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**Karel VOJTASÍK<sup>1</sup>, Eva HRUBEŠOVÁ<sup>2</sup>, Marek MOHYLA<sup>3</sup>, Jana STAŇKOVÁ<sup>4</sup>****AN ASSESSMENT OF STRESS STATE IN SECTION OF STEEL CONCRETE TUNNEL LINING  
BASED ON A THEORY OF SIMULTANEOUSLY WORKING RINGS****Abstract**

The paper describes a method of setting up a stress state in a section of the steel concrete tunnel lining. This method evolves from the theory of simultaneously working rings. The theory provides for the evaluation of strain parameters of the steel concrete tunnel lining section as they change themselves in the course of lining construction. The evaluation of stress state in section of the primary lining depends on the stress modulus of the homogenized lining section and on the radial stress transfer coefficients, which are outputs of the simultaneously working rings theory. On the basis of this method, the tangential stress redistribution coefficients redraft the stress state of the homogenized lining section for the real stress state in constituent section materials of the steel concrete tunnel lining, i.e. in the steel elements and the shotcrete.

**Keywords**

Tunnel primary lining, steel concrete lining, strain parameters, stress state in lining section.

**Abstrakt**

Článek popisuje metodu, kterou se zjišťuje stav napjatosti v průřezu ocelobetonového tunelového ostění. Metoda je odvozena z teorie spolupracujících prstenců. Tato metoda vyhodnocuje parametry ocelobetonového ostění tunelu, které se mění v průběhu výstavby. Vyhodnocení stavu napjatosti v průřezu primárního ostění spočívá na modulu pružnosti homogenizovaného průřezu a na přenosových koeficientech radiálního napětí, které jsou výsledkem teorie spolupracujících prstenců. Na základě uvedené metody lze stanovit přerozdělovací koeficienty tangenciálních napětí, které umožňují přepočítání napjatosti homogenizovaného průřezu ostění na skutečný stav napjatosti v průřezu tunelového ostění z oceli a betonu, tj. v ocelových prvcích a stříkaném betonu.

**Klíčová slova**

Primární ostění tunelu, ocelobetonová výztuž, přetvárné parametry, stav napětí v průřezu výztuže.

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## 1 INTRODUCTION

The steel concrete lining of underground work is a specific construction characterized by the concurrence of construction processes and execution of a stabilization function. The stabilization function begins at the moment when a concrete mixture is applied (shot) on the wall of the slope reinforced by steel elements. As reinforcement, expanded steel and steel frames from lattice, rolled or welded steel bars are generally used. Steel elements create an immediate stabilization reaction. Shotcrete integrates steel elements with rock mass. Integration ensures cooperation of OCB lining construction and rock mass, effective use of carrying capacity of the lining construction and mobilization of stable potential of geological environment. Thanks to shotcrete have OCB lining variable character; this results from two conditions. The first condition is objective and relates to solidification of the concrete mixture. The second, constructional condition, relates to execution of OCB lining, when the shotcrete is applied gradually, by spraying in two or several layers. Generally, there is a 24-hour period between the application of individual layers. The stated conditions are reason why parameters, and also the carrying capacity, of OCB lining develop during the construction until the moment the solidification of the concrete is completed. This characteristic of OCB lining is very important in terms of geological environment stabilization. A gradual increase of OCB lining solidity enables a release of stress from the geological environment. Steel elements play the main role after rock excavation and immediate installation of steel concrete lining. Lattice cases the slope; steel frames originate a stabilization reaction. The layer of shotcrete fulfills the role of a perfect filling. The low solidity of the OCB lining prevents no stress release in rock material around the cavity. The solidity and carrying capacity of OCB lining is at this moment given by the deformational and strength properties of the steel elements. In this phase, significant stress release generally occurs in the geological environment. Furthermore, the hardening concrete layer participates in the origination of stabilization reaction of the OCB lining. Completion of the OCB lining stabilization reaction growth is subject to completion of the shotcrete hardening process. Working (deformation) characteristic of OCB lining is given by development of solidity (stress modulus) of OCB lining section [3]. Stress inside the OCB lining section in its individual construction materials will represent the development of stiffness in the OCB lining section.

## 2 THEORY OF SIMULTANEOUSLY WORKING RINGS

The theory of simultaneously working rings enables a homogenization of a materially heterogeneous section of OCB lining. The theory results from an analytical model for calculation of stress-deformation condition in multilayer ring defined by professor Bulytchev [1]. This analytical model uses the theory of analytical functions of complex variable, the theory of complex potentials and function of Kolosov. The algorithm results from the assumption that an external load (normal and horizontal) of the ring is transferred by individual layers by means of so-called “transfer coefficients,” which generally result from the condition of continuity regarding the deformation on individual contacts of reinforcement layer. Such transfer coefficients are functions of layer thickness and strain characteristics of layer materials (Poisson’s ratio, stress modulus). The method results from the following shape of the external load (Illustration 1)

$$p = p_n = p_0 + p_2 \cos 2\theta$$

$$q = q_n = q_2 \sin 2\theta$$

where

$p_0$  – radial symmetric component of normal external load

$p_2$  – radial asymmetric component of normal external load

$q_2$  – component of external tangential load

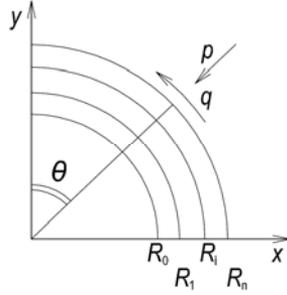


Illustration1: basic calculation scheme

Stress  $p_k, q_k$  on individual contacts of layers are defined by means of transfer coefficients as follows:

$$\begin{aligned}
 p_k &= p_0(k) + p_2(k) \cos 2\theta \\
 q_k &= q_2(k) \sin 2\theta \\
 p_0(k) &= \left( \prod_{i=k+1}^n K_0(i) \right) p_0 \\
 \begin{pmatrix} p_2(k) \\ q_2(k) \end{pmatrix} &= \left[ \prod_{i=k+1}^n \begin{pmatrix} K_{pp}(i) & K_{pq}(i) \\ K_{qp}(i) & K_{qq}(i) \end{pmatrix} \right] \begin{pmatrix} p_2 \\ q_2 \end{pmatrix}
 \end{aligned}$$

where  $K_0(i), K_{pp}(i), K_{pq}(i), K_{qp}(i), K_{qq}(i), i=1, \dots, n$  are transfer coefficients via  $i$  layer of the reinforcement (the transfer coefficients via the first (inner) layer equal zero). To such expressed values of radial stress on individual contacts of layers corresponds tangential stress and movements on contacts of layers. The tangential stress on the inner and external profile of the reinforcement ring can be expressed as follows:

inner profile of  $k$  layer:

$$\sigma_\theta(k, \text{inner}) = p_0(k)m_1 - p_0(k-1)m_2$$

external profile of  $k$  layer:

$$\sigma_\theta(k, \text{external}) = p_0(k)m'_1 - p_0(k-1)m'_2$$

where

$$m_1 = \frac{2c^2}{c^2 - 1}, m_2 = \frac{c^2 + 1}{c^2 - 1}, m'_1 = m_2, m'_2 = \frac{2}{c^2 - 1}, c = \frac{R_k}{R_{k-1}}$$

The stated calculation method represents a base for determination of quasi-homogenous stress modulus of non-homogenous lining. The non-homogenous lining with inner inserts from different material (e.g. steel inserts) can be divided into individual layers – some of these layers are homogenous and some are non-homogenous with evenly changing individual areas with varied solidity (filler, insert) (see Illustration 2). This entire lining can be regarded as a special case of multilayer lining and for determination of stress-deformation condition it can be resulted from the already-mentioned algorithm for solution of multilayer rings.

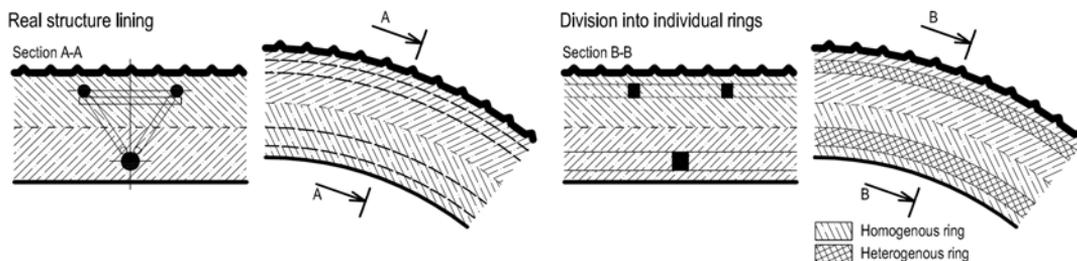


Illustration 2: breakdown of section of OCB lining on rings

Calculation for determination of the sole representative value of stress module regarding non-homogenous lining is divided in two steps:

- partial homogenization of materially non-homogenous rings (concrete-steel, steel-free space)
- global homogenization of partial rings executed gradually, always on two adjoining rings. In the first step, first two rings – inner ring and its neighbor – are homogenized. This process then repeats; to newly originated ring (from two previous rings), next partial rings is adjoined. The process repeats until all rings are adjoined in one complex.

The result of homogenization is the sole value of the stress module representing the structure and constructional materials of the lining profile and stress transfer coefficients. Redistribution coefficients, which serve for reverse calculation of stress values in actual materials (i.e. concrete and steel) in outer fibers of homogenous and non-homogenous rings, are further derived from the stress transfer coefficients.

Principle and basic relations regarding the theory of simultaneously working rings are described in detail in article [2]. A computer program was consequently developed regarding this theory; the program is called HOMO (Illustration 3). The program determined values of all the above-mentioned parameters that characterize the process of homogenization, i.e.: stress modulus of homogenized components of the lining section; coefficients of transfer regarding stress between the rings; material stress redistribution coefficients for outer fibers of homogenous and non-homogenous rings. The following example states results of the calculation of parameters regarding homogenization process and their use for determination of inner forces in concrete and steel for standard construction of OCB lining section.



Illustration 3: program HOMO

### 3 EXAMPLE

Illustration 4 captures stages of OCB lining section execution; OCB lining is constructed from steel lattice beam ASTA 95, the span of bar-shaped frames is one meter, and from shotcrete applied gradually in two identical layers with a one-day interval. The influence of steel lattice is not considered. Course regarding the relation of concrete hardening and time is stated in a graph on Illustration 5. From this relation, the actual values of concrete stress modulus are deduced for the given time of hardening.

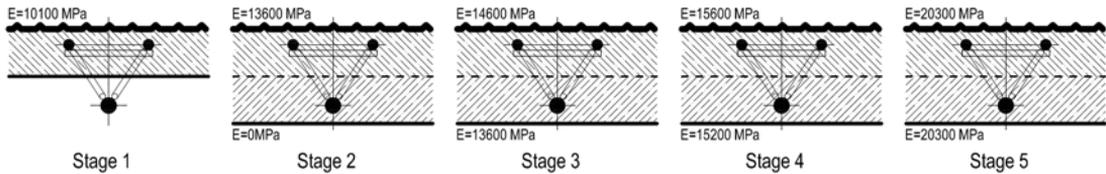


Illustration 4: stages for execution of OCB lining section

Calculation of OCB lining is divided into five stages. Each stage corresponds to a limit situation in execution of OCB lining section. First stage – initiation of construction, construction of steel frame and casing – application (spraying) of the first layer of concrete (1/2 day;  $E_{1v}=10100\text{MPa}$ ). Second stage – application (spraying) of the second layer of concrete (1 1/2 day;  $E_{1v}=13600\text{MPa}$ ;  $E_{2v}=0\text{MPa}$ ). In the third and fourth stage, the layers of shotcrete different have an stress modulus ( $E_{1v}>E_{2v}$ ), see Table 1. Lastly, the fifth stage represents a situation when stress modules equal in both layers ( $E_{1v}=E_{2v}$ ) and won't further increase.

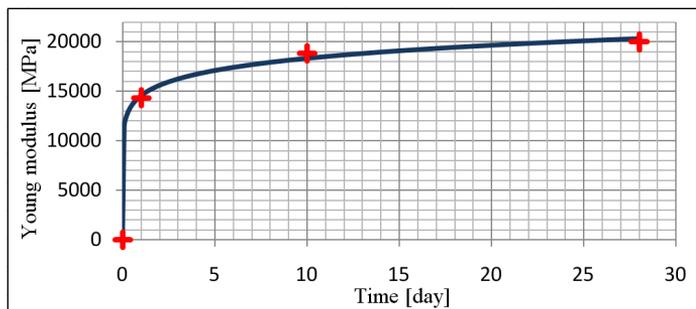


Illustration 5: relation of concrete hardening in time

In Table 1, values of stress modulus regarding homogenized section of OCB lining and redistribution coefficients of tangential stress are summarized for the given example of OCB lining – as calculated by HOMO program.

Stress modules of homogenized sections are determined for consequent static solutions of OCB lining in corresponding stages of construction. Redistribution coefficients of tangential stress are determined for calculation of condition regarding stress from homogenized section to condition of stress in OCB lining section in corresponding stage of construction.

Table 1: calculation results of modules regarding homogenized sections of OCB lining and redistribution coefficients of tangential stress ( $a_1$  – internal fibers of the ring;  $a_2$  – external fibers of the ring)

Ring num.	Material	Parameters	Stage				
			1	2	3	4	5
1	Shotcrete	Ebet. [MPa]	0	0	13600	15200	20300
		a1	0	0	0,866	0,893	0,927
		a2	0	0	0,865	0,892	0,926
2	Shotcrete	Ebet. [MPa]	0	0	13600	15200	20300
		a1	0	0	0,865	0,892	0,926
		a2	0	0	0,864	0,890	0,925
	Steel*)	a1	30,031	23,892	13,355	12,326	9,575
a2		29,989	23,859	13,337	12,309	9,562	
3	Shotcrete	Ebet. [MPa]	0	0	13600	15200	20300
		a1	0	0	0,866	0,892	0,926
		a2	0	0	0,865	0,891	0,926
4	Shotcrete	Ebet. [MPa]	10100	13600	14600	15600	20300
		a1	1,762	1,830	0,926	0,917	0,927
		a2	1,761	1,829	0,927	0,918	0,926
5	Shotcrete	Ebet. [MPa]	10100	13600	14600	15600	20300
		a1	1,761	1,829	0,927	0,918	0,928
		a2	1,762	1,830	0,928	0,915	0,929
	Steel*)	a1	36,543	28,13	13,156	12,144	9,437
a2		36,551	28,136	13,159	12,147	9,440	
6	Shotcrete	Ebet. [MPa]	10100	13600	14600	15600	20300
		a1	1,765	1,832	0,929	0,919	0,930
		a2	1,766	1,834	0,930	0,920	0,931
E <sub>homo.</sub> [MPa]			6500	8200	15600	17000	22000

\*) steel - E = 210 000 MPa

Graphs in Illustration 6 show conditions of stress in OCB lining section for the fourth stage of construction. It graphically expresses courses of stress in homogenized section and courses of stress in concrete and steel calculated with the use of corresponding redistribution coefficients of tangential stress.

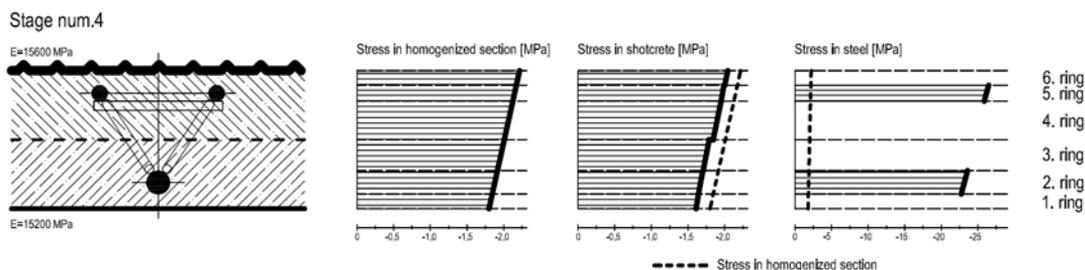


Illustration 6: course of stress distribution in materials, concrete and steel, in OCB lining section for the fourth stage

#### 4 CONCLUSION

The theory of simultaneously working rings and related homogenization of the section form base for determination of working characteristics and condition of stress in OCB lining characterized

by heterogeneous structure of the section, time versatility of deformational parameters of constructional materials and individual stages of the construction.

The stated approach and derived method solve only one part of a much more complex problem. The condition of stress in steel concrete reinforcement of underground works depends not only on their construction but also on the behavior of the geological environment, which is to a significant extent determined by the working characteristic (development of stiffness) of the reinforcement.

In our case, this eventuality was not considered, and the condition of stress in the section was determined only for one partial stage of OCB lining construction.

The submitted method enables complete inclusion of all stages of OCB lining construction. However, it requires knowledge of relations regarding interaction between reactions of the reinforcement, its variable stiffness and value of load from geological environment. If we know values of load for every corresponding stage of OCB lining construction, we will be able to determine development of stress in OCB lining from initiation of the construction until final stabilization of geological environment.

The theory of simultaneously working rings and related analytical method for determination of deformation parameters of steel concrete reinforcement sections represent alternative to standard procedures regarding evaluation of reinforced concrete sections or solution derived from numerical methods, which, compared with the proposed analytical method, strive for involvement of all elements of the complex in their actual parameters.

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