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Oldřich SUCHARDA[1], Jiří BROŽOVSKÝ[2]

MODELS FOR REINFORCEMENT IN FINAL FINITE ELEMENT ANALYSIS OF STRUCTURES

MODELY BETONÁŘSKÉ VÝZTUŽE PRO KONEČNĚPRVKOVOU ANALÝZU KONSTRUKCÍ

Abstract

Reinforced concrete belongs to one of the most important structural materials. The paper discusses modeling of steel reinforcement within non-linear finite element analysis. Selected models of reinforcement have been verified against numerical simulations and validated through experiments. Particular attention is paid to a reinforcement slip, for which a linearly elastic bond element is implemented in conjunction with elastic-plastic models for both reinforcement bars and concrete.

Keywords

Beam, concrete, reinforcement, bond element, finite elements, plasticity.

Abstrakt

Železobeton patří k nejdůležitějším konstrukčním materiálům. Článek se zabývá modelováním betonářské výztuže pro konečněprvkovou nelineární analýzu konstrukcí. Vybrané modely výztuže jsou ověřeny na numerických simulacích, které jsou srovnány s experimenty. Zvláštní pozornost je věnována prokluzu výztuže, pro který je implementován lineárně pružný spojovací prvek v kombinaci s pružnoplastickými modely výztuže a betonu.

Klíčová slova

Nosník, beton, výztuž, spojovací prvek, metoda konečných prvků, plasticita.

1 INTRODUCTION

Reinforced concrete ranks among most important construction materials. It combines two different materials. This is concrete with relative high compression strength and little tensile strength and a steel reinforcement. Various models of materials were developed to describe properly the behaviour of the reinforced concrete in static analyses. One of the models is the elastic-plastic model [16] which combines the Chen-Chen condition of plasticity [4] and Ohtani concept of hardening [9]. For details about the model and implementation of the model into *BSA* software which is used for numerical calculations see [16]. CONCRETE is a model based on ANSYS [15, 17]. It was used to verify certain calculations. Because CONCRETE is a model developed as a part of a comprehensive software system, this model of the concrete can be used in combined tasks [7]. Different models can

[1] Ing. Oldřich Sucharda, Department of Structural Mechanics, Faculty of Civil Engineering, VŠB-Technical University of Ostrava, Ludvíka Podéště 1875/17, 708 33 Ostrava - Poruba, phone: (+420) 597 32 1391, e-mail: oldrich.sucharda@vsb.cz.

[2] Doc. Ing. Jiří Brožovský, Ph.D., Department of Structural Mechanics, Fakulta stavební, VŠB-Technical University of Ostrava, Ludvíka Podéště 1875/17, 708 33 Ostrava - Poruba, phone: (+420) 597 321 321, e-mail: jiri.brozovsky@vsb.cz.

be used to model the reinforcement in the final element analysis. The models will be discussed below in the numerical analysis. In case of the finite element analysis of the reinforced concrete constructions pursuant to [3], the basis for the modelling can be models of smeared reinforcement, embedded reinforcement models or discrete reinforcement models. The reinforcement models can be divided, depending on the method used when taking into account the concrete-reinforcement slip. This issue is typically solved by special bond elements [6].

2 REINFORCEMENT

Reinforce concrete constructions are typically reinforced by concrete reinforcement made from reinforcing steel. The reinforcement looks typically as ribbed rods or imprinted rods. The reinforcement can also consist of wires. Welded networks are also used quite often. There are, however, special cases of reinforcement. They include solid reinforcements or prestressing reinforcements in case of prestressed constructions. The solid reinforcement combines rolled profiles with concrete [10].

For purposes of this paper, the concrete reinforcement is considered to be an isotropic material with a high strength which is same for both compression and tension. Typically, the strength is between 300 to 700 MPa. Basic material characteristics of steel are identified in tensile tests where working diagrams specify the uniaxial stress. It follows from the working diagram that behaviour of the steel is, at the beginning, linearly elastic. The linear elasticity exists up to the level which is identified as the limit of proportionality. An important value is the yield point - if it is exceeded, plastic deformations appear. The working diagram beyond the yield point depends on a specific composition of the steel, working procedure, technology adaptations or the selected degree of idealisation.

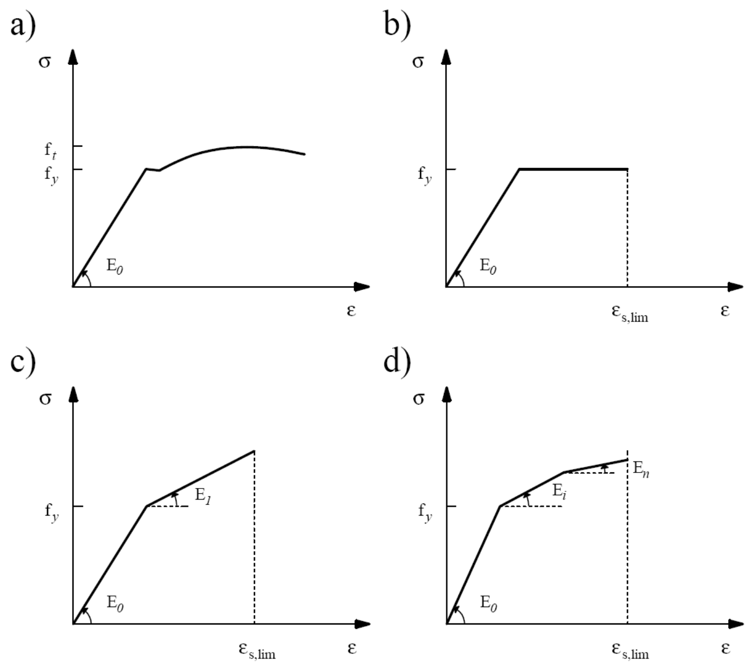

Fig. 1: Working diagrams – a) concrete reinforcement b) the ideal elastic-plastic behaviour c) the ideal elastic-plastic behaviour with linear reinforcement d) the multilinear behaviour

Typical working diagrams of the steel are the linear elastic, ideal elastic-plastic, ideal elastic-plastic behaviour with linear reinforcement and multilinear working diagrams. See Fig. 1 [6]. The real working diagrams correspond well to the idealised working diagram in case of the elastic-plastic behaviour with linear reinforcement [10]. This diagram requires good information about the yield point, initial modulus of elasticity, the modulus of elasticity for the reinforcement, and the maximum relative deformation. The condition of elasticity for the uniaxial state of stress in the elastic-plastic working diagrams simplifies the expression [10] as follows:

$$\sigma \le f_y \,, \tag{1}$$

where:

σ – is the stress for the uniaxial state of stress and

f_y – is the yield point of steel.

The relative deformation is limited by $\varepsilon_{s,\lim}$.

3 SMEARED STIFFNESS CONCEPT

The model of the smeared reinforcement [6] represents the best solution in wire concrete models. It can be used to model rather dense rod reinforcement where the rods are of a small diameter. The reinforcement stiffness matrix is created for each direction of the reinforcement. The basis for calculation is dimensions and properties of the basic material, this means, the concrete. Stiffness of each direction of the reinforcement can be described using the stiffness matrix

$$\mathbf{D}_{s,i} = \begin{bmatrix} pE_{s,i} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{2}$$

where p is the degree of reinforcement resulting from the reinforcement-area-to-total-cross-section ratio (the finite element) and where $E_{s,i}$ is the modulus of elasticity of the reinforcement. The stiffness matrix is transformed then into the correct coordination system, using the formula

$$\mathbf{D}_s = \mathbf{T}_\sigma^{-1} \mathbf{D}_{S,x} \mathbf{T}_\varepsilon \,, \tag{3}$$

where:

$\mathbf{T}_\sigma^{-1}$ – is the transformation matrix,

$\mathbf{D}_{S,x}$ – is the stiffness matrix for reinforcement and

$\mathbf{T}_\varepsilon$ – is the transformation matrix.

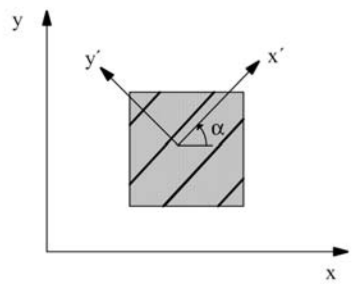

Fig. 2: Model of the scattered reinforcement

The transformation matrixes are defined [6] as follows

$$\mathbf{T}_{\sigma} = \begin{bmatrix} c^2 & s^2 & 2cs \\ s^2 & c^2 & -2cs \\ -cs & cs & c^2 - s^2 \end{bmatrix} \text{ and } \mathbf{T}_{\varepsilon} = \begin{bmatrix} c^2 & s^2 & cs \\ s^2 & c^2 & -cs \\ -2cs & 2cs & c^2 - s^2 \end{bmatrix}. \tag{4}$$

The transformation matrix [5] is dependant on inclination of the reinforcement within the coordinate system where c is cos α and s corresponds to sin α. The stiffness of the reinforcement is added to the matrix of stiffness of the material, D, for the concrete. The resulting matrix of stiffness for the material is

$$\mathbf{D} = \mathbf{D}_c + \sum_{i=1}^{n} \mathbf{D}_{s,i} \ , \tag{5}$$

where:

$\mathbf{D}_C$ – is the stiffness matrix for the concrete,

$\mathbf{D}_{S,i}$ – is the stiffness matrix for the reinforcement in individual directions.

In the model of the smeared reinforcement, it is assumed that the reinforcement and concrete co-act perfectly. This means that the model does not consider slippage between the reinforcement and concrete.

4 STEEL ELEMENT EMBEDDED IN CONCRETE ELEMENT

Another possibility is to introduce embedded models of the reinforcement which assume that the reinforcement acts perfectly together with the concrete. The models are used, for instance, in combination with rectangular or isoparametric finite elements. The finite elements of the reinforcement are embedded into the spatial and plane finite elements of the concrete. The calculation uses special transformation matrixes. The resulting matrix of stiffness for the finite element is:

$$\mathbf{K} = \mathbf{K}_c + \sum_{i=1}^{n} \mathbf{K}_{s,i} \ , \tag{6}$$

where:

$\mathbf{K}_C$ – is the stiffness matrix for the finite element of the concrete and

$\mathbf{K}_{S,i}$ – is the stiffness matrix for the finite element of the reinforcement in individual directions.

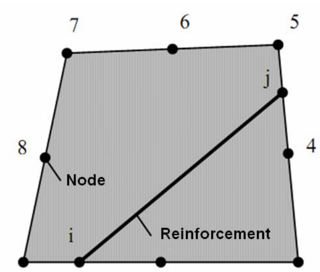

Fig. 3: The embedded model of the reinforcement (1-9 nodes of the concrete, i,j - nodes in the reinforcement)

It can be proved for a rectangular finite element and simple location of the reinforcement within the element that the stiffness of the finite element of reinforced concrete is same for the both models of the reinforcement. From the point of view of implementation of the reinforcement models into computations software, the process is rather complicated because a specific transformation matrix needs to be derived for each finite element separately. The use of the reinforcement model and specific transformation matrix for a rectangular finite element is given in [13]. The reinforcement models was, for instance, implemented with some isoparametric finite components in ANSYS [15].

5 DISCRETE MODEL OF REINFORCEMENT

A discrete reinforcement model combines two finite elements. There are the finite element for the concrete and that for the reinforcement. Some nodes are joint for the both finite elements. The finite elements for the reinforcement are one-dimension elements with two nodes in tasks describing the planar state of stress and deformation and with two degrees of freedom. This method requires a suitable grid of the finite elements which integrates then rod elements of the reinforcement. To model the rod reinforcement, various modified one-dimension finite elements can be used.

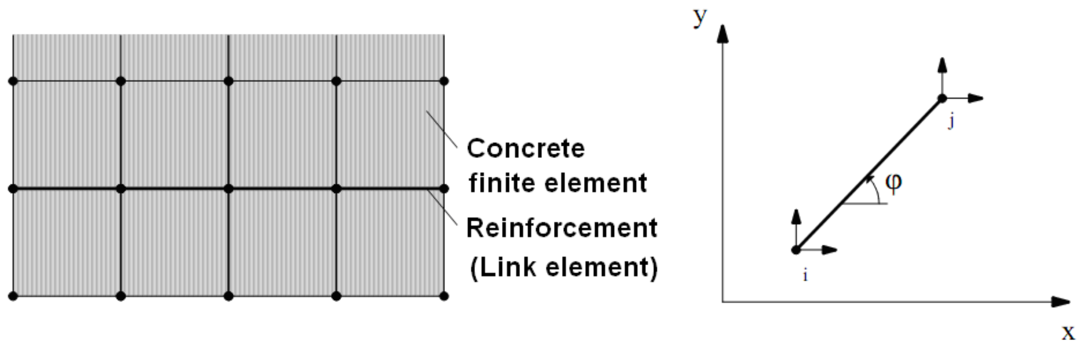

Fig. 4: The discrete model of the reinforcement

The matrix of stiffness for the finite element with two modes and two degrees of freedom is described as follows:

$$\mathbf{K} = \frac{E_S A}{L}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -c^2 & cs & s^2 \end{bmatrix}, \tag{7}$$

where:

E_S – is the modulus of elasticity of the steel,

A – is the cross-sectional area of the rod,

L – is the length of the link element,

c – is cos φ a

s – is sin φ.

The model is well suited, in particular, for the main load-carrying reinforcement. In order to take into account the concrete-reinforcement slippage, it is possible to use special bond elements and interfaces in the model.

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6 BOND ELEMENT

In reality, cohesion between the concrete and reinforcement is not perfect. The cohesion depends, in particular, on the surface and geometry of the reinforcement. In order to improve the cohesion, ribs or indents are made in the concrete reinforcement. Sometimes, plain reinforcement is used, but this is not the best solution, considering the concrete and steel cohesion.

When analysing the reinforced concrete structures where structural measures are made and specific reinforcing inserts are chosen to limit the slippage, ideal concurrence of the concrete and reinforcement are typically taken into account. This results in a higher total stiffness of the model. Influence of the slippage depends also on the chosen constitutive model of the concrete.

If it is necessary to investigate into influence of the slippage or if a detailed analysis is prepared, the best solution for the modelling of the concrete-reinforcement slippage is a discrete model of the reinforcement and specific bond elements [12]. The bond elements can be represented as small springs. For graphic representation see Fig. 5. The matrix of stiffness for the bond element is:

$$\mathbf{K} = k\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \tag{8}$$

where k is the coefficient of matrix of the bond elements and the individual matrix members correspond to the joint node for the concrete and reinforcement when the slippage is being modelled. For detailed derivation and use of the bond elements see [13]. The stiffness of the bond elements is determined on the basis of tests.

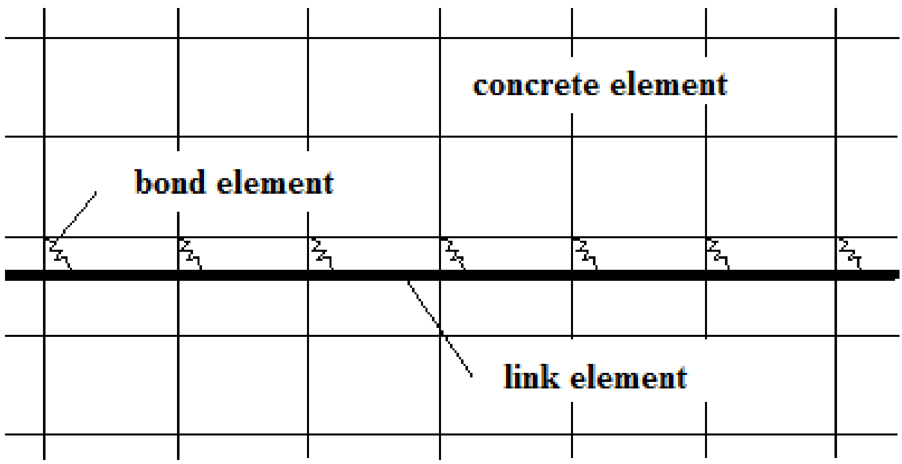

Fig. 5: The bond element between the concrete and reinforcement

When describing behaviour of the bond elements, it is necessary to differentiate between monotonous or cyclical loads. In designs of anchoring elements and cyclical loads, it is advisable to use three-dimension computational models with the connecting interfaces which take into account the type of ribs as well. In general analyses of the reinforced concrete structures with monotonous loads, those models provide little benefit only at expense of much time required for calculations, computational convergence and demanding collection of experimental data which are needed for that model.

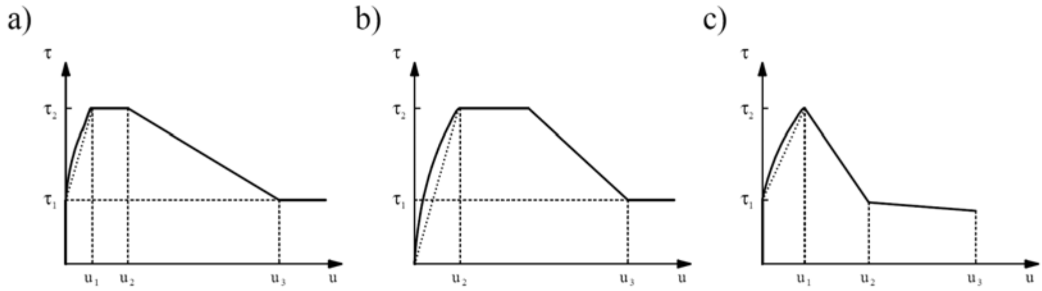

Fig. 6: The working diagram for the concrete-monotonous slippage a) Kwak and Filippa [5,11] b) CEB-FIB [3] c) Bigaj [2]

In order to determine the stiffness in practice, it is advisable to use the working diagrams specified in the recommendations CEB-FIB [3] or in research works [2, 5, 11]. The working diagrams are different, in particular, in the initial concrete-steel connection. The total load-carrying capacity of the structure is typically affected mostly by the shape of the ascending branch of the working diagram.

7 NUMERICAL EXAMPLE

The numerical analysis analyses a reinforced concrete beam for two models of the reinforcement and concrete [15, 16]. Results of the analysis are confronted then with [1]. Then, investigation is made into use of a bond element with an elastic-plastic model [16] which combines the Chen-Chen condition of plasticity [4] and Ohtani concept of hardening [9]. The reinforced concrete beam used in the experiment is of a rectangular shape. Its span is 3.6576 m and it is loaded with a single loading force. Fig. 7 shows the experiment principles, position of the single loading force, boundary conditions and reinforcement. The load is 200 kN. The initial material characteristics of the concrete in the experiment is the modulus of elasticity E_{cm} = 26,182 MPa [8]. This value was used for the both constitutive models of the concrete. In the CONCRETE model, the shear stiffness of failed concrete in tension is considered in the shear transfer coefficient for crack opening and $\beta_t = 0.5$ and shear transfer coefficient for crack closing $\beta_c = 0.9$. The working diagram of the steel is elastic-plastic with hardening. The initial modulus of stiffness is E_{s1} = 203,255 MPa. The yield point for steel is f_y = 309.36 MPa. After the yield point is achieved, the modulus of elasticity is $E_{s2} = E_{s1}/100$. The task was solved using the Newton-Raphson method [14].

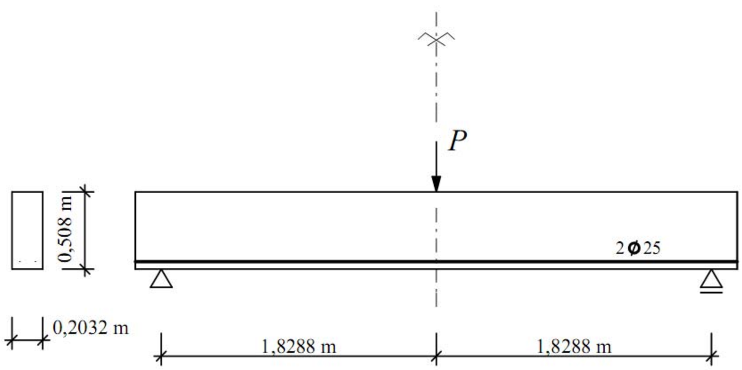

Fig. 7: Specification of the experiment

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The working diagrams for the specific types of the reinforcement and concrete structures are shown in Fig. 9 and 10. The chart shows deflection in the middle of the beam. The calculation correlates well with the experiments in three loading stages. The difference in the results for the scattered and discrete reinforcement can be regarded as little even if the working diagram is same. Only in the CONCRETE model, there is a difference in the initial stage of crack occurrence. In the elastic-plastic model, the biggest difference is in the curves at the initial plastification of the reinforcement and concrete crushing. The reached load is almost same, the difference being the maximum deformation. This, however, is closely connected with specific convergence rules. The calculations have also another thing in common: they slightly overestimate stiffness of the failing concrete. A reason would be that the calculation considered perfect concurrence of the reinforcement and concrete. A solution would be to use the below mentioned special bond elements between the reinforcement and concrete. To give some background for the calculation, let us take the resulting cracks modelled in the CONCRETE module in ANSYS. See Fig. This combines a model of smeared reinforcement in the initial stage of crack occurrence and the situation just before end of the calculation. For graphic results for the elastic-plastic model from *BSA* see [16].

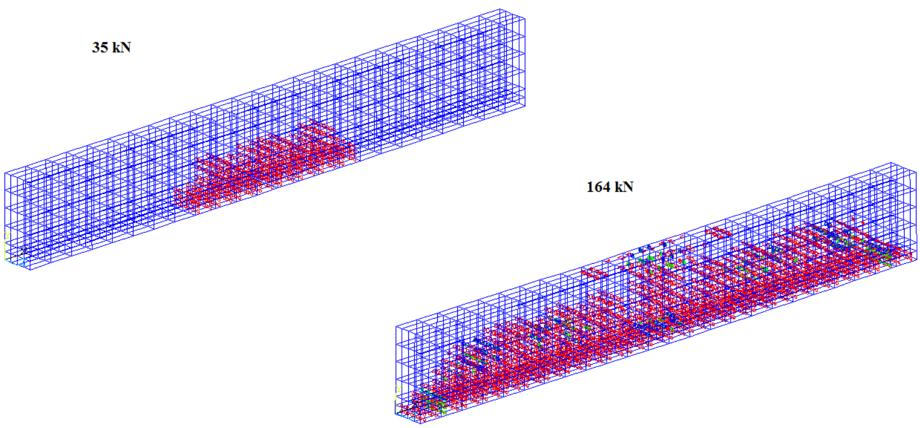

Fig. 8: Development of cracks the initial cracks (35 kN and 164 kN loads)

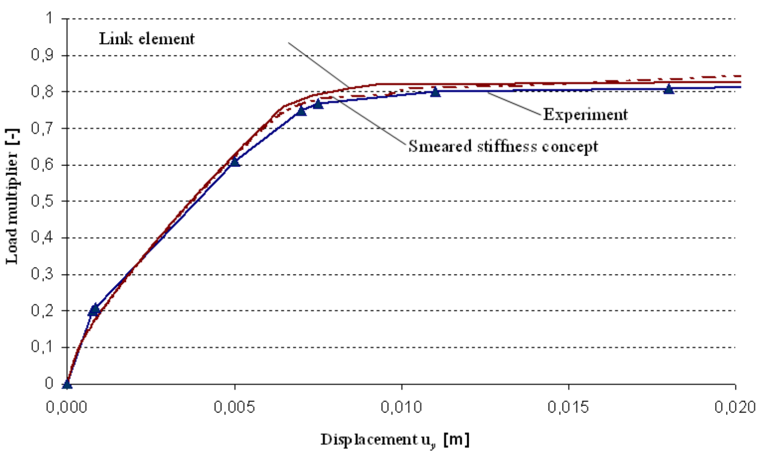

Fig. 9: Numerical analysis results: the elastic-plastic model of the concrete

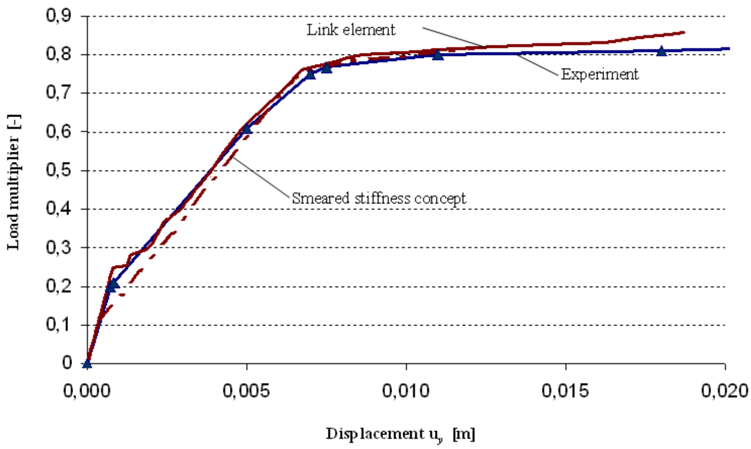

Fig. 10: Numerical analysis results: the CONCRETE model

Fig. 11 shows the working diagrams for the calculations which used in *BSA* a discrete model of reinforcement, special bond elements and an elastic-plastic model. The input parameters for the bond element study are same as in the previous calculation.

The stiffness is assumed to be constant for the bond elements. The working diagram used for the model is taken from the recommendation [3]. This is a prudent approach which provides prudent information about influence of the bond element stiffness on the computational process. If the stiffness of the bond elements is lower, the calculation needed more time and was less stable. If the stiffness of the bond element is $k = 10^{12}$, such connection between the reinforcement and concrete can be regarded as almost perfect. The best option seems to be 10^8: for this value, the calculation correlates at best to the experiment. The chart shows the calculation (the dot-and-dash line) where the variable stiffness of the bond element was determined on the basis of recommendations and information specified in [5, 11, 2].

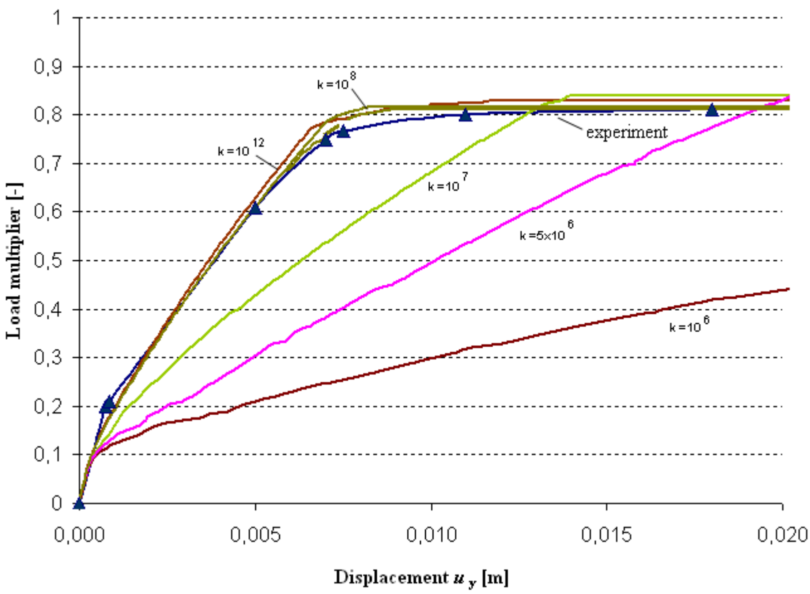

Fig. 11: Numerical analysis results: the bond elements

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8 CONCLUSION

The paper presents results of two approaches used in the modelling of reinforcement. Confrontation with an experiment provides very similar results. The models included scattered and discrete reinforcements. The reason for the difference between the working diagrams resulting from the calculations and experiment can be the assumed perfect connection of the reinforcement and concrete. The results were then confronted with the solution based on a discrete reinforcement model and bond elements. The use of the bond elements is given in the comparison study for various parameters of stiffness of the bond element. In order to introduce the special bond elements, the best solution seems to be to use input date from experiments or from recommendations specified in [2, 3, 5, 11].

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REFERENCES

[1] BARZEGAR, F. *Layering of RC Membrane and Plate Elements in Nonlinear Analysis.* Journal of the Struct. Div. 1988, Vol. 114, No.11. p. 2474-2492.

[2] BIGAJ, A. J. *Structural Dependence of Rotation Capacity of Plastic Hinges in RC Beam and Slabs*. PhD Thesis. Delf Univerzity of Technology, ISBN 90-407-1926-8.

[3] CEB - FIP Model Code 1990: Design Code. by Comite Euro-International du Beton, Thomas Telford, 1993. ISBN: 978-0727716965.

[4] CHEN, A. C. T., CHEN, W. F. *Constitutive Relations for Concrete*. Journal of the Engineering Mechanics Division ASCE, 1975

[5] FILIPPOU, F. C. *A Simple Model for Reinforcing Bar Anchorages Under Cyclic Excitations*. ASCE, Journal of Structural Engineering. 1986, Vol. 112, No. 7, pp. 1639-1659.

[6] HOFSTETTER, G., MANG, H., A. Computational Mechanics of Reinforced Concrete Structures. Wiesbaden: Vieweg, 1995. ISBN 3-528-06390-4.

[7] KRÁLIK, J., HUKEL, D. Nelineárna analýza interakcie oceľovej výstelky a železobetónovej steny šachty lokalizácie havárie za extrémneho pôsobenia tlaku a teploty. In *Modelování v mechanice 2010*. Mezinárodní konference. Ostrava: VŠB – TU Ostrava, 2010. ISBN 978-80-248-2234-1.

[8] MEGUID HEKAL, S. A. *Solution of Plate Structures Respecting Real Material Properties*. Disertační práce. Brno: Brno University of Technology, 1992.

[9] OHTANI, Y., CHEN, W. F., *Multiple Hardening Plasticity for Concrete Materials*. Journal of the EDM ASCE,1988.

[10] PROCHÁZKA, J. a kol. Navrhování betonových konstrukcí 1. 2 vyd. Praha: ČVUT v Praze, 2006. ISBN 80-903807-1-9.

[11] KWAK, H. G. *Material Nonlinear Finite Element Analysis and Optimal Design of Reinforced Concrete Structures*. Ph.D. Dissertation. Department of Civil Engineering, KAIST, Korea. 1990.

[12] RAVINGER, J. Modelovanie nelineárneho pôsobenia železobetónového nosníka použitím MKP. Stavebnický časopis. 35, 1987-8, 571-589.

[13] RAVINGER, J. Programy – statika, stabilita a dynamika stavebných konštrukcií. Alfa, Bratislava, 1990.

10.2478/v10160-011-0032-9

[14] RAVINGER, J., PSOTNÝ, M. *Analýza konštrukcií, Nelineárne úlohy*. Bratislava: STU v Bratislavě, 2007. ISBN 978-80-227-2713-6.

[15] RELEASE 11 DOCUMENTATION FOR ANSYS, SAS IP, INC., 2007.

[16] SUCHARDA, O., BROŽOVSKÝ, J. *Elastic-plastic Modelling of Reinforced Concrete Beam: Implementation and Comparison with the Experiment*. Transactions of the VŠB - Technical University of Ostrava. Construction Series. 2011, ISSN 1804-4824.

[17] WILLAM, K. J., WARNKE, E. P. *Constitutive Model for the Triaxial Behavior of Concrete,* Proceedings of International Association for Bridge and Structural Engineering, Bergamo, 1975.

Reviewers:

Dr.h.c. prof. Ing. Ján Ravinger, DrSc., Department of Structural Mechanics, Faculty of Civil Engineering, Slovak University of Technology in Bratislava.

Prof. Ing. Jiří Šejnoha, DrSc., Department of Mechanics, Faculty of Civil Engineering, Czech Technical University in Prague.