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ELASTIC-PLASTIC MODELLING OF REINFORCED CONCRETE BEAM: IMPLEMENTATION AND COMPARISON WITH THE EXPERIMENT

PRUŽNOPLASTICKÉ MODELOVÁNÍ ŽELEZOBETONOVÉHO NOSNÍKU: IMPLEMENTACE A SROVNÁNÍ S EXPERIMENTEM

Abstract

The paper deals with the finite element method analysis of the reinforced concrete beam. Within the analysis a non-linear behaviour of concrete is considered. The constitutive model of concrete is based on the elastic-plastic behaviour. Reinforcement is modeled as smeared. The article also includes a comparison of the numerical calculations with the experiment. For the calculation, BSA software was used.

Keywords

Beam, concrete, finite element method, reinforcement, plasticity.

Abstrakt

Příspěvek se zabývá analýzou železobetonového nosníku metodou konečných prvků. Při analýze se předpokládá nelineární chování betonu a je užit rovinný výpočetní model. Konstitutivní model betonu uvažuje pružno-plastické chování betonu, které představuje fyzikální nelinearitu. Výztuž je modelována jako rozmazaná a předpokládá se jednoosý stav napjatosti. Příspěvek také obsahuje srovnání numerických výpočtů s experimentem. Popisovaný model betonu bude dále využit pro předpověď únosnosti podobných železobetonových konstrukcí.

Klíčová slova

Nosník, beton, metoda konečných prvků, výztuž, plasticita.

1 INTRODUCTION

It has been becoming more and more important to determine exactly the load-carrying capacity of reinforced concrete structure [2], as this information is among most relevant for precise specification of the service life. In the past forty years, a number of theories and constitutive models has been developed for the concrete and reinforced concrete [14], [15]. The difference among the models is precision, computational complexity and the scope of input data. In general, the more quantity precise is the model, the higher are the demands in terms of quantity and quality of the complex data, this making, in turn, the computational process more demanding [18]. A non-linear

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analysis of concrete and reinforced concrete structures is often combined with heat transmission models [5] or is used in dynamic load simulations [6].

Because the calculations often need to be repeated (either for individual time sequences or in simulation calculations) and because many input data are required, it is recommended to use such a model which will be a good compromise between the computational complexity, the number of the input data and the required precision of the final results.

This paper presents an elastic-plastic model which seems to be a good solution for the peak requirements above. Using this model, it is possible to model the behaviour up to the load-carrying capacity, which is, for most purposes, sufficient. The model is based on Chen-Chen condition of plasticity [3] and Ohtani concept of hardening [12]. Because the general Ohtani model allows to use a number of different approached, implementation of Ohtani concept in the article is described in more detail.

This model of the concrete is used in the Finite Element Method Analysis [8]. A numerical example presented here models a reinforced concrete beam by means of isoparametric four-node wall finite elements [16], [7] and confronts the results with experimental measurements pursuant to [4].

2 THE ELASTIC-PLASTIC CONSTITUTIVE MODEL OF THE CONCRETE

The constitutive model of the concrete combines the Chen-Chen condition of plasticity [3] and Ohtani concept of hardening [12]. The Chen-Chen condition of plasticity was formulated for the concrete on the basis of experiments carried out, among others, by Kupfer [9].

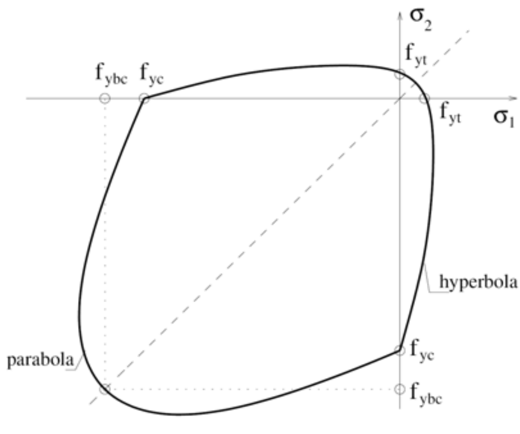

Fig. 1: Chen-Chen condition of plasticity [1]

In the compression-compression area, the condition of plasticity is described as follows:

$$J_2 + \frac{A_{yc}}{3} I_1 - \tau_{yc}^2 = 0 \tag{1}$$

In other areas, the function is following:

$$J_2 + \frac{1}{6} I_1^2 \frac{A_{yt}}{3} I_1 - \tau_{yt}^2 = 0\,, \tag{2}$$

where:

I_1 – is the first invariant of the stress tensor,

J_2 – is the second invariant of the stress deviator.

$A_{yc}, \tau_{yc}, A_{yt}, \tau_{yt}$ – is determined using the material constants f_{yc}, f_{ybc} and f_{yt}.:

$$A_{yc} = \frac{f_{ybc}^2 - f_{yc}^2}{2f_{ybc} - f_{yc}}, \tag{3}$$

$$\tau_{yc}^2 = \frac{f_{ybc} f_{yc}(2f_{yc} - f_{ybc})}{3(2f_{ybc} - f_{yc})}, \tag{4}$$

$$A_{yt} = \frac{f_{yc} - f_{yt}}{2}, \tag{5}$$

$$\tau_{yt}^2 = \frac{f_{yc} f_{yt}}{6}, \tag{6}$$

The condition is defined by means of the plastic limit of the material in a uniaxial compression, f_{yc}, the plastic limit in a biaxial compression, f_{ybc}, and the plastic limit in a uniaxial tension, f_{yt}. See Fig. 1.

The Ohtani hardening model is based on the Chen-Chen condition of plasticity. It was developed by Ohtani and Chen [12]. The hardening model is defined by three parameters-namely by the equivalent stress in the uniaxial compression, biaxial compression and uniaxial tension.

The hardening function looks as follows:

$$\psi = \alpha_1 \left\{\frac{\partial f}{\partial \sigma_c}\right\}^T \left\{\frac{\partial \sigma_c}{\partial \varepsilon_{pc}}\right\} + \alpha_2 \left\{\frac{\partial f}{\partial \sigma_{bc}}\right\}^T \left\{\frac{\partial \sigma_{bc}}{\partial \varepsilon_{pbc}}\right\} + \alpha_3 \left\{\frac{\partial f}{\partial \sigma_t}\right\}^T \left\{\frac{\partial \sigma_t}{\partial \varepsilon_{pt}}\right\} \tag{7}$$

or can be expressed in an abbreviated way:

$$\psi = \alpha_1 Q_1 H_c + \alpha_2 Q_2 H_{bc} + \alpha_3 Q_3 H_t . \tag{8}$$

The parameters α_1, α_2 and α_3 describe influence of the individual equation terms (8) and depend on the area subject to the stress. For recommended values see [12]. The hardening parameters H_c, H_{bc} and H_t are differentiation of the function. They describe development of tension depending on the equivalent plastic deformation. The hardening parameters can be obtained from material tests for various types of loads. Using the Ramberg-Osgood function, the hardening parameters can be approximated, giving the form in [17]:

$$H = \frac{E_0}{kn}\left(\frac{\sigma}{E_0}\right)^{1-n}, \tag{9}$$

where E_0, k and n are determined from the test cycle working charts. It is necessary to know the initial modulus of elasticity and two points from the load-displacement diagram. A typical shape of the final approximation is in Fig. 2.

Q_1, Q_2 and Q_3 are material constants which depend on the equivalent stress in the uniaxial compression, biaxial compression and uniaxial tension. They are determined using the current state of stress provided that the equations (1) or (2) are fulfilled. An iterational method is used for the calculation. Calculation of the equivalent stress can be divided into following steps:

1) Computation of I_1 (the invariant of the stress tensor) and of J_2 (the second invariant of the stress deviator).
2) Identifying the stress area and plastic/failure limits.
3) Testing the material (elastic, plastic of failed material) for a specific state of stress.
4) Identifying the area of stress for the material, if plasticized.
5) Expressing the equations (1) or (2) for the current level of the stress.

6) Executing the zero step of the iteration process (it is possible, for instance, to use the initial condition of plasticity, the failure condition or the plasticity condition obtained in the previous calculation step).
7) Iterating to determine the equivalent stress, until the stress (the invariant of the stress tensor and the second invariant of the stress deviator) and the equivalent stress fulfil the equation expressed in step 5.

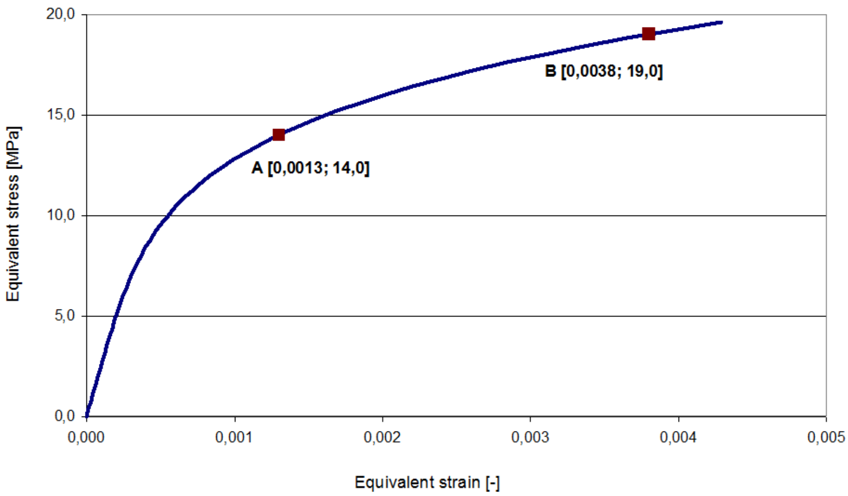

Fig. 2: Working diagram of the concrete subject to a single-axis compression. Ramberg-Osgood approximation.

If the material is plasticised, the parameters needed for description of the plasticity in a certain point, will be determined using the equations (3) through (6).

The model shows good compliance with experiments, but too many input parameters are needed.

3 REINFORCEMENT

When analysing the reinforced concrete structures, reinforcement can be included into the calculation by various methods. The reinforcement is included in the calculation by using a model of smeared reinforcement which assumes perfect contact of the reinforcement and concrete. This model is suitable for scattered reinforcement (such as fibre-reinforced concrete) or for standard reinforcing linings [14]. This means that in case of perfect contact the model does not consider slippage between the reinforcement and concrete.

Stiffness of each direction of the reinforcement can be described using the rigidity matrix

$$\mathbf{D}_{s,i} = \begin{bmatrix} pE_{s,i} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad (10)$$

where p is the degree of reinforcement resulting from the reinforcement-area-to-total-cross-section ratio (the finite element) and where $E_{s,i}$ is the modulus of elasticity of the reinforcement. The stiffness of the reinforcement is added to the matrix of stiffness of the material, $\mathbf{D}$, for the concrete. The resulting matrix of stiffness for the material is

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$$\mathbf{D} = \mathbf{D}_c + \sum_{i=1}^{n} \mathbf{D}_{s,i} \,, \tag{11}$$

where:

$\mathbf{D}_C$ – is the stiffness matrix for the concrete,

$\mathbf{D}_{S,i}$ – is the stiffness matrix for the reinforcement in individual directions.

In case of the reinforcement, it is assumed that the single-axis state of stress is ideal elastic-plastic or elastic-plastic with linear reinforcement. The condition of elasticity for the uniaxial state of stress is

$$\sigma \leq f_{sy} \,, \tag{12}$$

where:

σ – is the stress in a single-axis state of stress,

f_{sy} – is the yield limit of steel.

The relative deformation is limited by $\varepsilon_{s,\lim}$.

4 NUMERICAL EXAMPLE

The constitutive model of the concrete was used and confronted with results of an experiment when a beam was loaded with a single force of 200 kN [4]. The calculation was performed in BSA – this software is intended for analysing of concrete and reinforced concrete structures [19].

The beam spam is 3.6576 m and its cross-section is a rectangle. The height is 0.508 m and width is 0.2032 m. Fig. 3 shows the beam under testing, loads and reinforcement. The load is applied in the middle of the span on the upper edge. The initial material characteristic of the concrete is the modulus of elasticity E_{cm} = 26,182 MPa [11]. The reinforcement consists of two steel rods, dia. 25 mm. The modulus of elasticity of the steel reinforcement is E_{s1} = 203,255 MPa and yield limit is 309.36 MPa. In case of steel, the residual plastic stiffness modulus is $E_{s2} = E_{s1}/100$.

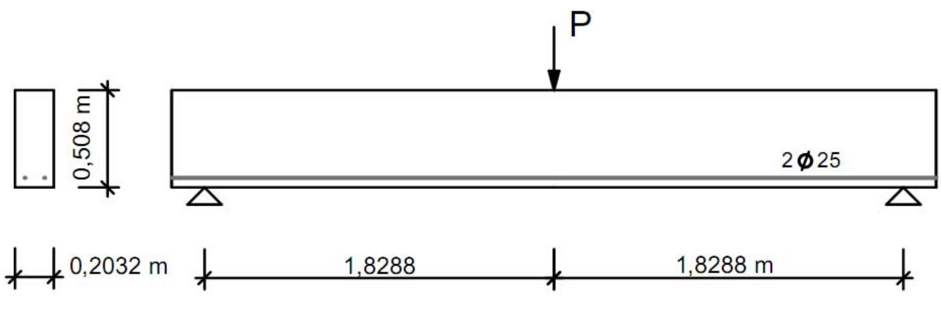

Fig. 3: Geometry of the beam under testing

The compression strength of the concrete for this model was determined using the formula from [13], where the modulus of elasticity is

$$E_{cm} = 22(f_{uc}/10)^{0,3} \,. \tag{13}$$

The tensile strength of the concrete was calculated using the compression strength and is

$$f_{ut} = 0.1\, f_{uc} \tag{14}$$

The strength of the concrete in biaxial compression is

$$f_{ubc} = 1.2\, f_{uc}. \tag{15}$$

Below is the plastic limit in the constitutive model for concrete subject to compression and tension in different types of stress:

$$f_{yc} = 0.5\, f_{uc}, \tag{16}$$

$$f_{ybc} = 0.5\, f_{ubc}, \tag{17}$$

$$f_{yt} = 0.5\, f_{ut}. \tag{18}$$

The Finite Element Method with a calculation model for plane stress state was chosen. The calculation model is a regular mesh of 400 isoparametric finite elements with four integration points. Detailed derivation of the finite element is given in [7]. The calculation model has got 451 nodes and it is shown in Fig. 4. A smeared model was used for the reinforcement. The model with reinforcement is crosshatched in Fig. 5. The Newton-Raphson method was used.

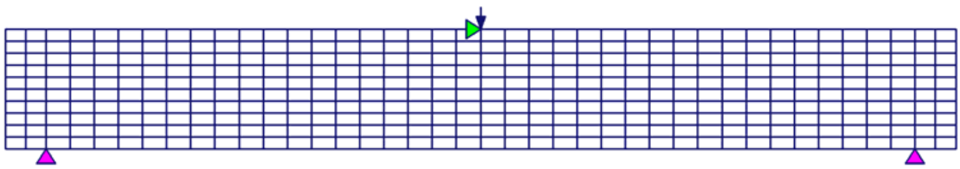

Fig. 4: Calculation model

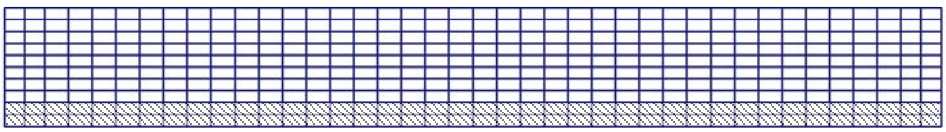

Fig. 5: Calculation model (the crosshatched area – concrete + reinforcement)

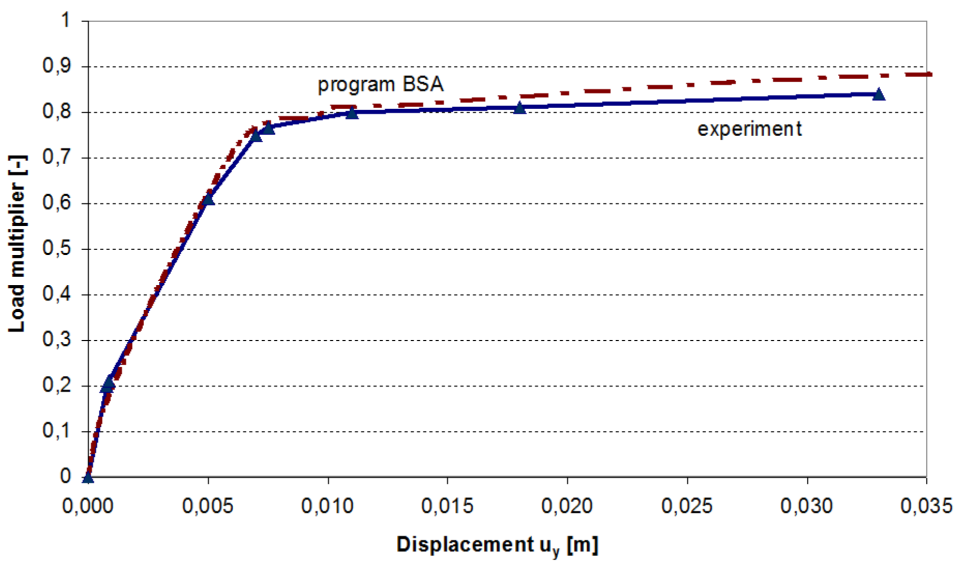

Fig. 6 Numerical analysis results

Fig. 6 shows results of the non-linear analysis of the reinforced concrete beam for the chosen constitutive model of the concrete. The results are confronted with the experiment [4]. The chart shows deflection in the middle of the span, depending on the load multiplier.

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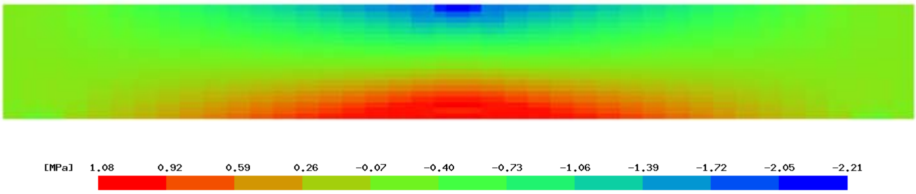
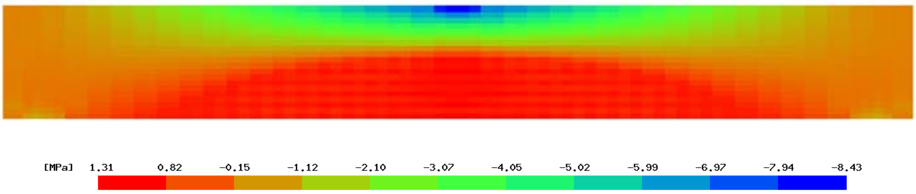
Fig. 7 shows gradual development of the area under tension during the loading test. The calculated results correlate well with the experiment. Only in a higher level of the load, the results are slightly overvaluated.

The load multiplier = 0.09 (Max = 1.08 MPa; Min = -2.21 MPa)



The load multiplier = 0.30 (Max = 1.31 MPa; Min = -8.43 MPa)



The load multiplier = 0.65 (Max = 1.73 MPa; Min = -14.25 MPa)

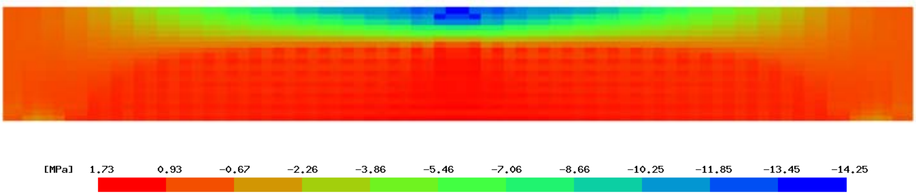


Fig. 7: Stress, σ_x, during the loading [MPa]

5 CONCLUSIONS

New opportunities have been appearing for the use of the non-linear analysis in simulations of the real behaviour of concrete and reinforced concrete structures [10]. Currently, this analysis is used typically for determination of global resistance of constructions, total load-carrying capacity determination or in analysis of problematic structural details. Many approaches are available for modelling of real behaviour of concrete and reinforced concrete structures. The constitutive model of the concrete described in this paper assumes elastic-plastic behaviour of the material. It combines the Chen-Chen condition of plasticity [3] and Ohtani concept of hardening [12].

In order to validate the applicability of the constitutive model of the concrete, the computed data were confronted with experiment data. The confrontation of the experiment and BSA calculation proves that the results correlate well. The higher load-carrying capacity calculated in the numerical analysis in the subsequent phase of the loading might be the consequence of the chosen modelling concept, i.e. the elastic-plastic concrete with hardening, and the chosen model of the reinforcement. This model of the concrete will be chosen to forecast the load-carrying capacity of similar reinforced concrete constructions.

It should be pointed out that the results are affected not only by the constitutive model of the concrete, but also by details and quality of the finite element network, model of boundary conditions, type of the finite element, the method used to solve the system of linear and non-linear equations and other aspects.

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