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# EFFECT OF SELECTED PARAMETERS OF NON-LINEAR ANALYSIS OF CONCRETE STRUCTURES

# VLIV VYBRANÝCH PARAMETRŮ NELINEÁRNÍ ANALÝZY BETONOVÝCH KONSTRUKCÍ

**Abstract**

The paper deals with a comparison of numerical calculations with experiment for different parameters of initial plasticity conditions by non-linear analysis. The paper also deals with a verification of the geometric non-linearity effect as per the theory of 2nd order and structural weight. The used constitutive model combines Chen-Chen condition of plasticity and Ohtani concept of hardening. The modelled experiment chosen for parametric study is reinforced concrete beam which is loaded by two forces.

**Keywords**

Beam, reinforced concrete, plasticity condition, structural weight, non-linear analysis.

**Abstrakt**

Příspěvek se zabývá srovnáním numerických výpočtů s experimentem pro různé parametry počáteční podmínky plasticity u fyzikálně nelineární analýzy. Dále se příspěvek zabývá ověřením vlivu geometrické nelinearity dle teorie 2. řádu a vlastní tíhy konstrukce na výpočet. Použitý konstitutivní model betonu kombinuje Chen-Chenovu podmínku plasticity a model zpevnění vypracovaný Ohtanim. Modelovaným experimentem zvoleným pro parametrickou studii je železobetonový nosník, který je zatížený dvěma silami.

**Klíčová slova**

Nosník, železobeton, podmínka plasticity, vlastní tíha, nelineární analýza.

## 1 INTRODUCTION

Many input parameters exist in the non-linear analysis of the reinforced concrete structures. Those input parameters may influence results. They include geometric non-linearity, own weight of the construction and, in case of an elastic-plastic model [3], the initial conditions of plasticity. The paper determines effects of the parameters above on a reinforced concrete beam with a rectangular cross-section which is loaded with two forces. The reason is that the geometric non-linearity and/or influence of the own weight is often neglected in analyses [9]. The initial condition of plasticity of the concrete – is not often available from experimental data or from information provided by the concrete production plant. Parameters of the initial condition of plasticity of the concrete are plastic limit of the material in a uniaxial compression, the plastic limit in a biaxial

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compression, and the plastic limit in a uniaxial tension. Plastic limit of the material are expressed by means of agreed values from failure condition of concrete.

One of most preferred numerical methods – the finite element method (FEM) – was chosen for the analysis of the building structures. Planar computational models and an isoparametric four – node finite elements were selected for the analysis [15]. The model of the smeared reinforcement [18] was chosen for the reinforcement.

## 2 THE ELASTIC-PLASTIC MODEL OF THE CONCRETE

Many constitutive models [1, 3, 5, 8, 21, 22] are available for the non-linear analysis of reinforced concrete structures. The constitutive model of the concrete combines the Chen-Chen condition of plasticity [2] and Ohtani concept of hardening [10]. The Chen-Chen condition of plasticity [3] was formulated exactly for the concrete on the basis of experiments carried out, among others, by Kupfer [6]. The mentioned constitutive model of the concrete was implemented in BSA (Building and Structural Analysis), this software being developed in VŠB-TU Ostrava [17, 19]. The Chen-Chen condition of plasticity [2] is defined by means of the plastic limit of the material in a uniaxial compression, $f_{yc}$, the plastic limit in a biaxial compression, $f_{ybc}$, and the plastic limit in a uniaxial tension, $f_{yt}$.

In order to use the constitutive model of concrete, it is also necessary to define a failure condition which is given for the concrete by the uniaxial compression strength, $f_{uc}$, biaxial compression strength, $f_{ubc}$, and uniaxial tension strength, $f_{ut}$. The plasticity limit can be described using the $\alpha$ coefficient for each type of load as follows:

$$f_{yc} = \alpha f_{uc}, \tag{1}$$

$$f_{ybc} = \alpha f_{ubc}, \tag{2}$$

$$f_{yt} = \alpha f_{ut}. \tag{3}$$

Coefficients $\alpha$ ranges from 0 to 1. Typically, $\alpha$ is chosen from the interval 0.3 [3] to 0.5 [17]. A parametric study was carried out for four $\alpha$ coefficients. The initial yield surface, subsequent loading surfaces and failure surface are shown in the level of main tension in Fig. 1.The figure shows also the working diagram for concrete.

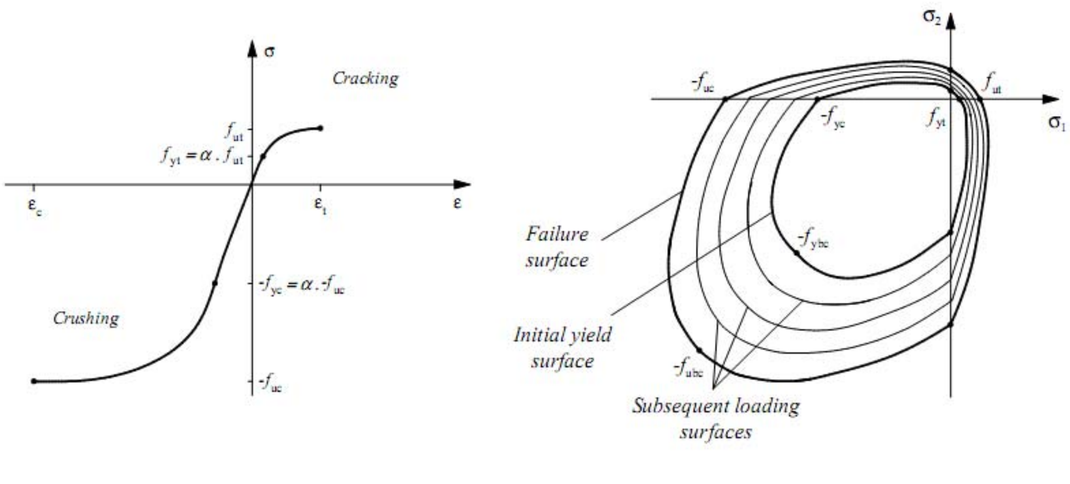


Fig.1: The working diagram of concrete and Chen-Chen condition of plasticity (failure) [3]

## 3 IMPLEMENTING THE EFFECTS OF GEOMETRIC NON-LINEARITY IN LINE WITH THE 2ND ORDER THEORY INTO FEM CALCULATION

When analysing reinforced-concrete beams subject to cross loads, effects of the physical non-linearity typically prevail over those of the geometric non-linearity. For this reason, the effects of the geometric non-linearity are often neglected [9]. In order to verify effects of the geometric non-linearity, BSA which is being developed, includes a modified algorithm which respects the 2$^{nd}$ order theory. An incremental procedure is used again to solve the system of non-linear equations [13, 14]. In the 2$^{nd}$ order theory, conditions of balance are based on a distorted construction [20].

In equilibrium, the following condition holds good:

$$\mathbf{f}_{ext} - \mathbf{f}_{\mathrm{int}} = 0 \ , \tag{4}$$

where: $\mathbf{f}_{ext}$ is the vector of external forces and

$\mathbf{f}_{\mathrm{int}}$ is the vector of internal forces.

In the geometric non-linearity calculation, the matrix of rigidity of a construction changes as a result of load deformation:

$$\mathbf{K}' = \mathbf{K}(\mathbf{u}). \tag{5}$$

The equilibrium ceases to exist and a residual vector is created:

$$\mathbf{f}_{ext} - \mathbf{f}_{\mathrm{int}} = \mathbf{r} \ . \tag{6}$$

A deformation increment, $\Delta\boldsymbol{\alpha}$, resulting from the residual vector is solved using:

$$\mathbf{K}'\Delta\boldsymbol{\alpha} = \mathbf{r} \ . \tag{7}$$

The final deformation vector $\tilde{\mathbf{u}}$ is:

$$\tilde{\mathbf{u}} = \mathbf{u} + \Delta\boldsymbol{\alpha} \,, \tag{8}$$

where: $\mathbf{u}$ is the vector of deformation caused by external loading and

$\Delta\boldsymbol{\alpha}$ is the vector of incremental deformation caused by geometric non-linearity.

In the incremental approach, the calculation is divided into several steps. The implementation of the 2$^{nd}$ order consists in updating of the geometry of the calculation model for each load step or iteration.

## 4 INFLUENCE OF INITIAL CONDITIONS OF PLASTICITY

For purposes of experiments in the parametric study with the initial condition of plasticity, a reinforced concrete beam loaded with two forces was chosen. The experiment was tested and published by Gaston, Siess and Newmark [4]. Source data relating to the experiment are taken from [7]. Fig. 2 shows the experiment.

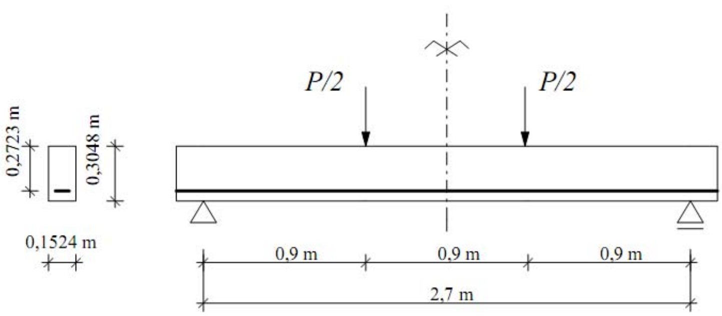


Fig. 2: Experiment

The reinforced concrete beam is made from concrete with the compressive strength of 32.3 MPa. The modulus of elasticity of concrete, $E_c$, is 27.1 GPa and Poisson coefficient is 0.17. Because the compression strength and modulus of elasticity of the concrete correlate little only for plain concrete, the tensile strength of concrete was determined on the basis of recommendations [12]:

$$f_{ut} = 0.3(0.7 f_{uc}^{(2/3)}), \tag{9}$$

where $f_{uc}$ is the compression strength of the concrete. The strength of the concrete in biaxial compression is:

$$f_{ubc} = 1.2 f_{uc}. \tag{10}$$

Coefficients $\alpha = 0.3$, 0.4, 0.45 and 0.5 were used for the calculations of parameters (1), (2), (3) which describe the original condition of plasticity. The degree of reinforcement, $\rho$, of the reinforced concrete cross section relating to the efficient height of the cross-section is 0.0062. It is assumed that the working diagram of the reinforcement is ideally elastic-plastic. The yield point of steel, $f_y$, is 323.6 MPa and the modulus of elasticity of steel, $E_s$, is 198.0 GPa. Fig. 4 shows a planar calculation model. For the sake of clarity, the calculation model is visualised in space in the ANSYS pre-processor ANSYS [16] and shown in Fig. 3.

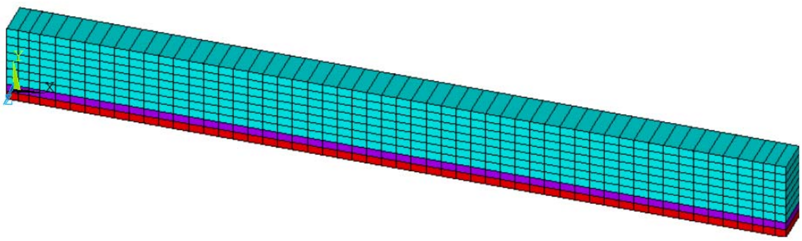

Fig.3: Calculation model

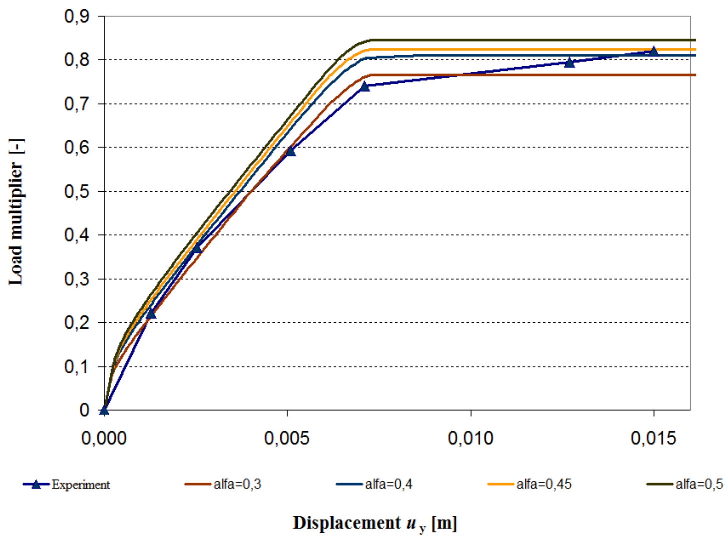


Fig.4: Calculation results

The comparison of the experiment and numerical calculation results proves that in most calculations the initial rigidity of the calculation model is slightly overestimated. Only for o $\alpha = 0.30$, the calculation model rigidity is slightly underestimated. During the loading, the difference between the working diagrams resulting from the numerical calculations and experiment is small. The development of the working diagram for the experiment and maximum load carrying capacity of the beam are described at best for $\alpha = 0.45$. Coefficients $\alpha = 0.40$ also proves reasonable accordance of the working diagram. Fig. 5-7 show results of the direct stress for the condition of plasticity $\alpha = 0.45$. The figures show the initial condition for development of the plastic area in the drawn area where compression concentrates on the upper edge. Further loading results in plasticizing of the reinforcement: concrete crushes and the calculation is over.

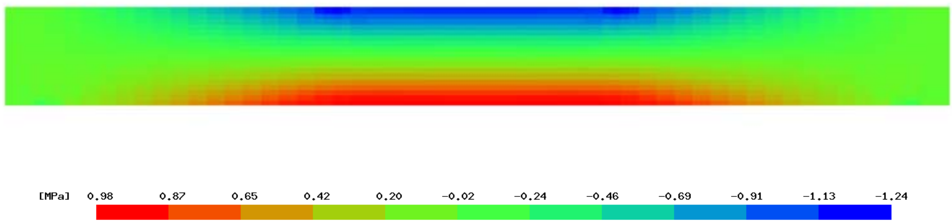


Fig. 5: Stress, $\sigma_x$, during the loading [MPa]

The load multiplier = 0.10 (Max = 0.98 MPa; Min = -1.24 MPa)

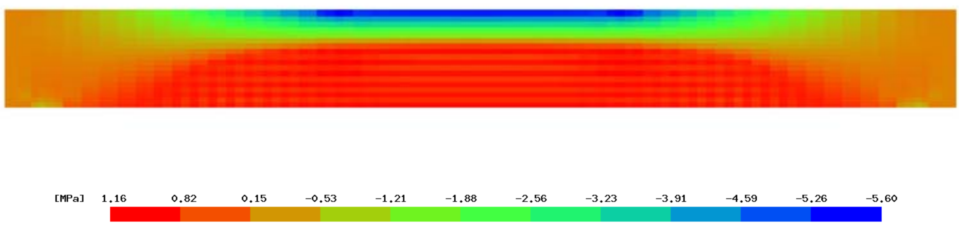


Fig. 6: Stress, $\sigma_x$, during the loading [MPa]

The load multiplier = 0.30 (Max = 1.16 MPa; Min = -5.60 MPa)

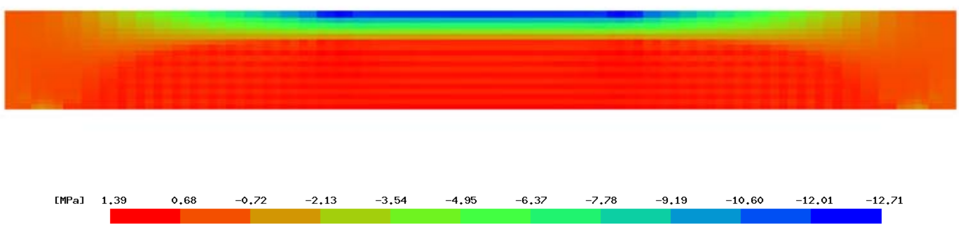


Fig. 7: Stress, $\sigma_x$, during the loading [MPa]

The load multiplier = 0.62 (Max = 1.39 MPa; Min = -12.71 MPa)

## 5 INFLUENCE OF THE 2$^{ND}$ ORDER THEORY

The analysis below deals with effects of the 2$^{nd}$ order theory for a beam from the previous parametric study. The initial plasticity of $\alpha = 0.45$ was chosen for the calculation.

When analysing regular reinforced-concrete constructions which are subject mostly to the bending, effects of the physical non-linearity typically prevail over those of the geometric non-linearity. For this reason, the effects of the geometric non-linearity are often neglected [9]. In order to verify effects of the geometric non-linearity, BSA which is being developed, includes a modified algorithm which is based on the Newton-Raphson method [14]. Small deformation only is likely for this type of the geometric non-linearity.

In case of the geometric non-linearity, the approach is based on the $2^{nd}$ order theory where the equilibrium conditions are created in nodes of the calculation model of a distorted construction. The implementation of the $2^{nd}$ order theory consists in updating of the geometry of the calculation model for each load step or iteration. A similar modification of geometry in the calculation model of a construction is used also in another calculation software: ATENA [11].

BSA calculations are supported by results obtained in SCIA [20], ATENA [11] and ANSYS [16]. In alternative software applications, the calculation is made for the geometric non-linearity and physical lineal calculation. Attention was paid to deflection in the middle of the beam. The calculation was carried out in five versions. For details see Table 1.

Tab. 1: Calculation parameters and calculation SW applications

| | Calculation parameters | | | Calculation programmes | | | |
|---|---|---|---|---|---|---|---|
| Calculations | Geometric nonlinearity | Own weight of construction | Physical non-linearity | BSA | SCIA | ANSYS | ATENA |
| Calculation 1 | no | no | no | yes | yes | yes | yes |
| Calculation 2 | yes | no | no | yes | yes | yes | yes |
| Calculation 3 | no | no | yes | yes | no | no | no |
| Calculation 4 | yes | no | yes | yes | no | no | no |
| Calculation 5 | yes | yes | yes | yes | no | no | no |

In the BSA calculation, the reinforcement is included also into the rigidity of the calculation model of the construction. In ANSYS [16], big deformations are set in the non-linear solver.

Because results of the linear calculation are slightly different for each software application, focus was also placed on the ratio of maximum deformations:

$$u_1 = \frac{u_{geo.non.}}{u_{geo.lin.}} \text{ [-]} \tag{11}$$

and difference in maximum deformations:

$$u_2 = u_{geo.non.} - u_{geo.lin.} \text{ [m]}, \tag{12}$$

where:

$u_{geo.non}$ is the vertical deformation for the geometric non-linear calculation [m] and

$u_{geo.lin.}$ is the vertical deformation for the geometric linear calculation [m].

The maximum vertical difference, $u_2$, for all computational SW applications is $2.10^{-6}$ m. Table 2 shows the maximum deformation ratios, $u_1$, for the vertical deflection in the middle of the beam for the geometric linear and non-linear calculations in BSA, SCIA [20], ANSYS [16] and ATENA [11]. The physical non-linearity is not included into the calculation. The calculations used are 1 and 2.

Tab. 2: Deformation ratios, $u_1$, for calculations the with linear and non-linear geometry

| Software | BSA | SCIA | ANSYS | ATENA |
|---|---|---|---|---|
| $u_1$ [-] | 1.0008410 | 1.0009528 | 1.0009040 | 1.0008610 |

The remaining calculations were carried out in BSA only. Each calculation took into account the physical non-linearity. Table 3 shows the deformation ratios, $u_1$, for the vertical deflection for both the linear and non-linear calculation in BSA. The calculation did not take into account the own weight of the construction. The calculations used are 3 and 4. The maximum loading multiplier obtained in the calculation 4 was 0.83.

Tab. 3: Deformation ratios, $u_1$, for calculations the with linear and non-linear geometry - BSA

| Load multiplier | 0.3 | 0.6 | 0.8 |
|---|---|---|---|
| $u_1$ [-] | 1.000281 | 1.000868 | 1.001593 |

## 6 INFLUENCE OF THE OWN WEIGHT OF THE CONSTRUCTION

Influence of the own weight of the construction was verified using a non-linear calculation in BSA. For purposes of calculation of the own weight of the beam construction, the volume density of concrete was 2,500 kg/m$^3$ and that of steel was 7,850 kg/m$^3$. The physical non-linearity was taken into account. The geometric non-linearity was not included into the calculation which dealt with the load only. The geometric non-linearity was included into the calculation which dealt with both the load and own weight of the construction. Similarly as with the previous calculations, the maximum deformation ratio was calculated:

$$u_3 = \frac{u_{weight+load}}{u_{load}} \text{ [-]}, \tag{13}$$

where:

$u_{weight+load}$ is the vertical deformation from the own weight of the construction and load [m] and

$u_{load}$ is the vertical deformation from the load [m].

Table 4 shows the deformation ratios, $u$, for the vertical deflection inside the beam span calculated in BSA for certain load multipliers. The calculations used are 3 and 5. The maximum load multiplier was 0.80 for the calculation 5.The difference in the working diagrams for the vertical deflection in the middle of then span for the calculations 3 and 5 was less than 5.0%. Only in case of the load multiplier = 0.8, it was 6.44%.

Table 4: Deformation ratios, $u_3$, for calculations the with linear and non-linear geometry - BSA

| Load multiplier | 0.3 | 0.6 | 0.8 |
|---|---|---|---|
| $u_3$ [-] | 1.038484 | 1.032328 | 1.064438 |

## 7 CONCLUSION

This paper compares selection of parameters for the initial conditions of plasticity in an elastic-plastic analysis of reinforced concrete structures. The parametric study was carried out for four $\alpha$ coefficients. The best parameter seems to be the initial plasticity of $\alpha = 0.45$, this value being in line with recommendations [12].

This paper also deals with the influence of geometric non-linearity and own weight in the elastic-plastic analysis. The geometric non-linearity was included into the calculation performed on the basis of $2^{nd}$ order theory. In those calculations, the effect of the geometric non-linearity on a rectangular cross-section beam loaded with two forces was not more than units of per mile. In the calculation which included effects of the own weight of the construction, the effects for specific values of the working diagram was less than 6.44%.

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