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BENDING TEST-BASED DETERMINATION OF EFFECTIVE CROSS-SECTION STIFFNESS

STANOVENÍ EFEKTIVNÍ TUHOSTI PRŮŘEZU OCELOVÉHO PRVKU Z OHYBOVÝCH ZKOUŠEK

Abstract

For majority of materials the stress-strain relation becomes non-linear as soon as the normal stress exceeds its limit value. The non-linear behavior manifests itself through the change of the cross-section stiffness *EI*. In this paper the effective (secant) stiffness *EI* is determined as a function of the relative cross-section rotation dφ from the bending-test. To this end, the displacement method is utilized along with an iterative procedure.

Keywords

Cross-section stiffness, bending test, physical nonlinearity, displacement method.

Abstrakt

Pro většinu používaných materiálů platí, že při překročení limitní hodnoty napětí materiálu již není závislost mezi vektorem napětí a vektorem deformací v určitých úsecích lineární. Vliv fyzikálně nelineárního chování se projeví u konstrukce změnou tuhosti materiálu *EI*. V tomto článku je uveden postup určení efektivní (sečnové) tuhosti *EI* průřezu ocelového prvku z ohybových zkoušek v závislosti na relativním natočení průřezu dφ. K určení *EI* použijeme obecnou deformační metodu a iterační postup výpočtu.

Klíčová slova

Tuhost materiálu, ohybová zkouška, fyzikální nelinearita, deformační metoda.

1 INTRODUCTION

Physical behaviour and non-linear geometry of structures can be affected by non-elastic behaviour of materials and changes in rod section shapes in the structure. Such changes result in changes of the bending stiffness of the rods. In [1], the efficient stiffness was derived as a function of a bending moment - *M*. If the geometry of the bends is non-linear, [3], as many as three deformation conditions of the construction may exist for one load (this means for one bending moment - *M*). Even if the normal force is zero or constant, the bending moment cannot define clearly the effective stiffness - *EI*. With rather big deformations, a single bending moment can be related to as many as two values of the effective stiffness (even if the normal forces are same). It follows from observations that *EI* cannot be defined unambiguously as a function of a change in its displacement - dφ.

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2 PROCEDURE

The effective (secant) stiffness *EI* with a zero normal force will be derived for a simple one meter long rod 1 consisting of a P-28 profile. The beam is divided along its length into *n* segments. The assumption is that the structure and each cross-section of the structure is made from an elastic-plastic material and Navier-Bernoulli's hypothesis apply: the flatness of the cross-section is maintained and does not change during the loading process. Effects of dislocating forces, stress, tilting and loss of stability are neglected. A general deformation method [2] and an iteration calculation will be used for determination of the effective stiffness. The calculation takes into account the geometric non-linearity.

The effective stiffness, *EI*, will be determined for relative angular displacement of individual cross-sections $d\varphi$. Input values for the calculation are the values resulting from the bending test in Fig. 3:

1. The beam is divided into *n* segment (considering the height of the profile, *n* is chosen to be 16 - then, the length of the two segments in the centre is 125 mm which is comparable with the height, *h*, of the profile P-28)
2. The force *P* is determined on the basis of deflection of the beam centre w_s (chapter 2.1)
3. The force *P* will be compared with the force P_{zk} which corresponds to the deflection w_s pursuant to the bending test (Fig. 3).
4. If the resulting force is different, the effective stiffness *EI* of the two central segments of the beam (Fig. 1) needs to be changed, until the resulting force corresponds to the force P_{zk} pursuant to the bending test with the required accuracy ε_2 (in the first calculation step, only *EI* of the two central segments is changed, then, *EI* is changed for all segments of the beam).
5. This procedure is applied to all values of the deflection in the centre of the beam $w_{s.}$
6. This method results in the effective stiffness values, *EI*, for each relative angular displacement of the segments $d\varphi$ is allocated (the stiffness curve is available after drawing the values into the chart).
7. The entire procedure is repeated. But in the subsequent steps, the stiffness is changed for all segments of the beam depending of the stiffness determined in the previous step. But the stiffness of the two central segments is modified continuously in line with the steps 3 and 4.
8. The calculation is completed if the values of the bending stiffness in the next steps will correspond to each other within the required accuracy, ε_3 .

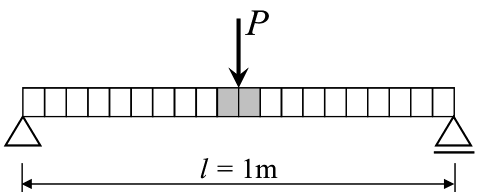

Fig. 1: The beam under load

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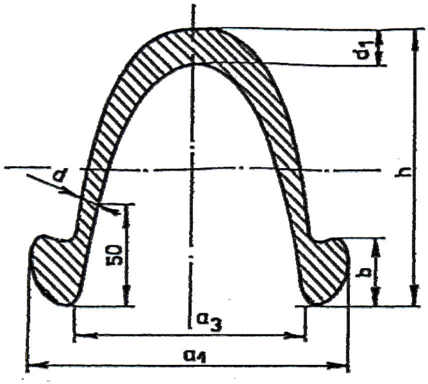

a_1 [mm]	148
a_3 [mm]	109
b [mm]	31
d [mm]	7,9
d_1 [mm]	14,5
h [mm]	127,5

Fig. 2: P-28 profile

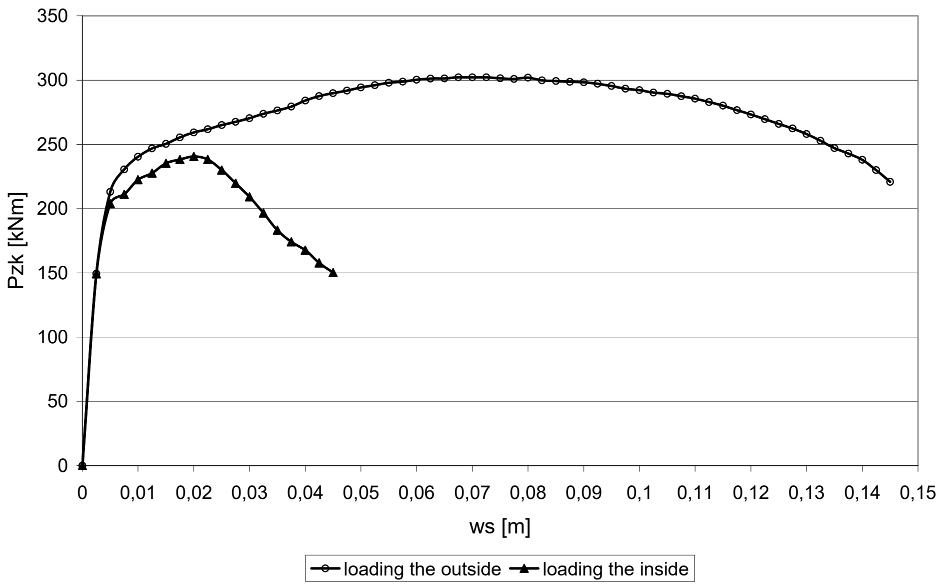

Fig. 3: Bending tests – loading the inside and outside of the profile root, P-28

2.1 Determining the force *P* on the basis of deflection of the beam centre w_s

The beam is divided into n straight-line segments. Each segment will be regarded as a both-side monolithic rod. This means, there are $n_p = [3\cdot(n+1)-3]$ unknown parameters of the deformation vector $\{r\}$. The unknown deformations represent horizontal dislocation u_i, vertical dislocation w_i and angular displacement φ_i for individual points in the beam. The deformations can be calculated by solving a set of equations

$$[K]\cdot\{r\}=\{F\}, \tag{1}$$

where $[K]$ is the total matrix of the rod stiffness. It can be calculated by localisation of the global stiffness matrices of the individual segments $[k_i]$. $[K]$ is the matrix for non-linear solution of the vector functions, $\{F\}$. $\{F\}$ is the loading vector which contains the only one non-zero force - P, which is the mean value of the vector.

$$\{F\}=\{0 \quad \cdots \quad P \quad \cdots \quad 0\}^T = P\cdot\{0 \quad \cdots \quad 1 \quad \cdots \quad 0\}^T = P\cdot\{\overline{F}\} \tag{2}$$

$$[k_i^*]=\begin{bmatrix} \frac{EA}{ds_i} & 0 & 0 & -\frac{EA}{ds_i} & 0 & 0 \\ 0 & \frac{12EI_i}{ds_i^3} & -\frac{6EI_i}{ds_i^2} & 0 & -\frac{12EI_i}{ds_i^3} & -\frac{6EI_i}{ds_i^2} \\ 0 & -\frac{6EI_i}{ds_i^2} & \frac{4EI_i}{ds_i} & 0 & \frac{6EI_i}{ds_i^2} & \frac{2EI_i}{ds_i} \\ -\frac{EA}{ds_i} & 0 & 0 & \frac{EA}{ds_i} & 0 & 0 \\ 0 & -\frac{12EI_i}{ds_i^3} & \frac{6EI_i}{ds_i^2} & 0 & \frac{12EI_i}{ds_i^3} & \frac{6EI_i}{ds_i^2} \\ 0 & -\frac{6EI_i}{ds_i^2} & \frac{2EI_i}{ds_i} & 0 & \frac{6EI_i}{ds_i^2} & \frac{4EI_i}{ds_i} \end{bmatrix} \tag{3}$$

$$[T_i]=\begin{bmatrix} \cos\varphi_i & \sin\varphi_i & 0 & 0 & 0 & 0 \\ -\sin\varphi_i & \cos\varphi_i & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\varphi_i & \sin\varphi_i & 0 \\ 0 & 0 & 0 & -\sin\varphi_i & \cos\varphi_i & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

$$[k_i]=[T_i]^T\cdot[k_i^*]\cdot[T_i] \tag{5}$$

When dealing with this task, the known value is the vertical dislocation of the beam centre, w_s, and the unknown value is the force, P. This means, this is a mixed task which needs to be solved by solving the following equations:

$$[K]\cdot\{r\}=P\cdot\{\overline{F}\} \tag{6}$$

$$\begin{bmatrix} k_{11} & \cdots & k_{1s} & \cdots & k_{1n_p} \\ \vdots & \ddots & & \iddots & \vdots \\ k_{s1} & & k_{ss} & & k_{sn_p} \\ \vdots & \iddots & & \ddots & \vdots \\ k_{n_p1} & \cdots & k_{n_ss} & \cdots & k_{n_pn_p} \end{bmatrix}\cdot\begin{Bmatrix} d_1 \\ \vdots \\ w_s \\ \vdots \\ d_{n_p} \end{Bmatrix}=P\cdot\begin{Bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{Bmatrix} \tag{7}$$

$$\begin{bmatrix} k_{11} & \cdots & 0 & \cdots & k_{1n_p} \\ \vdots & \ddots & & \iddots & \vdots \\ k_{s1} & & 1 & & k_{sn_p} \\ \vdots & \iddots & & \ddots & \vdots \\ k_{n_p1} & \cdots & 0 & \cdots & k_{n_pn_p} \end{bmatrix}\cdot\begin{Bmatrix} d_1 \\ \vdots \\ -P \\ \vdots \\ d_{n_p} \end{Bmatrix}=-w_s\cdot\begin{Bmatrix} k_{1s} \\ \vdots \\ k_{ss} \\ \vdots \\ k_{n_ps} \end{Bmatrix} \tag{8}$$

$$[K_{\overline{F}}]\cdot\{r_P\}=-w_s\cdot\{K_s\} \;\Rightarrow\; \{r_P\}=[K_{\overline{F}}]^{-1}\cdot(-w_s)\cdot\{K_s\} \tag{9}$$

$\{\overline{F}\}$ – the loading vector induced by the unit force $P = 1$

$\{K_s\}$ – the vector containing elements from the central column of the original stiffness matrix $[K]$

$[K_{\overline{F}}]$ – the modified stiffness matrix where the central column is replaced with the vector $\{\overline{F}\}$

$\{r_P\}$ – the modified deformation vector where w_s is replaced with a negative P

Once this equation is solved, we receive not only the values of all beam deformations, r, but also the vertical force, P, which induces the dislocation, w_s. After all deformations are known, new coordinates of the beam points need to be determined and the new stiffness matrix, $[K]$, of the distorted beam should be solved. Then, the entire task is repeated.

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The calculation for w_s is performed by iterations, until the required accuracy of the solution is reached. The accuracy depends on the magnitude of the loading in the subsequent k^{th} iterations.

$$\varepsilon_1 = \left|\frac{(P_k - P_{k-1})}{P_k}\right| \tag{10}$$

2.2 Determining the stiffness *EI* depending on the relative angular displacement $d\varphi$

We calculate the force P for the specific w_s. Then, the force P will be compared with the force P_{zk} which corresponds to the deflection w_s pursuant to the bending test (Fig. 3). If the force is different, the effective stiffness, *EI,* of the two central segments should be adjusted.

$$\begin{aligned} EI_{n/2}' &= EI_{n/2+1}' = \frac{1}{2}\left(EI_{n/2} + \widetilde{EI}\right) \\ F < F_{zk} &\Rightarrow \widetilde{EI} = EI_{n/2}(1+\varepsilon_2) \\ F > F_{zk} &\Rightarrow \widetilde{EI} = EI_{n/2}(1-\varepsilon_2) \end{aligned} \tag{11}$$

The stiffness of the central segments is made more precise, until the force P_{zk} resulting from the bending test is determined with the required accuracy, ε_2.

$$\varepsilon_2 = \left|\frac{(P_{zk} - P)}{P}\right| \tag{12}$$

This procedure is applied to all values of the deflection, w_s. This process results in the effective stiffness values, EI_i, for each relative displacement of the central segments $d\varphi_i$ is allocated (the stiffness curve is available after drawing the values into the chart).

$$d\varphi_i = \left|\frac{1}{ds_i}(\varphi_i - \varphi_{i+1})\right| \tag{13}$$

$$ds_i \approx \sqrt{(x_i - x_{i-1})^2 + (z_i - z_{i-1})^2} \tag{14}$$

The entire procedure is repeated. But in the subsequent j^{th} steps, the effective stiffness is changed for all i^{th} segments of the beam depending of the stiffness determined in the previous step. At the same time, the stiffness of the two central segments is adjusted continuously by the method described above (depending on the force P_{zk} resulting from the bending test).

$$d\varphi_{i,j} \geq d\varphi_{m-1,j-1} \cap d\varphi_{i,j} < d\varphi_{m,j-1} \Rightarrow EI_{i,j} = EI_{m-1.j-1} - \frac{(d\varphi_{i,j} - d\varphi_{m-1,j-1})}{(d\varphi_{m,j-1} - d\varphi_{m-1,j-1})}\left(EI_{m-1,j-1} - EI_{m,j-1}\right) \tag{15}$$

When changing the effective stiffness, it is necessary to check all the time where EI_i decreases only in the subsequent steps. If the stiffness starts increasing again, the increased stiffness should be replaced with the lower one from the previous step.

The calculation is repeated, until the required accuracy of the solution, ε_3, is reached. The accuracy depends on the stiffness calculated in the subsequent steps.

$$\varepsilon_3 = \max\left|\frac{(EI_{i,j} - EI_{i,j-1})}{EI_{i,j}}\right| \tag{16}$$

2.3 Resulting values of the effective stiffness *EI*

The aforementioned method results in the effective stiffness *EI* for the P-28 profile loaded on the inside and outside the root. The curve was drawn using the calculated values. Figures 4 through 6 show the curves with the required accuracy $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.0001$. In the linear area (with the geometric linearity and physical linearity) the stiffness *EI* is constant. The linear area forms only a small part of the curve. If the relative angular displacement $d\varphi$ increases, the stiffness *EI* decreases.

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The reason for such a decrease in the stiffness is that the yield point of the material has been exceeded and the geometry of the profile has changed.

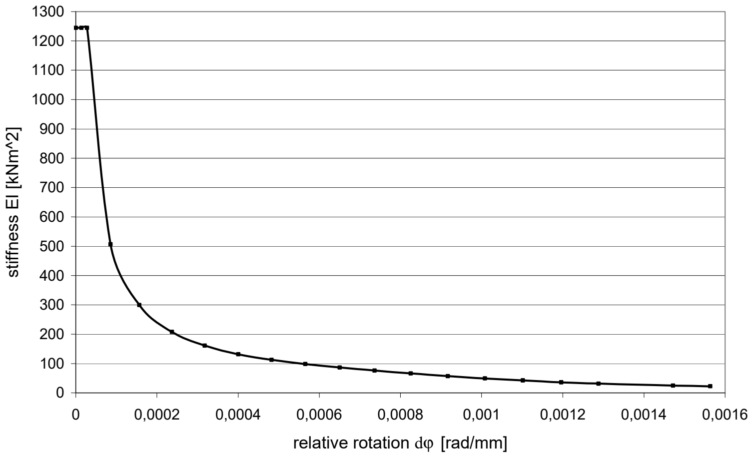

Fig. 4: Stiffness curves - loading the inside of the profile root, P-28

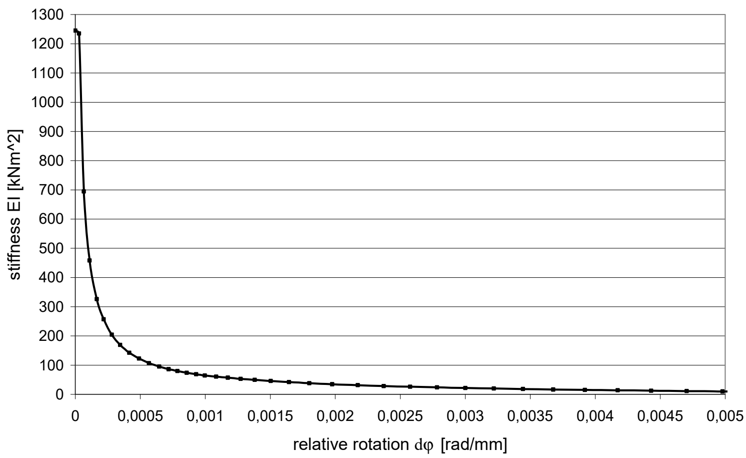

Fig. 5: Stiffness curves - loading the outside of the profile root, P-28

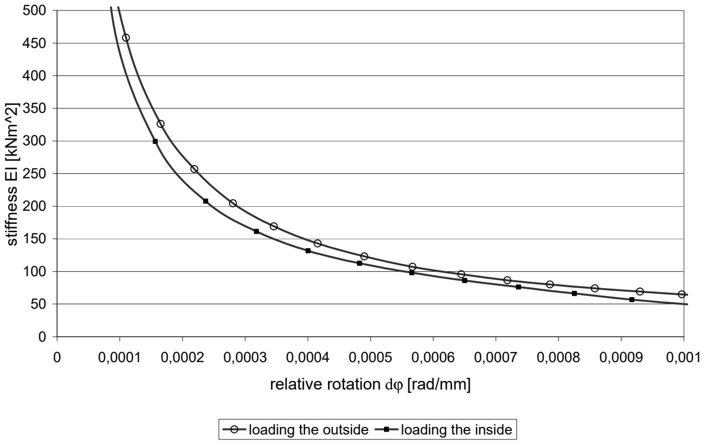

Fig. 6: Comparison of the resulting stiffness - loading the inside and outside of the profile root, P-28

2.4 Confronting the results with those calculated in ANSYS

Using the models available in ANSYS, the effective stiffness was determined for the P-28 profile. The effective stiffness is a function of the bending moment *M* [4]. If the functional dependence of the effective stiffness *EI* on the bending moment *M* is available, it is possible to calculate the relative angular displacement dφ and to draw development of the stiffness *EI* depending on the relative angular displacement dφ (Fig. 7).

$$|d\varphi| = \left|\frac{M}{EI}\right| \tag{17}$$

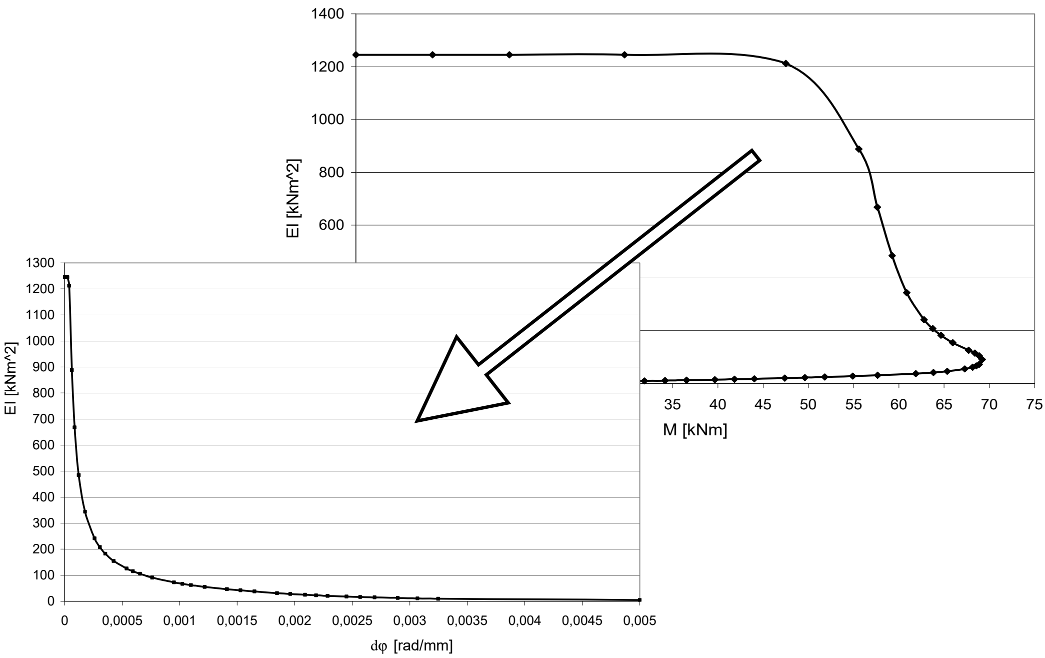

Fig. 7: Curves *EI* stiffness vs. *M* and dφ

Fig. 7 shows the stiffness curve for the P-28 profile loaded from the outside of the profile root. Fig. 8 confronts that curve with that resulting from the bending test and calculated using the general deformation method.

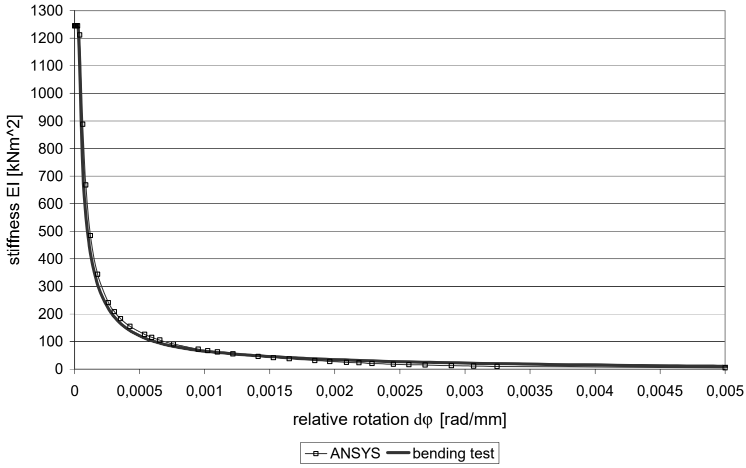

Fig. 8: Confronting the ANSYS and bending test curves

3 CONCLUSION

This paper describes determination of the effective (secant) stiffness of the cross-section *EI* of a steel profile P-28, using the bending tests made with a one-meter long sample. The effective stiffness *EI* was calculated as dependence on the relative angular displacement dφ. The values of *EI* were calculated using the general deformation method and iterations. Then, the curves were drawn. The results were confronted with the values calculated in ANSYS.

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REFERENCES

[1] JANAS, P. *Spolehlivost ocelových výztuží dlouhých důlních děl při rázovém zatížení.* Final report of project GA ČR 105/04/0458. Ostrava, 2007.

[2] KADLČÁK,J., KYTÝR,J. *Statika stavebních konstrukcí II.* Brno, 2001. ISBN 80-214-1648-3.

[3] RANDÝSKOVÁ, L., JANAS, P. Numerické geometricky nelineární řešení soustavy kruhových oblouků pomocí deformační metody. In *Modelování v mechanice 2010*. International conference. Collection of papers, Ostrava, 2010. ISBN 978-80-248-2234-1.

[4] MARKOPOLOULOS, A., JANAS, P., PODEŠVA, J. Efektivní ohybová tuhost profilu P-28 se zahrnutím osové síly. In *Modelování v mechanice 2010*. International conference. Collection of papers, Ostrava, 2010. ISBN 978-80-248-2234-1.

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