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CALCULATION OF FATIGUE CRACK PROPAGATION USING DOPROC METHOD

VYUŽITÍ METODY POPV K VÝPOČTU ŠÍŘENÍ ÚNAVOVÝCH TRHLIN

Abstract

Probabilistic calculation of steel structures and bridges using DOProC method, leads to the probabilities of three basic random events in dependence on years of bridge's operation and fatigue crack propagation. On the basis of that calculation for each individual year, determined by analysis of reliability function, the dependence of the failure probability on time of the bridge‘s operation is specified. When the limit reliability is known, it is possible to determine times of the structure's inspections using conditional probability.

Keywords

Direct Optimized Probabilistic Calculation, DOProC, software package ProbCalc, fatigue crack, weakened cross section area, linear fracture mechanics, acceptable size, detectable size, initial size, propagation from the edge, propagation from the surface, conditional probability, inspection of structure.

Abstrakt

U ocelových konstrukcí a mostů namáhaných únavou lze stanovit pravděpodobnosti pro základní jevy, které mohou nastat v libovolném čase t životnosti konstrukce a souvisí s růstem únavové trhliny. Tyto pravděpodobnosti, určené na základě analýzy funkce spolehlivosti pro každý rok provozu konstrukce např. metodou POPV, jsou výchozím podkladem pro stanovení času prohlídek cyklicky namáhané ocelové konstrukce nebo mostu s využitím podmíněné pravděpodobnosti.

Klíčová slova

Přímý Optimalizovaný Pravděpodobnostní Výpočet, POPV, programový systém ProbCalc, únavová trhlina, plocha oslabení, lineární lomová mechanika, přípustný rozměr, měřitelný rozměr, iniciační rozměr, šíření z okraje, šíření z povrchu, podmíněná pravděpodobnost, prohlídka konstrukce.

1 INTRODUCTION

Reliability of the load-bearing structure has been significantly influenced by degradation resulting, in particular, from the fatigue of the basic materials. Wöhler's curves are used when designing such structures. The service life can be limited until a failure occurs. The failure is,

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however, very difficult to determine. For purposes of the modeling, the amplitude oscillation is considered to be constant, and a certain number of load cycles is taken into account. The method has been developed to provide procedures describing real conditions, all this making the work of design engineers easier. As fatigue cracks appear randomly on existing structures (in crane rails and bridges), it is believed that the designing method is imperfect to a certain extent. Methods are under development that would be able to reveal potential defects and damage resulting from initiation cracks that accelerate considerably the propagation of fatigue cracks. Linear fracture mechanics is among alternative methods. Machinery experts have been dealing with such issues for many years. Results have been gradually taken over and implemented into designs of the loading structures in buildings. This approach is typically used for the determination of times of inspection and analyses of inspection results. If cracks are not found, a conditional probability exists that they might appear later on.

Attention is paid to fatigue damage of building steel structures and bridges where the acceptable fatigue crack size is assessed. The acceptable crack size plays a key role in degradation of an element dimensioned for an extreme loading combination that is exposed to variable operation loads. It represents a possible degradation of an element in an ultimate limit state that can be still monitored.

The outcome is procedures that should clarify currently acceptable methods used for the designing of the fatigue crack in the context of the safe service life and acceptable failure rate. A flange of the composite reinforced concrete bridge has been chosen for applications of the theoretical solution. This tension is exposed, in particular, to tension. Depending on location of an initial crack, the crack may propagate from the edge (such as [4] or [9]) or surface (such as [10] or [11]). Regarding the frequency, weight and concentration of stresses, those locations rank among those with the major hazard of fatigue cracks appearing in the steel structures and bridges.

he tasks are based on the Direct Optimized Probabilistic Calculation ("DOProC" - see, for instance [12] to [19]) that determines the P_f failure probability and time intervals for regular inspections of the construction.

2 PROBABILISTIC APPROACH TO THE PROPAGATION OF FATIGUE CRACKS

Occurrence of initiation cracks and crack propagation in structures subject to fatigue load has been known for a long time. The process is closely connected with fabrication of the steel structures and, in particular, with creation of details which tend to be damaged by fatigue. The key difference is between initiation of cracks resulting from steelmaking inclusions and those created during fabrication of structural details. Regarding the former, it takes a long time until it reaches the surface, while the latter is at the surface from the beginning of the loading. Standardized approaches of previous EC standards suppose that surface cracks were not present there. The acceptable damage method which is described in the new standard admits random occurrence of surface cracks. The major difference is that a fatigue crack might not be fragile, but could be ductile. In real components of steel structures and bridges, the latter is more frequent that the former which is used in experimental measurements in processed small test-pieces. This fact is not a new phenomenon. It has been known for a long time and has been mentioned, for instance, by T.L.Anderson [1]. During the designing, fabrication and processing of details, nobody, however, paid attention to random occurrence of initiation cracks from surface areas (from the surface or from the edge).

Three sizes are important for the characteristics of the propagation of fatigue cracks. These are the initiation size, the detectable size and the final size which occurs prior to failure caused by a fragile or ductile crack.

The fatigue crack damage depends on a number of stress range cycles. This is a time factor in the course of reliability for the entire designed service life. In the course of time, the failure rate increases, while the reliability drops.

The topic is discussed in two levels that affect each other: the probabilistic solution to the propagation of the fatigue crack and uncertainties in determination of quantities used in the calculation.

When investigating into the propagation, the fatigue crack that deteriorates a certain area of the structure components is described with one dimension only: *a*. In order to describe the propagation of the crack, the linear elastic fracture mechanics is typically used. It is based on the Paris-Erdogan law:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C.(\Delta K)^m \ , \tag{1}$$

where C, m are material constants, a is the crack size and N is the number of loading cycles.

The initial assumption is that the primary design should take into account the effects of the extreme loading resulting from the ultimate state of carrying capacity method. Then, the fatigue resistance should be assessed. This means, the reliability margin in the technical probability method is:

$$g_{(R,S)} = G = R - S \ , \tag{2}$$

where R is the random resistance of the element and S represents random variable effects of the extreme load.

When using (1), the condition for the acceptable crack length a_{ac} is:

$$N = \frac{1}{C}\int_{a_0}^{a_{ac}} \frac{\mathrm{d}a}{\Delta K^m} > N_{cel} \ , \tag{3}$$

where N is the number of cycles needed to increase the crack from the initiation size a_0 to the acceptable crack size a_{ac}, and N_{tot} is the number of cycles throughout the service life.

The equation for the propagation of the crack size (1) needs to be modified for this purpose. The state of stress near the crack face is described using (the stress intensity coefficient) which depends on the loading (bending, tension), size and shape of the fatigue crack, and geometry of the load-bearing component. If the stress range and axial stress-load of the flange are constant, the following relation applies:

$$\Delta K = \Delta\sigma.\sqrt{\pi.a}.F_{(a)} \ . \tag{4}$$

$F_{(a)}$ is the calibration function which corresponds to propagation behavior of the crack. Once (1) is modified using (4), the formula is following:

$$\int_{a_1}^{a_2} \frac{\mathrm{d}a}{\left(\sqrt{\pi.a}.F_{(a)}\right)^m} = \int_{N_1}^{N_2} C.\Delta\sigma^m.\mathrm{d}N \ . \tag{5}$$

While the left side of the formula describes the reliability of the structure - R, the right side defines accumulated load effects - S.

It is possible to define the reliability function, where the analysis of the reliability function gives the failure probability, P_f :

$$G_{fail(Z)} = R_{(a_2)} - S \ , \tag{6}$$

where Z is a vector of random physical properties such as mechanical properties, geometry of the structure, load effects and dimensions of the fatigue crack. The failure probability equals to:

$$P_f = P\left(G_{fail(Z)} < 0\right) = P\left(R_{(a_2)} < S\right). \tag{7}$$

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3 METODOLOGY OF FATIGUE CRACK PROPAGATION

A tension flange has been chosen for applications of the theoretical solution suggested in the studies. Depending on location of an initial crack, the crack may propagate from the edge (see Fig 1) or from the surface (see Fig. 2). Regarding the frequency, weight and stress concentration, those locations rank among those with the major hazard of fatigue cracks appearing in the steel structures and bridges.

A flange without stress concentration is used for confronting the both cases depending on the location of the crack initiation. The cases are different in calibration functions - $F_{(a)}$ - and in weakened surfaces which are appearing during the crack propagation.

3.1 Propagation of the fatigue crack from the edge

For the crack propagating from the edge, the calibration function is:

$$F_{(a)} = 1{,}12 - 0{,}231\left(\frac{a}{b}\right) + 10{,}55\left(\frac{a}{b}\right)^2 - 21{,}72\left(\frac{a}{b}\right)^3 + 30{,}39\left(\frac{a}{b}\right)^4 , \qquad (8)$$

where a is the length of the crack and b is the width of the flange (see Fig.1).

The acceptable crack size - a_{ac} – can be described then by a formula resulting from the deduced weakening of the cross-section area of the flange:

$$a_{ac} = b.\left(1 - \frac{\sigma_{max}}{f_y}\right) . \qquad (9)$$

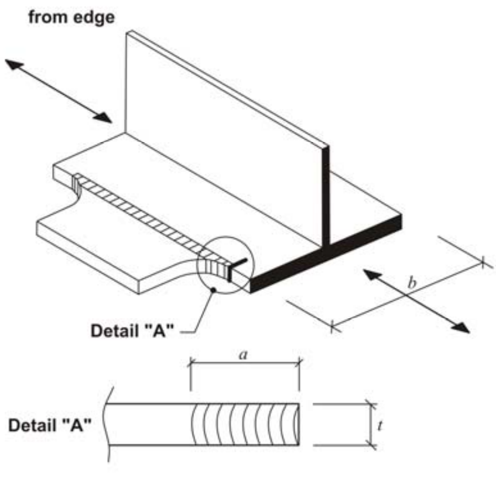

Fig. 1: Characteristic propagation of cracks from the outer edge

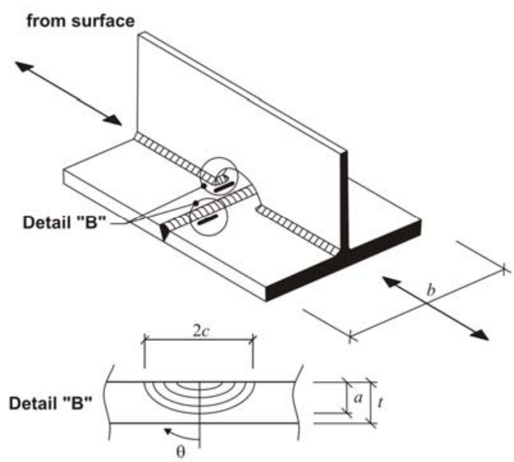

Fig. 2: Characteristic propagation of cracks from the surface

3.2 Propagation of the fatigue crack from the surface

A similar approach can be used to determine the acceptable size of a crack propagating from the surface. The bending component can be neglected for welded steel two-axis symmetric I-profiles where the fatigue crack appears in the lower tension flange. The flange is loaded only by the normal stress resulting from the axial load – tension: $\sigma_m = \sigma$.

It is rather difficult to deduce analytically the acceptable size of the crack propagating from the surface. In accordance with [2], the shape is replaced with a semi-elliptic curve where the ellipsis axes are a (the crack depth) and c (a half of the crack width) – see Fig. 2. The area of the surface crack depends on the number of N loading cycles and is described by the following formula:

$$A_{cr(N)} = \frac{1}{2}\pi . a_{(N)} . c_{(N)} \; . \tag{10}$$

During propagation of the fatigue crack from the surface, it is not enough to monitor only one crack size (which would be sufficient, for instance, for a crack propagating from the edge). In that case, the crack size needs to be analyzed for directions of the both semi-axes: a and c. The propagation of the fatigue crack from the surface in the a direction depends on the propagation in the c direction. Crack velocity propagation is described by (1). In [20] there is a formula for calculation of the crack depth - Δa – as a result of an increased width of the Δc crack:

$$\Delta a = \left\{ \frac{1}{\left[1{,}1 + 0{,}35 \left(\frac{a}{t} \right)^2 \sqrt{\frac{a}{c}} \right]} \right\}^m \Delta c \; . \tag{11}$$

The crack sizes for a and c are during the propagation limited by upper limit values:

$$2.c \le 0{,}8 b_f \text{ and } a \le 0{,}8 t_f \; , \tag{12}$$

If these upper limit values are exceeded, the fatigue crack propagates differently.

[20] gives also the formula for the mutual dependence of the sizes in a and c:

$$c = \frac{0{,}3027}{t} . a^2 + 1{,}0202 . a + 0{,}00699 . t \; . \tag{13}$$

When determining the acceptable crack size, a modified relation, (10), should be taken as a basis. After modification:

$$\sigma_{\max} . \frac{b_f t_f}{b_f t_f - \frac{1}{2}\pi a \left(\frac{0{,}3027}{t_f} . a^2 + 1{,}0202 . a + 0{,}00699 . t_f \right)} \le f_y \tag{14}$$

It is difficult to describe the a crack size directly explicitly. In order to calculate the acceptable crack size - a_{ac}, it is necessary to use a numerical iteration approach where restrictions resulting from (14) should be taken as a basis.

4 PROBABILISTIC CALCULATION OF FATIGUE CRACKS PROPAGATING FROM THE SURFACE

The fatigue reliability of the structure with cracks propagating from the surface was calculated using the Direct Optimized Probabilistic Calculation ("DOProC" - see, for instance [12] to [19]). For the probabilistic calculation of the fatigue crack propagation from the surface, the structure reliability - $R_{(a_d)}$ a $R_{(a_{ac})}$ - should be calculated, and the left side of the equation (5) should be taken into account. Another variable which affects the propagation of the fatigue cracks in the structure is S. This is a loading component which can be determined for each year of operation of the construction using the right-hand side of the formula (5). Finally, it is necessary to calculate P_f using (7) for individual years of operation. This value is the quantille in a negative section of the reliability function histogram, G_{fail} .

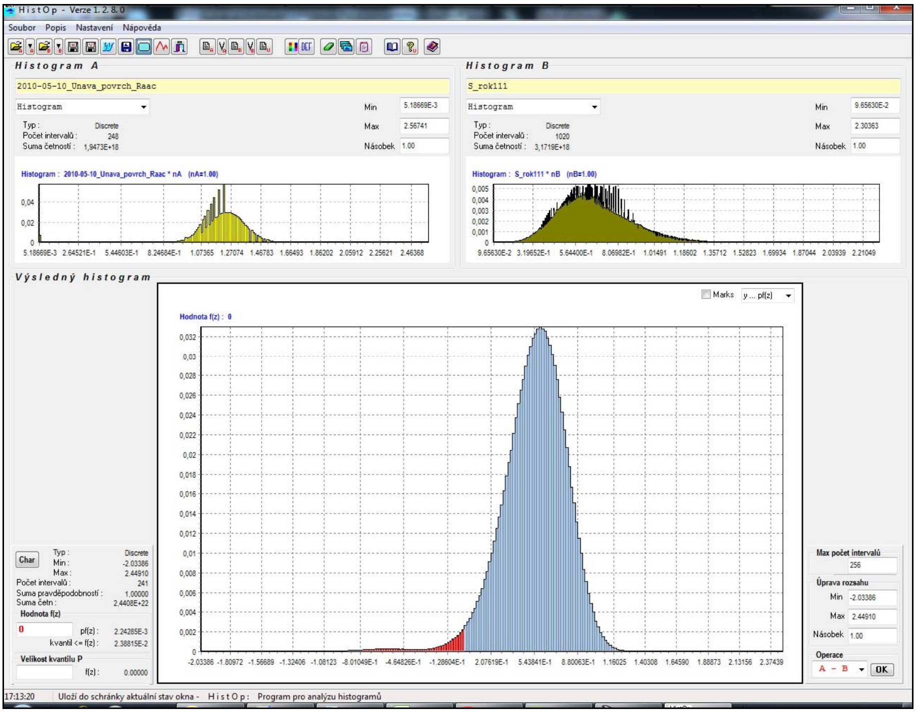

Fig.3: Histogram of reliability function $G_{fail} = R(a_{ac}) - S$ for total number oscillation of stress peak per 111 years, probability of failure $P(G_{fail} < 0) = 2,\ 38815.10^{-2}$ (output from HistOp program)

These failure probabilities are used to determine the probability of random phenomena - U, D and F – which could occur anytime, this means in the „t" time, throughout the service life of the construction. They are defined, for instance, in [8]:

- ***U*(*t*) phenomenon:** No fatigue crack failure has not been revealed within the t–time and the fatigue crack size $a(t)$ has not reached the detectable crack size, a_d . This means:

$$a(t) < a_d \ . \tag{15}$$

- ***D*(*t*) phenomenon:** A fatigue crack failure has been revealed within the t–time and the fatigue crack size $a(t)$ is still below the acceptable crack size a_{ac}. This means:

$$a_d \leq a(t) < a_{ac} \ . \tag{16}$$

- ***F*(*t*) phenomenon:** A failure has been revealed within the t–time and the fatigue crack size $a(t)$ has reached the acceptable crack size a_{ac}. This means:

$$a(t) \geq a_{ac} \ . \tag{17}$$

The three random phenomena are linked to the propagation of the fatigue crack and define a full space for occurrence of such cracks. The probabilities of the random phenomena have been calculated for each year of the service life of the construction. Fig. 4 shows an example for the time t = 70 to 120 years.

Using the calculated probability of the F failure, P_f, and the required reliability, it was possible to define the time for the first inspection of the bridge. The required reliability was expressed by the designed failure probability P_d = 0,02277. Fig. 5 shows the dependence of the failure probability, P_f , on years of operation of the construction. In case of the steel bridge under investigation, it was calculated that the first inspection and check for a fatigue crack on the surface should take place in the 111th year of operation of the bridge.

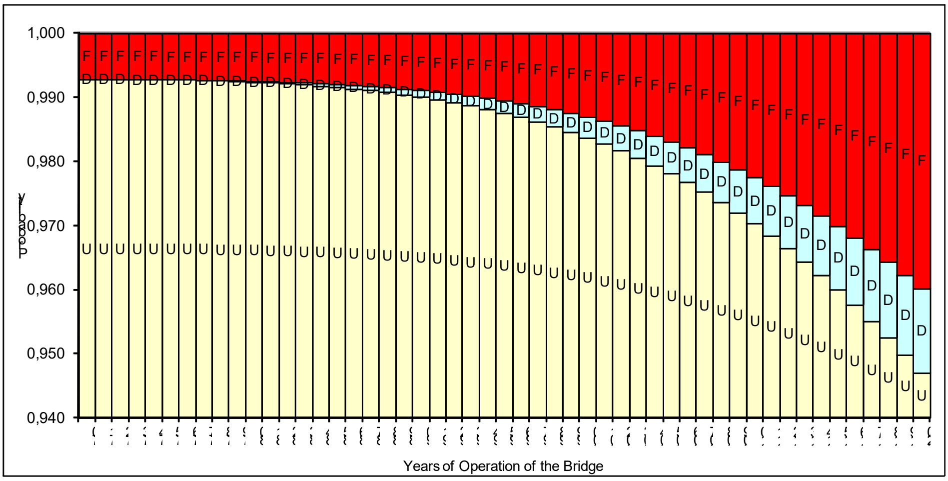

Fig.4: Probability of the *U*, *D* and F phenomena depending on the years of operation of the bridge (the period between 70 to 120 years)

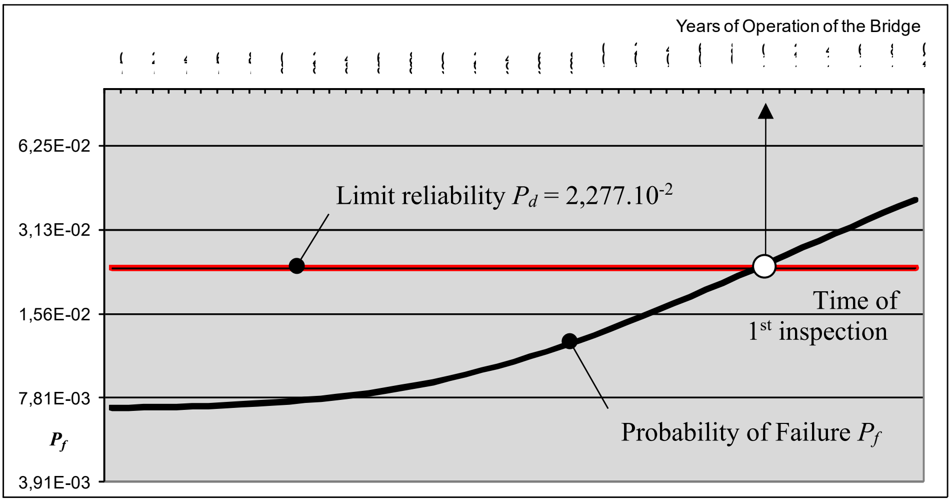

Fig.5: Dependence of the failure probability P_f on the years of operation of the bridge needed for the time of the first inspection of the bridge structure

Comparison with the propagation of the fatigue crack from an edge proved that velocity of propagation of the fatigue crack from the surface is considerably slower. If this velocity is confronted to the time of the first inspection that is the 54th year of operation for the fatigue crack from the edge, the fatigue crack from the surface propagates two times slower.

5 CONCLUSION

This paper provides theoretical background of propagation and practical introduction into the fatigue cracks. A particular attention is paid to the maximum acceptable crack size. The final fatigue crack size may contribute to a division made between the critical crack size and acceptable crack size. The acceptable crack size comprises safety margins for the critical crack size that may occur in consequence of a brittle fracture and, more often in steel structures, in consequence of a ductile fracture.

The acceptable fatigue crack is the size that might be achieved, in cross-sections and elements of steel structures and steel bridges dimensioned for the combined extreme loads, as a result of gradual degradation when the required reliability is reached at the end of the designed service life of the structure.

The new method is the acceptable damage method, and the name itself explains the approach. Damage is caused by a potential defect that has not been corrected and becomes an initiation crack. The expected crack size or non-existence should be revealed during a special system of inspections. Those inspections are considerably more important than standard inspections. This relates both to individual time and quality of inspections.

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