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DETERMINATION OF INSPECTIONS OF STRUCTURES SUBJECT TO FATIGUE

STANOVENÍ SYSTÉMU PROHLÍDEK KONSTRUKCÍ NAMÁHANÝCH NA ÚNAVU

Abstract

The paper describes in detail and gives examples of the probabilistic assessment of a steel construction subject to fatigue load, particular attention being paid to cracks from the edge and those from surface. This information is used as a basis for proposing a system of inspections. The newly developed method Direct Optimized Probabilistic Calculation (DOProC) is used for solution.

Keywords

Direct Optimized Probabilistic Calculation, DOProC, software package ProbCalc, fatigue crack, linear fracture mechanics, acceptable size, detectable size, initial size, fatigue crack propagation from the edge, fatigue crack propagation from surface, conditional probability, inspection of structure.

Abstrakt

V příspěvku je detailně zpracovaná a na příkladu demonstrovaná metodika pravděpodobnostního posouzení ocelové konstrukce namáhané únavou s ohledem na vznik únavových trhlin z okraje a povrchu, která vede k návrhu systému prohlídek konstrukce. K řešení je využita nově vyvíjená metoda Přímého Optimalizovaného Pravděpodobnostního Výpočtu – POPV.

Klíčová slova

Přímý Optimalizovaný Pravděpodobnostní Výpočet, POPV, programový systém ProbCalc, únavová trhlina, lineární lomová mechanika, přípustný rozměr, měřitelný rozměr, iniciační rozměr, šíření z okraje, šíření z povrchu, podmíněná pravděpodobnost, prohlídka konstrukce.

1 INTRODUCTION

The Direct Optimized Probabilistic Calculation ("DOPRoC") consists in numerical integration of a convolution integral without applying any simulation techniques. DOPRoC can be used to solve efficiently probabilistic tasks [8]. DOPRoC has proved to be a good solution, among others, in fatigue crack progression in constructions subject to cyclical loads [13]. This paper describes in detail and gives examples of the probabilistic assessment of a construction subject to fatigue load (a detail subject to fatigue from [4]), particular attention being paid to cracks from the edge and those from surface and limit of strength for the basic material. This information is used as a basis for proposing a system of inspections.

Using the Direct Optimised Probabilistic Calculation ("DOPRoC"), probabilities can be determined for basic phenomena in steel structures and steel bridges which are subject to fatigue. Such phenomena are connected with propagation of fatigue cracks which may occur in any t time of

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the service life. The probabilities determined on the basis of the reliability function analysis for each year of operation of the construction are the starting point for the conditionally probabilistic definition of inspection times for such steel structures or bridges which are subject to cyclical loads.

2 PROPAGATION OF THE FATIGUE CRACK

Reliability of the bearing structure has been significantly influenced by degradation resulting, in particular, from the fatigue of the basic materials. Wöhler's curves [1, 2] are used when designing such structures. The service life can be limited until a failure occurs. For purposes of the modelling, the amplitude oscillation is considered to be constant, and a certain number of load cycles is taken into account. Attention to the fatigue cracks in steel structures and bridges has been paid for a long time.

Methods are under development that would be able to reveal potential defects and damage resulting from initiation cracks that accelerate considerably the propagation of fatigue cracks. Linear fracture mechanics is among alternative methods. Machinery experts have been dealing with such issues for many years. Results have been gradually taken over and implemented into designs of the loading structures in buildings.

Three sizes are important for the characteristics of the propagation of fatigue cracks. The first size is the *initiation size of the crack* that corresponds to a random failure in an element subject to random loads. Existence of the initiation cracks during the propagation should be revealed, along the *measurable length of the crack*, during inspections. The third important size has been referred to so far as the critical size – it is the final recorded size before a brittle fracture results in a failure. It would be advisable to use another method to specify the acceptable final size. Building structures and bridges are sized for extreme loads. Fatigue loads are investigated into only in details that are liable to fatigue cracks caused by variable operation loads. If the load-bearing element is designed with a reasonable designed reliability margin for effects of the extreme load, then a crack will negatively influence the designed condition.

The fatigue crack damage depends on a number of stress swing cycles. This is a time factor of reliability in the course of reliability for the entire designed service life. It is assumed that in the course of time the failure rate increases, while the reliability drops. If the propagation of the fatigue crack is included into the failure rate, it is necessary to investigate into the fatigue crack and define the maximum acceptable weakening. The weakening depends on the acceptable crack size which comprises safety margins for the critical crack size that may occur in consequence of a brittle fracture and, more often in steel structures, in consequence of a ductile fracture. The reason for this type of degradation of a load-bearing element in the course of time is the random existence of the initiation crack and propagation of the crack in the consequence of variable load effects. The result is the weakening of the element that has been sized for extreme load effects. The crack propagates in a stable way until it reaches the acceptable size that is a limit for the required reliability.

The probabilistic methods should be used for the investigation into the propagation rate of the fatigue crack until the *acceptable size* is reached because the input variables include uncertainties and reliability should be taken into account [6]. The most important inputs are the initiation crack size and the acceptable crack size. The definition of the acceptable crack size/index is a necessary, but not the only one, condition because the initiation crack size is most important for the crack propagation.

The description below focuses on clarification and specification of certain requirements set forth in the standards that make it possible to reach the required reliability if the acceptable damage method is used. The method describes here is valid for propagation of a crack up to the size which is the specified acceptable size for real structural details set forth, for instance, in [3], as well as in the conditions applicable to the deterministic approach.

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3 PROBABILISTIC APPROACH TO THE PROPAGATION OF FATIGUE CRACKS

The topic is discussed in two levels that affect each other: the probabilistic solution to the propagation of the fatigue crack and uncertainties in determination of quantities used in the calculation.

When investigating into the propagation, the fatigue crack that deteriorates a certain area of the structure components is described with one dimension only *a*. In order to describe the propagation of the crack, the linear elastic fracture mechanics [5] is typically used. This method defines the limit of propagation rate of the crack $\frac{\mathrm{d}a}{\mathrm{d}N}$ and swing of the stress rate coefficient in the face of the crack using the Paris-Erdogan law (e.g.. [21]):

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C.\Delta K^{m} \tag{1}$$

where *C*, *m* are material constants, *a* is the crack size and *N* is the number of loading cycles. The fatigue crack will propagate in a stable way only if the initial crack a_0 exists in the place where the stress is concentrated. This place is located at the edge or on the surface of the element.

The primary assumption is that the primary design should take into account the effects of the extreme loading and the fatigue resistance should be assessed then. This means, the reliability margin in the technical probability method is:

$$g(R,S) = G = R - S\ , \tag{2}$$

where *R* is the random resistance of the element and *S* represents random variable effects of the extreme load. If such element is subject to the operating load, following cases can occur:

a) **safe service life -** the fatigue effects do not degrade the element by means of the fatigue crack,
b) **acceptable failure rate** - the fatigue effects degrade the element and decrease the load-bearing capacity of the element,
c) **acceptable failure rate** - fatigue effects are expressed as stress changes.

The calculation model of the fatigue crack propagation defines the stress when the maximum acceptable crack results in the constant resistance of the structure, *R*, that corresponds to the stress in the yield point f_y. The approach c) is more demonstrative and has been preferred to the approach b) because it describes the non-linear growth of the both stresses in the element under degradation.

When using (1), the condition for the a_{ac} acceptable crack length is:

$$N = \frac{1}{C}\int_{a_0}^{a_{ac}} \frac{\mathrm{d}a}{\Delta K^{m}} > N_{tot}\ , \tag{3}$$

where *N* is the number of cycles needed to increase the crack from the initiation size a_0 to the acceptable crack size a_{ac}, and N_{tot} is the number of cycles throughout the service life. The equation (3) cannot be used, because the initiation crack size is not known.

The equation for the propagation of the crack size (1) needs to be modified for this purpose. If the stress swing $\Delta\sigma$ is known, the swing of the stress rate coefficient ΔK is:

$$\Delta K = \Delta\sigma.\sqrt{\pi.a}.F_{(a)}\ , \tag{4}$$

where $F_{(a)}$ is the calibration function which represents the course of propagation of the crack. After the change of the number of cycles from N_1 to N_2, the crack will propagate from the length a_1 to a_2. Having modified (1) and using (4), the following formula will be achieved:

$$\int_{a_1}^{a_2} \frac{\mathrm{d}a}{\left(\sqrt{\pi.a}.F_{(a)}\right)^m} = \int_{N_1}^{N_2} C.\Delta\sigma^m.\mathrm{d}N \quad . \tag{5}$$

If the length of the crack a_1 equals to the initial length a_0 (this is the assumed size of the initiation crack in the probabilistic approach) and if a_2 equals to the final acceptable crack length a_{ac} (this is the acceptable crack size which replaces the critical crack size a_{cr} if the crack results in a brittle fracture), the left-hand side of the equation (5) can be regarded as the resistance of the structure - R:

$$R_{(a_{ac})} = \int_{a_0}^{a_{ac}} \frac{\mathrm{d}a}{\left(\sqrt{\pi.a}.F_{(a)}\right)^m} \quad . \tag{6}$$

Similarly, it is possible to define the cumulated effect of loads that is equal to the right side (randomly variable effects of the extreme load) (5):

$$S = \int_{N_0}^{N} C.\Delta\sigma^m.\mathrm{d}N = C.\Delta\sigma^m.(N - N_0) \ , \tag{7}$$

where N is the total number of oscillations of stress peaks ($\Delta\sigma$) for the change of the length from a_0 to a_{ac}, and N_0 is the number of oscillations in the time of initialisation of the fatigue crack (typically, the number of oscillations is zero).

It is possible to define a reliability function G_{fail}. The analysis of the reliability function gives a failure probability p_f:

$$G_{fail(Z)} = R_{(a_{ac})} - S \ , \tag{8}$$

where Z is a vector of random physical properties such as mechanical properties, geometry of the structure, load effects and dimensions of the fatigue crack.

The failure probability p_f equals to:

$$P_f = P\left(G_{fail(Z)} < 0\right) = P\left(R_{(a_{ac})} - S < 0\right) . \tag{9}$$

4 USING THE CONDITIONED PROBABILITY TO DETERMINE TIMES TO INSPECT THE CONSTRUCTION

Because it is not certain in the probabilistic calculation whether the initiation crack exists and what the initiation crack size is and because other inaccuracies influence the calculation of the crack propagation, a specialised inspection is necessary to check the size of the measureable crack in a specific period of time. The acceptable crack size influences the time of the inspection. If no fatigue cracks are found, the analysis of inspection results give conditional probability during occurrence (e.g. [22, 25]).

While the fatigue crack is propagating, it is possible to define following random phenomena that are related to the growth of the fatigue crack and may occur in any time, t, during the service life of the structure. Then:

- ***U*(*t*) phenomenon:** No fatigue crack failure has not been revealed within the t–time and the fatigue crack size $a(t)$ has not reached the detectable crack size, a_d . This means:

$$a(t) < a_d \ , \tag{10}$$

- ***D*(*t*) phenomenon:** a fatigue crack failure has been revealed within the t–time and the fatigue crack size $a(t)$ is still below the acceptable crack size a_{ac}. This means:

$$a_d \le a(t) < a_{ac} \ , \tag{11}$$

- ***F*(*t*) phenomenon:** a failure has been revealed within the *t*–time and the fatigue crack size $a(t)$ has reached the acceptable crack size a_{ac}. This means:

$$a(t) \geq a_{ac} \ . \tag{12}$$

If the crack is not revealed within the *t*-time, this may mean that there is not any fatigue crack in the construction element. This might be an initiative phase of nucleation of the fatigue crack (when a crack appears in the material) and this phenomenon is not taken into account in the fracture mechanics. Even if the fatigue crack is not revealed it is likely that it exists but the fatigue crack size is so small that it cannot be detected under existing conditions.

Using the phenomena above, it is possible to define probability for their occurrence in any *t*-time. Those three phenomena cover the complete spectrum of phenomena that might occur in the *t*-time. This means:

$$P(U(t)) + P(D(t)) + P(F(t)) = 1 \ . \tag{13}$$

The probabilistic calculation is carried out in time steps where one step typically equals to one year of the service life of the construction. When the failure probability $P(F(t))$ reaches the designed failure probability p_d, an inspection should be carried out in order to find out fatigue cracks, if any, in the construction element. The inspection provides information about real conditions of the construction. Such conditions can be taken into account when carrying out further probabilistic calculations. The inspection in the *t* time may result in any of the three mentioned phenomena. Using the inspection results for the *t* time, it is possible to define the probability of the mentioned phenomena in another times: $T > t_I$. For that purpose, the conditional probability should be taken into consideration.

In order to determine the time for the next inspection, it is necessary to define the conditional probabilities $P(F(T)|U(t_I))$ and $P(F(T)|D(t_I))$, which can be expressed using the full probability law (e.g.. [25]) as follows:

$$P(F(T)|U(t_I)) = \frac{P(F(T)) - P(F(t_I)) - P(D(t_I)).P(F(T)|D(t_I))}{P(U(t_I))} \ , \tag{14}$$

$$P(F(T)|D(t_I)) = \frac{P(F(T)) - P(F(t_I)) - P(U(t_I)).P(F(T)|U(t_I))}{P(D(t_I))} \ . \tag{15}$$

If re-distribution of stress from a point that is weakened by the crack is not taken into account, the crack propagation crack is usually rather high in the practical range of measurable values. If a fatigue crack is found during the inspection, it is necessary to monitor the safe growth of the crack or to take actions that will slow down or stop further propagation of the fatigue crack. In order to time the inspections well, the equation (14) is most important. It defines the failure probability in $T>t_I$ provided that no fatigue cracks have been revealed during the last inspection. It is clear from the equation that the results of the failure probability are influenced by mutual relations between the three crack sizes - the initiation crack size, measurable crack size and acceptable crack size.

The probability in the equation (14) can be calculated in any time $T > t_I$ using the available software [7, 9] and DOProC [14, 15, 23, 24] or using Monte Carlo (the results achieved if these two methods with slightly different input parameters are used) was carried out in [25]). When the failure probability P(F(t) / U(tI)) reaches the designed failure probability P_{fd}, an inspection should be carried out in order to reveal fatigue cracks, if any, in the construction component. The inspection may result in one of the mentioned phenomena with corresponding probabilities. The entire calculation can be repeated in order to ensure well-timed inspections in the future.

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5 PROBABILISTIC CALCULATION OF THE PROPAGATION OF THE FATIGUE CRACK

Fatigue cracks appear most frequently in decks of railway or road bridges. The fatigue cracks may occur easily because each normal force represents one loading cycle (such as [20]). The loading effects are more evident if the construction element is located close to the point of loading application.

An important factor influencing occurrence of the fatigue crack is the weld itself because internal tension and initiation cracks may appear because of poor workmanship. Where the cross-section changes suddenly (Fig. 1), the fatigue damage can be influenced by differences during the real tension (where peaks appear in the weld) and designed tension (an even tension in the flange).

Depending on location of an initial crack, the crack may propagate from the edge [19] or from the surface [16, 17, 18]. Regarding the frequency, weight and stress concentration, those locations rank among those with the major hazard of fatigue cracks appearing in the steel structures and bridges. The cases are different in calibration functions -$F_{(a)}$- and in weakened surfaces which are appearing during the crack propagation. In the calculation this influences the fatigue resistance of the construction.

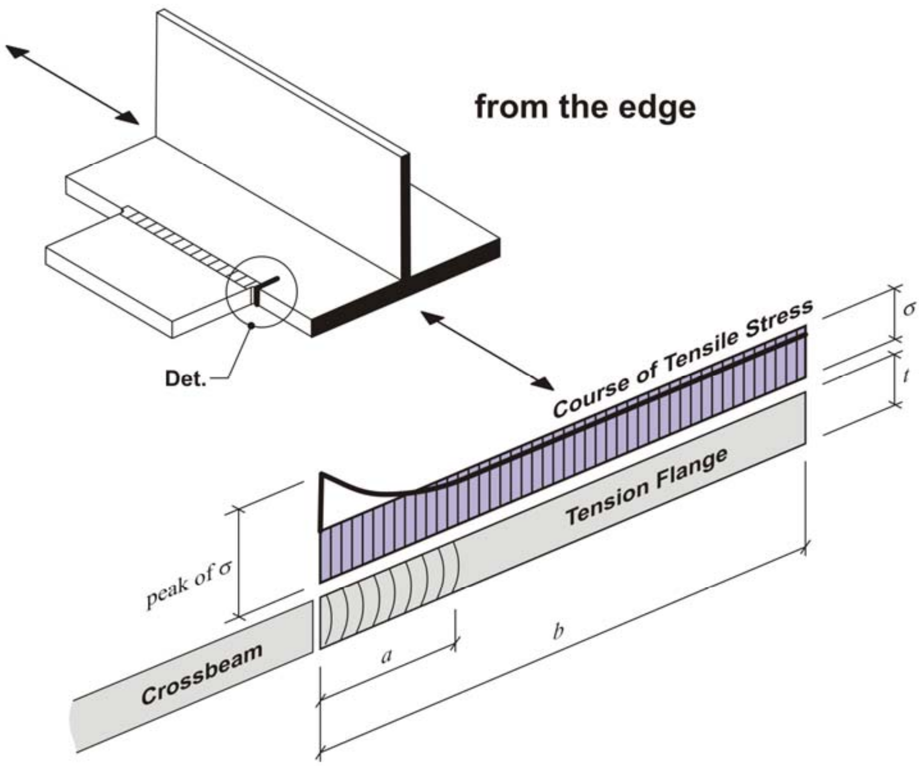

Fig. 1. Detail of a bridge structure which is subject to fatigue damage

In case of the fatigue crack from the edge, the acceptable crack size a_{ac} is:

$$a_{ac} = b.\left(1 - \frac{\sigma_{\max}}{f_y}\right) . \qquad (16)$$

It is difficult to express the acceptable crack size from the surface a_{ac} directly and explicitly [10, 11]. The crack size is calculated using the numerical iteration:

$$\frac{1}{2}\pi.a_{ac}\cdot\left(\frac{0,3027}{t}.a_{ac}^{2}+1,0202.a_{ac}+0,00699.t\right)-b.t\left(1-\frac{\sigma_{\max}}{f_y}\right)=0 \qquad (17)$$

The fatigue reliability of the structure with cracks propagating from the surface was calculated using the DoPRoC. First, it is necessary to determine the resistance of the structure $R(a_d)$ and $R(a_{ac})$. For that purpose the left side of the equation (6) is used and the respective upper limit of a_2 and(16) and (17) are applied. If the crack propagates from the edge, the resulting histogram of the structure resistance $R(a_d)$ and $R(a_{ac})$ is in Fig. 2 and 3.

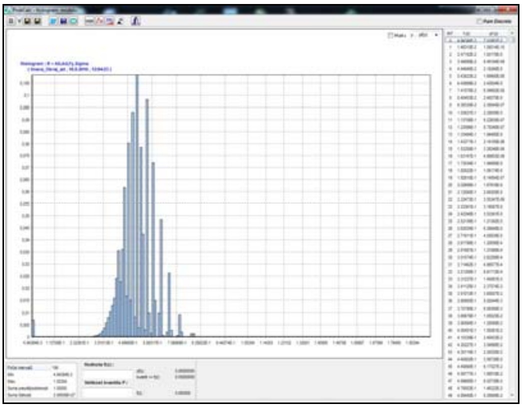

Fig. 2: Histogram – structural reliability $R(a_d)$

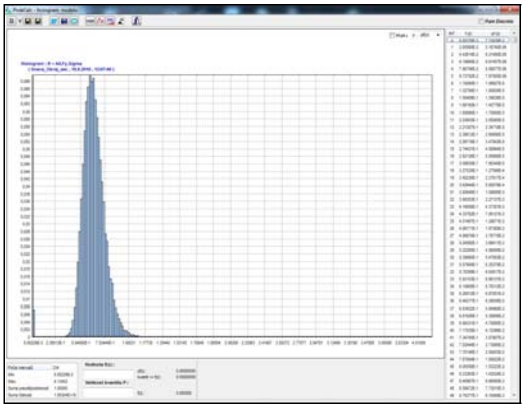

Fig. 3: Histogram – structural reliability $R(a_{ac})$

Another quantity that is important for the reliability of the structure is the loading effect S (7). When calculating the loading effect, two deterministically material characteristics C and m and two pairs of quantities with the parametric distribution of probabilities are used: the oscillation of stress peaks $\Delta\sigma$ [MPa] and the number of oscillations of stress peaks N used. This quantity is determined for each year of operation of the construction. Fig. 4 shows the histogram of the accumulated loading effects for S and for the set number of the stress peak oscillations in 54 years of operation.

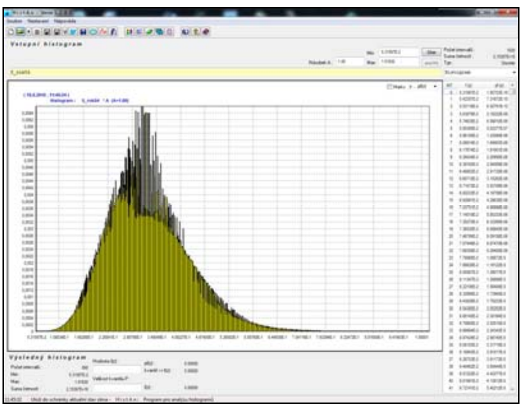

Fig. 4: Histogram – accumulated loading effects S for the total number of oscillation of stress peaks after 54 year of operation of the bridge

Fig. 5: Histogram - reliability function G_{fail} in 54 years – propagation from the edge

Using (9), we obtain the failure probability P_f for each year of operation of the construction. If the crack propagates from the surface, the resulting histogram of reliability function for the 54th year, for instance, of the bridge operation is that in Figure 5 (the failure probability $P_f = P(G_{fail} < 0) = 7.76732.10^{-2}$).

Using the calculated probability of the failure, P_f, and the required reliability P_d, it was possible to define the time for the first inspection of the bridge. Fig. 6 and Fig. 7 show the failure probability p_f depending on the years of operation for the both types of the fatigue damage. The required reliability is expressed in the technical practice as a reliability index $\beta = 2$, that corresponds to the failure rate of cca. 0.02277.

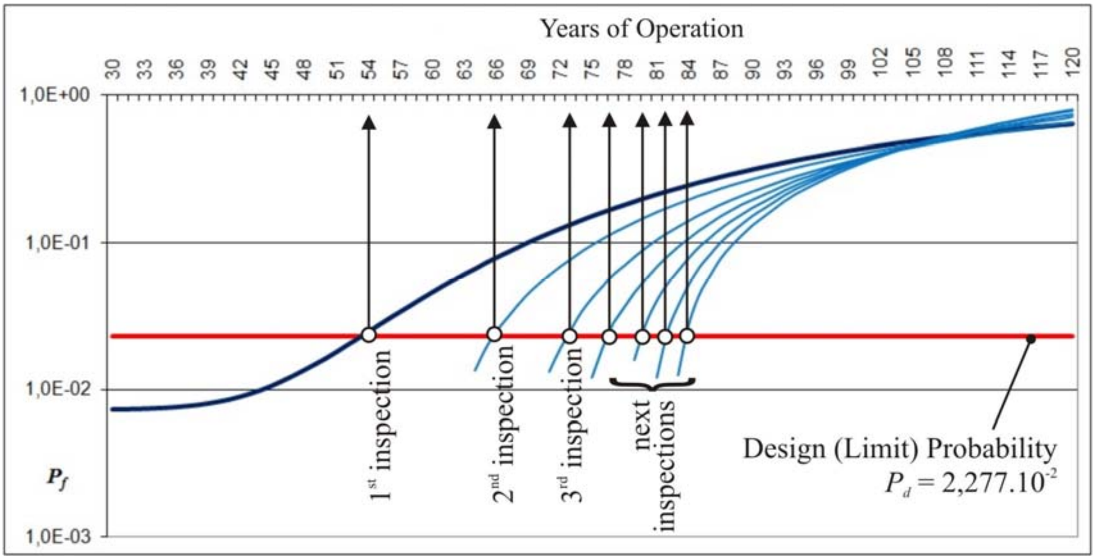

Fig. 6: Failure probability p_f depending on the years of operation of the bridge (30 to 120 years) and times for inspection with the focus on fatigue crack from the edge

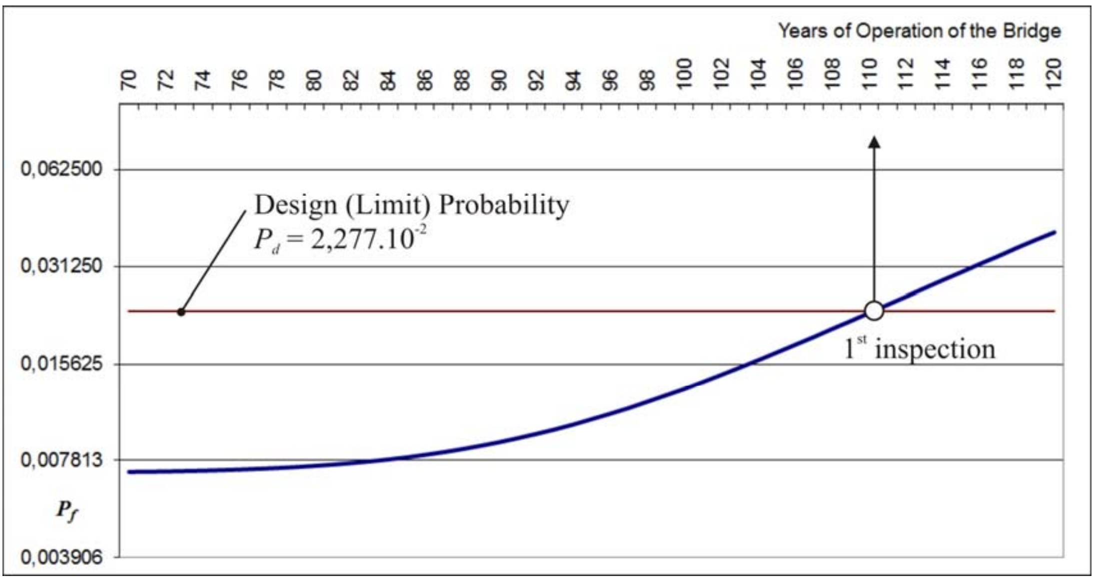

Fig. 7: Failure probability p_f depending on the years of operation of the bridge (70 to 120 years) and the time for the 1st inspection with the focus on fatigue crack from the surface

In case of the steel bridge under investigation, it was calculated that the first inspection and check for a fatigue crack from the edge should take place in the 54th year of operation of the bridge. Regarding the fatigue crack propagating from the edge, the first inspection should be in the 111st year of operation (because of the advanced time for the first inspection, the other inspections were not determined using the conditional probability).

If the edge crack cannot be measured during the first inspection, the next inspection will take place in the 66th year of operation. And if the crack is not identified again, the next year of inspection is 73 on the basis of the conditional probability. After that year, the inspection intervals will become shorter considerably (operation years: 77, 80, 82 and 84) if the crack is not identified during the 85th year, it can be assumed that if the input values have not changed (in particular, the intensity and efficiency of the operation load), the medium value of the initial crack will be less than the expected crack or there is not any fatigue crack at all.

Comparing the both types of the fatigue cracks has proved, among others, that velocity of propagation of the fatigue crack from the surface is considerably slower than that from the edge. If this velocity is considered in the context of the first inspections, the propagation of the fatigue crack from the surface is more that twice slower.

6 CONCLUSION

This paper provides theoretical background for propagation and practical introduction into the fatigue cracks in steel structures and bridges which are subject to cyclic loads. A particular attention is paid to the maximum acceptable crack size.

Propagation of the fatigue cracks and possible forecast of such propagation in the course of time since the start of variable loading effects is the case when probabilistic methods must be used (such as [12]) because too many uncertainties influence the determination of the input values. The uncertainties include both loading effects and construction resistance (for instance, the stochastic response to effects of the variable operation form by oscillation of stress in locations which are susceptible to fatigue damage). In the global context, it is the size of the expected initial crack which is managed with most difficulties.

The model is based on a linear fracture mechanics. Using the conditioned probability, it is possible to propose a regular system of inspections for the structure.

The calculation uses the newly developed Direct Optimized Probabilistic Calculation ("DOPRoC") which is suitable for several probabilistic calculations. Examples of the probabilistic methods used in calculations have been proving that the method is suitable not only for the reliability assessment, but also for other probabilistic calculations, including the propagation of the fatigue cracks. DOProC appears to be a very efficient tool that results in the solution affected by a numerical error and by an error resulting from the discretising of the input and output quantities only.

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