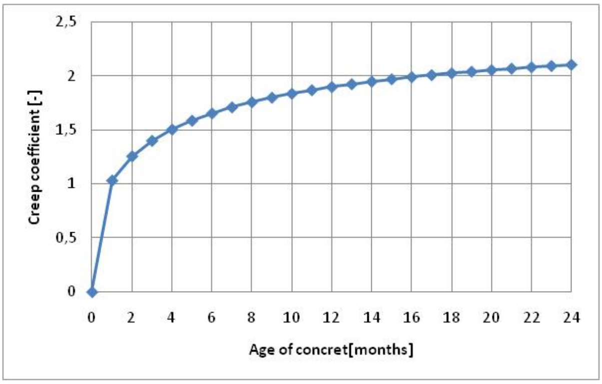

Fig.5: The creep coefficient as a function of time for a relative humidity of 50%, 0-24 months

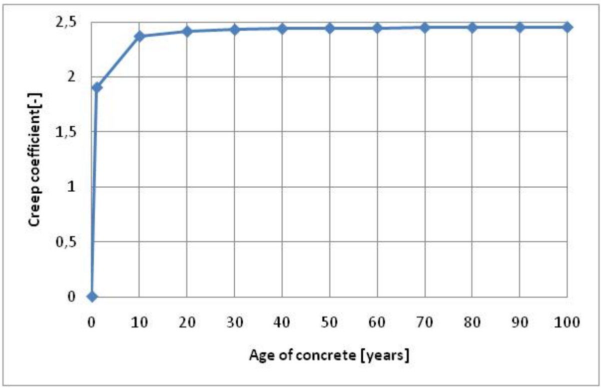

Fig.6: The creep coefficient as a function of time for a relative humidity of 50%, 0-100 years

3.4 Determination of deflection

Total deflection (14) is the sum of long-term (15) and short-term deflection (16), and while the long-term load consists of prestressed strength, own weight and other dead loads, the short-term load consists of a live load. The creep effect is expressed with an effective modulus of elasticity (17).

$$f_{tot} = f_{lt} + f_{st} \tag{14}$$

$$f_{lt} = -\frac{P_1.(d_1 - z_{Ih}).L^2}{8.E_{c,eff}I_I} - \frac{P_2.(d_2 - z_{Ih}).L^2}{8.E_{c,eff}I_I} - \frac{P_3.(d_3 - z_{Ih})}{8.E_{c,eff}I_I} + \frac{5}{384}.\frac{g_k.L^4}{E_{c,eff}I_I} \tag{15}$$

$$f_{st} = +\frac{5}{384}.\frac{q_k.L^4}{E_{cm}I_I} \tag{16}$$

$$E_{c,eff} = \frac{E_{cm}}{1+\varphi(t,t_0)} \tag{17}$$

where:

f_{tot} – total deflection [m],

f_{lt} – long-term deflection [m],

f_{st} – short-term deflection [m],

E_{cm} – modulus of elasticity [GPa],

$E_{c,eff}$ – effective modulus of elasticity [GPa],

$\varphi(t,t_0)$ – creep coefficient as a function of time t and time of action initiation t_0 [-].

Total deflection is influenced by the dead and live load ratio and creep coefficient. Deflection as a function of time for a different ratio between dead and live loads and the creep coefficient fixed for the relative humidity RH_1 = 50% is in Table 5; for the relative humidity RH_2 = 30%, in Table 6.

Tab. 5: Deflection as a function of dead and live load ration, RH = 50%

Ratio g_k/q_k			0,6/0,4			0,75/0,25			1/0		
Age of concrete	φ	$E_{c,eff}$	f_{lt}	f_{st}	f_{TOT}	f_{lt}	f_{st}	f_{TOT}	f_{lt}	f_{st}	f_{TOT}
years		GPa	m	m	m	m	m	m	m	m	m
1 month	1,033	17,711	-0,003	0,014	0,011	0,009	0,009	0,018	0,028	0	0,028
1	1,898	12,424	-0,004	0,014	0,010	0,013	0,009	0,022	0,040	0	0,040
2	2,103	11,600	-0,004	0,014	0,010	0,013	0,009	0,022	0,043	0	0,043
5	2,291	10,939	-0,005	0,014	0,009	0,014	0,009	0,023	0,045	0	0,045
10	2,370	10,682	-0,005	0,014	0,009	0,015	0,009	0,024	0,047	0	0,047
100	2,452	10,429	-0,005	0,014	0,009	0,015	0,009	0,024	0,048	0	0,048
Max. deflection divergence			Δ=12 mm			Δ=15 mm			Δ=20 mm		

Tab. 6: Deflection as a function of dead and live load ration, RH = 30%

Ratio g_k/q_k			0,6/0,4			0,75/0,25			1/0		
Age of concrete	φ	$E_{c,eff}$	f_{lt}	f_{st}	f_{TOT}	f_{lt}	f_{st}	f_{TOT}	f_{lt}	f_{st}	f_{TOT}
years		GPa	m	m	m	m	m	m	m	m	m
1 month	1,195	16,401	-0,003	0,014	0,011	0,010	0,009	0,019	0,030	0,000	0,030
1	2,196	11,264	-0,005	0,014	0,009	0,014	0,009	0,023	0,044	0,000	0,044
2	2,434	10,483	-0,005	0,014	0,009	0,015	0,009	0,024	0,047	0,000	0,047
5	2,651	9,860	-0,005	0,014	0,009	0,016	0,009	0,025	0,050	0,000	0,050
10	2,742	9,621	-0,005	0,014	0,009	0,016	0,009	0,025	0,052	0,000	0,052
100	2,837	9,382	-0,006	0,014	0,008	0,017	0,009	0,026	0,053	0,000	0,053
Max. deflection divergence			Δ=11 mm			Δ=16 mm			Δ=23 mm		

3.5 Discussion

The total deflection limit in EC 2 is considered according to an equation (18), the deflection from other loads in the building component is considered according to equation (19). Though the determined deflection does not reach the limit values, it is evident in Table 5 and Table 6 that the deflection varies over time; the maximum difference in deflection for the least acceptable conditions could be up to 23 mm. These differences could be the reason for the failure of the adjacent structures.

$$f_{\mathrm{lim}} = \frac{L}{250} = \frac{16000}{250} = 64 \text{ mm} \tag{18}$$

$$f_{\mathrm{lim}2} = \frac{L}{500} = \frac{16000}{500} = 32 \text{ mm} \tag{19}$$

$$\Delta_{\max} = f_{TOT,100yeras} - f_{lt,1month} \tag{20}$$

4 CONCLUSION

In the paper an analysis is conducted of the limit state of the load-bearing capacity and limit state of serviceability of prestressed purlin with a span of 16 m.

The limit state of the load-bearing capacity is focused on the ultimate bending moment of the carrying capacity. The carrying capacities formerly valid under CSN 731201 and EC 2 are compared. The possibility of using a simpler limit equilibrium method is discussed.

Furthermore, the serviceability limit state is analysed, taking into consideration the occurrence of cracks and the size of deflection. Though the determined deflections do not reach the limit values, the variation of deflection owing to a variation in loading and concrete creep are significant and could be the source of the failure of the adjacent structures during the service life of the building.

AKNOWLEDGEMENTS

This outcome has been achieved with the financial support of the Ministry of Education, Youth and Sports of the Czech Republic, project No. 1M0579, within activities of the CIDEAS research centre.

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Reviewer:

Jaroslav Navrátil, MSc., PhD., docent, IDEA RS s.r.o., South Moravian Innovation Centre, Brno, Czech Republic.