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STATISTICAL DEPENDENCE OF INPUT VARIABLES IN DOPROC METHOD

STATISTICKÁ ZÁVISLOST VSTUPNÍCH VELIČIN V METODĚ POPV

Abstract

In probabilistic tasks, input random variables are often statistically dependent. This fact should be considered in correct computational procedures. In case of the newly developed Direct Optimized Probabilistic Calculation (DOProC), the statistically dependent variables can be expressed by the so-called multidimensional histograms, which can be used e.g. for probabilistic calculations and reliability assessment in the software system ProbCalc.

Keywords

Direct Optimized Probabilistic Calculation, DOProC, random variable, statistical dependence, probability, double histogram, HistAn2D, triple histogram, HistAn3D.

Abstrakt

V pravděpodobnostních úlohách jsou často vstupní náhodné proměnné veličiny statisticky závislé. Tato skutečnost by se měla v korektních výpočetních postupech respektovat. Statistická závislost může být vyjádřena např. korelačním koeficientem, resp. korelační maticí. V případě nově vyvíjené metody Přímého Optimalizovaného Pravděpodobnostního Výpočtu (zkráceně POPV), lze statisticky závislé veličiny vyjádřit pomocí tzv. vícerozměrných histogramů, které lze využít např. při pravděpodobnostních výpočtech a posudku spolehlivosti s využitím programového systému ProbCalc.

Klíčová slova

Přímý Optimalizovaný Pravděpodobnostní Výpočet, POPV, náhodná proměnná, statistická závislost, pravděpodobnost, dvojný histogram, HistAn2D, trojný histogram, HistAn3D.

1 INTRODUCTION

The Direct Optimized Probabilistic Calculation (DOProC) is the probabilistic method which has been developed since 2002. Detailed information about this method is available in several publications, for instance, in [3, 4, 8]). Attention to the statistic dependence of the random input quantities has been paid within DOProC, for instance, in [15]. Investigations have resulted to the algorithm which can be used for creation of multidimensional histograms of the statistically dependent variables.

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Using this technique, DOProC methods can be reliably extended to cover calculations where statistically independent as well as the statistically dependent input variables are used (as in [9, 10, 11, 14, 17, 18]).

The multidimensional histogram of the statistically dependent variables enters the calculation as a whole. For each interval/class, it is possible to generate, with a certain non-zero probability, the values of those dependent variables which identify the interval.

2 THEORETICAL ANALYSIS

In each standard histogram A, one axis includes the a_j class which is limited by $a_{\min}$ and $a_{\max}$, while the other axis shows typically the probability, p_{aj}, of occurrence of that class, a_j. The sum of probabilities for each class a_j in the histogram is $\sum p_{aj} = 1$. In the double histogram of two random variables, Z_1 and Z_2, the quantity z_1 is limited again by $z_{1,\min}$ and $z_{1,\max}$, while z_2 is limited by $z_{2,\min}$ and $z_{2,\max}$.

The values can be divided, using the step Δz_1, into N_1 intervals for random quantities Z_1, or, using the step Δz_2, into N_2 intervals for the random quantities Z_2. The number of intervals are as follows:

$$N_1 = \frac{z_{1,\max} - z_{1,\min}}{\Delta z_1} \tag{1}$$

and

$$N_2 = \frac{z_{2,\max} - z_{2,\min}}{\Delta z_2} \; . \tag{2}$$

If the input variable z_1 is in the j^{th} class of $z_{1,j}$ in theory, z_2 could acquire following values: $z_{2,1}$, $z_{2,2}$, … , $z_{2,j}$, … , $z_{2,N2}$. This means, it can acquire N_2 values. The double histogram of the random quantities z_1 and z_2 can contain $N_1 \cdot N_2$ classes. This means, each class is determined by two values, $z_{1,j}$ and $z_{2,j}$, and by the probability of occurrence of that class, $p_{z,1,j,z,2,j}$.

And again: $\sum p_{z,1,j,z,2,j} = 1$. The number of classes with the non-zero histogram can reach the product of $N_1 \cdot N_2$. If the random quantities are dependant, the number of classes in the histogram with the non-zero probability can be considerably lower than the product $N_1 \cdot N_2$. This is not, however, the general case but parametric calculations with statistically dependant input variables have been proving this (see below).

The occurrence of intervals with the non-zero probability plays a major role in DOProC calculations because, in case of double or multidimensional histograms, it is not necessary to enter the non-zero probability classes. The number of classes, T_C , in such a histogram will be same as the number of classes with the non-zero probability of occurrence. For each class T_s, a certain probability exists for occurrence, p_{Ts} , of all random variables in the class. In case of the double histogram, in each class T_s there are random variables $z_{1,Ts}$ and $z_{2,Ts}$, which are characterized by the mean value from the interval.

If M random variables enter the probabilistic calculation, in the multidimensional histogram there will be not more than $T_M = N_1 \cdot N_2 \cdot \ldots \cdot N_m \cdot \ldots \cdot N_{M-1} \cdot N_M$ classes. Because these are mostly dependent quantities, the number of non-zero classes will be, typically:

$$T_C << T_M \; . \tag{3}$$

The source for creation of a multidimensional histogram of M independent random quantities is the primary data which can be obtained by measurements or observations. For each random quantity z_n , the number of interval N_n, will be determined. Each random quantity can have a different number of intervals. The total number of classes (incl. zero classes), which can be formed from the chosen number of intervals for each random dependent input quantity, will be T_M. When the zero-probability classes are eliminated, the number of classes will go down to T_C.

The primary data of random quantity should be checked for statistic dependence. This can be done using the Pearson's correlation coefficient, Spearman's rank correlation coefficient or Kendall correlation coefficient [13]. The correlation coefficient of the primary data should correspond to the correlation coefficient obtained after creation of the histogram.

It follows from the analyses that this depends on the chosen number of classes for each random input quantity. The more classes exist, the more the correlation coefficient converges to the correlation coefficient of the primary data.

Using the same interval for the dependent random quantity, z_n, the step Δz_n will be as follows:

$$\Delta z_n = \frac{z_{\max} - z_{\min}}{N_n} \ . \tag{4}$$

Each interval is given the mean (average) value of the interval for z_n. That value is $z_{n,p}$. Calculation of the values is very simple because:

$$z_{n,p,1} = z_{\min} + \frac{\Delta z_n}{2} \, , \ z_{n,p,2} = z_{n,p,1} + \Delta z_n \ , \cdots$$

$$\cdots \, , \ z_{n,p,m} = z_{n,p,m-1} + \Delta z_n \ , \cdots \, , \ z_{n,p,M} = z_{n,p,M-1} + \Delta z_n \ . \tag{5}$$

The dependent input variable z_1, which will be divided in the multidimensional histogram into N_1 intervals, will occur in the first interval of the multidimensional histogram in $N_2 \cdot ... \cdot N_m \cdot \ldots \cdot N_{M-1} \cdot N_M = T_M / N_1$ classes. The situation will be similar in all intervals of the dependent input quantity z_i. It is advisable to determine the class sequence for both reduced and unreduced multidimensional histograms. In the unreduced multidimensional histogram, there will be classes with the zero probability, while the reduced multidimensional histogram will comprise only classes with the non-zero probability. In the unreduced histogram, there will be T_M classes, while the reduced histogram will include T_c classes only.

In order to include a group of the random dependent variables into the global unreduced multidimensional histogram, it is necessary to calculate the number (sequence) of the class P in the unreduced histogram. The calculation is simple:

$$P = a_1 \cdot N_2 \cdot \ldots \cdot N_m \cdot \ldots \cdot N_{M-1} \cdot N_M + a_2 \cdot N_3 \cdot \ldots \cdot N_m \cdot \ldots \cdot N_{M-1} \cdot N_M +$$

$$+ a_3 \cdot N_4 \cdot \ldots \cdot N_m \cdot \ldots \cdot N_{M-1} \cdot N_M + \ldots + a_M \ . \tag{6}$$

In (6) the coefficients a_1, a_2, a_3 , … , a_M result from the following equation:

$a_1 = \text{celá část podílu } \dfrac{z_{1,s,k} - z_{1,\min}}{\Delta z_1}$, or $a_1 = \dfrac{z_{1,s,k} - z_{1,\min}}{\Delta z_1} - 1$, if $\dfrac{z_{1,s,k} - z_{1,\min}}{\Delta z_1}$ is an integer.

In analogy:

$a_2 = \text{celá část podílu } \dfrac{z_{2,s,k} - z_{2,\min}}{\Delta z_2}$, or $a_2 = \dfrac{z_{2,s,k} - z_{2,\min}}{\Delta z_2} - 1$, if $\dfrac{z_{12s,k} - z_{2,\min}}{\Delta z_2}$ is an integer…

For a_M :

$$a_M = \text{celá část podílu } \frac{z_{m,s,k} - z_{m,\min}}{\Delta z_m} + 1 \ .$$

The same sequence can exist for several groups of primary data of the random dependent input quantities. Some sequences (a majority of them, in case of the dependent random quantities) will not occur there at all. The probability of occurrence of a class is given by the sum of occurrences of same sequences divided by the sum of total number of groups of the random independent input quantities.

If the coefficients a_1, a_2, a_3 , … , a_M are known for the P sequence of the class in the global histogram (the coefficients can be determined from the P sequence), it is possible to determine the mean values of the dependent input quantities in the class within the corresponding sequence. If the sequence of the class is P in the global unreduced histogram, the following equations are valid for the coefficients and mean values of the class:

$$z_{1,p} = z_{1,\min} + \Delta z_1 \cdot \left(a_1 + \frac{1}{2}\right), \quad z_{2,p} = z_{2,\min} + \Delta z_2 \cdot \left(a_2 + \frac{1}{2}\right), \ldots$$

$$\ldots, \; z_{M-1,p} = z_{M-1,\min} + \Delta z_{M-1} \cdot \left(a_{M-1} + \frac{1}{2}\right), \; z_{M,p} = z_{M,\min} + \Delta z_M \cdot \left(a_M - \frac{1}{2}\right). \qquad (7)$$

Finally, it should be pointed out that the sets of random dependent input quantities for the probabilistic calculations can be depicted in charts in a form of multidimensional histograms, if these are two dependent input quantities. If there are more dependent input quantities, it is impossible to depict them in charts.

It should be also emphasized that even the independent random input quantities can be entered into the calculation in a form of double or multidimensional histograms. This means, it is not necessary to define criteria for dividing the input data clearly into statistically dependent and independent ones. If it is assumed that the input data could be statistically dependent, this fact should be considered during calculations. Computational algorithms in DOProC can be used in calculations using the statistically independent random input quantities as well as using the data where statistical dependence could be taken into account.

3 SOFTWARE: HISTAN2D AND HISTAN3D

Having applied the technique described above, primary data have been analyzed for cross-section parameters of rolled sections, IPE 140. These data were published in [16]. Special software applications HistAn2D [6] and HistAn3D [7] (see Fig. 1 and 2) were developed for creation of the double and triple histograms which describe the statistical dependence between two or three random variables.

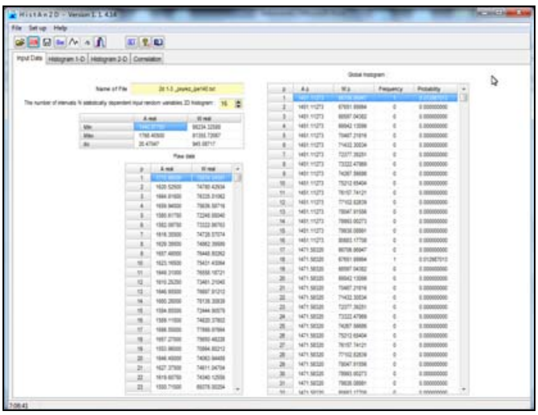

Fig. 1: Desktop in HistAn2D for analysis of statistical dependence of two random quantities and double histograms

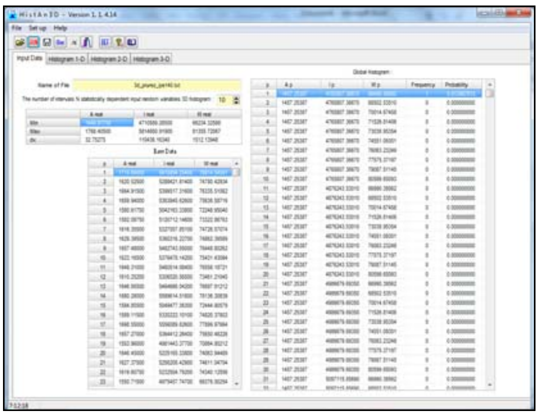

Fig. 2: Desktop in HistAn3D for analysis of statistical dependence of three random quantities and triple histograms

Once the text file with the statistically dependent primary data is read, it is necessary to enter only the number of interval/classes of the double or triple histogram which should be created. Using the software, it is possible to view for each random variable a simple histogram with non-parametric (empirical) distribution of probability (Fig. 3 and Fig. 4) as well as a multidimensional histogram which describes the statistical dependence between the quantities. The double histogram can be shown in a chart (see Fig. 6). When creating the histogram, the primary data need to be divided into

16^2 intervals. If the double histogram were created from an histogram for a cross-section area *A* (Fig. 3) and histogram of a cross-section module *W (*Fig. 4) (this means, for two statistically independent quantities), the double histogram would correspond to the chart in Fig. 5 which is very different from a double histogram of two dependent quantities (Fig. 6). It is impossible to view a triple histogram (this was mentioned at the end of the previous chapter), but HistAn3D can show this histogram in layers.

Correlation coefficients suggested by Pearson and Spearman (see Table 1) were calculated for the primary data and for double histograms of the dependent input data with different number of intervals. It is clear that the correlation coefficients of the double histograms are different and that they converge towards the correlation coefficients of the primary data with the increasing number of intervals. See Fig. 7 and 8. Using the statistically dependent input quantities (see chapter 4), 16^2 intervals were chosen for creation of a double histogram. For that number of interval, the calculated correlation coefficients do not differ too much from the correlation coefficients of the primary data (see Fig. 7 and 8).

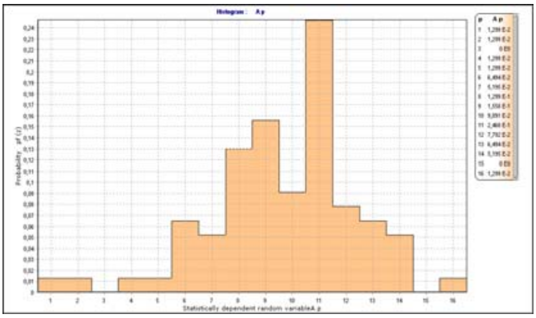

Fig. 3: Histogram of the cross-section area *A*

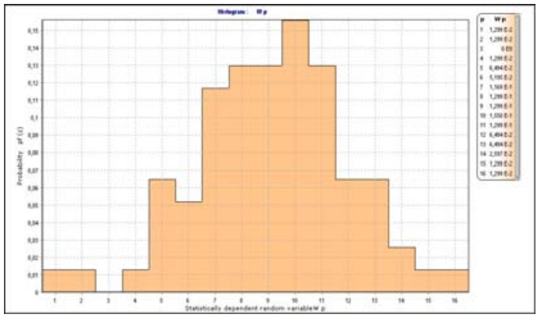

Fig. 4: Histogram of the cross-section modulus W_y

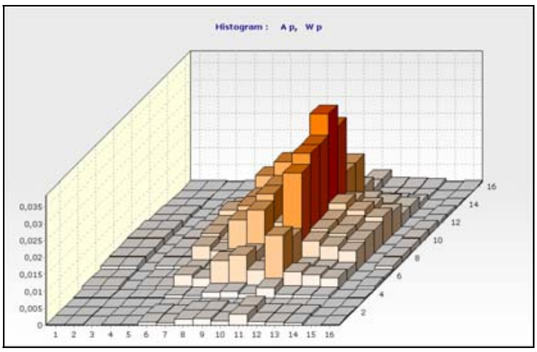

Fig. 5: Behavior of two independent random quantities - cross-section area *A* and cross-section modulus W_y

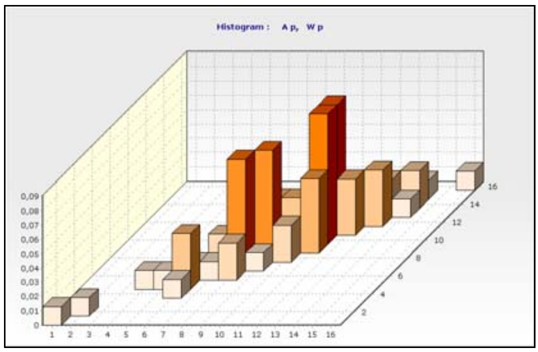

Fig. 6: Double histogram for two statistically random quantities – cross-section area *A* and cross-section modulus W_y

The calculations prove that the correlation coefficients in a double histogram of the independent random quantities in Fig. 5 are zero. Table 2 lists the numbers of classes of the zero probability double histograms for a total number of intervals. For the double histogram in Fig. 5, the number of zero probability classes is considerably lower than the number of a double histogram based on the same primary data which are, however, regarded as statistically dependent (Fig. 6). This fact is shown also in a chart in Fig. 9. The multidimensional histograms can be used for description of the statistical dependence of the random input variables in those probabilistic calculations which are solved using DOProC techniques. This includes, for instance, ProbCalc [2, 5].

Table 1: Correlation coefficients of a double histogram of the statistically dependent quantities with different numbers of intervals (Pearson's correlation coefficient for primary data is 0.96452015; Spearman correlation coefficient for primary data is 0.94989221)

Number of intervals in a double histogram	*Pearson's correlation coefficient*	*Spearman's rank correlation coefficient*	*Number of intervals in a double histogram*	*Pearson's correlation coefficient*	*Spearman's rank correlation coefficient*
$4^2 = 16$	0,79985097	0,79507798	$18^2 = 324$	0,95267109	0,94023800
$6^2 = 36$	0,86661900	0,86360377	$20^2 = 400$	0,96046634	0,94378886
$8^2 = 64$	0,91530000	0,91194405	$22^2 = 484$	0,95940904	0,94355084
$10^2 = 100$	0,93984931	0,92352904	$24^2 = 576$	0,95903334	0,94989866
$12^2 = 144$	0,94381175	0,93613068	$26^2 = 676$	0,96464064	0,95260826
$14^2 = 196$	0,95443331	0,93939308	$28^2 = 784$	0,96017017	0,94660574
$16^2 = 256$	0,94876401	0,93694950	$30^2 = 900$	0,95938019	0,94245225

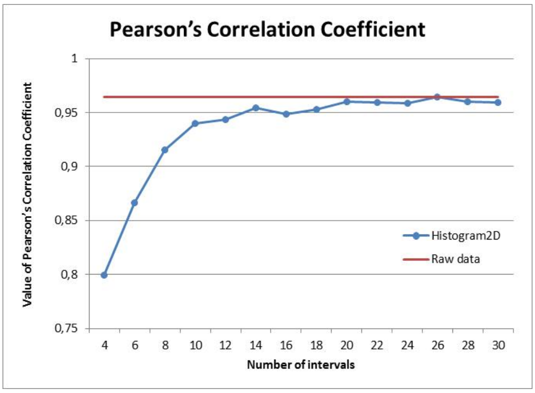

Fig. 7: Pearson's correlation coefficient of a double histogram vs. number of intervals

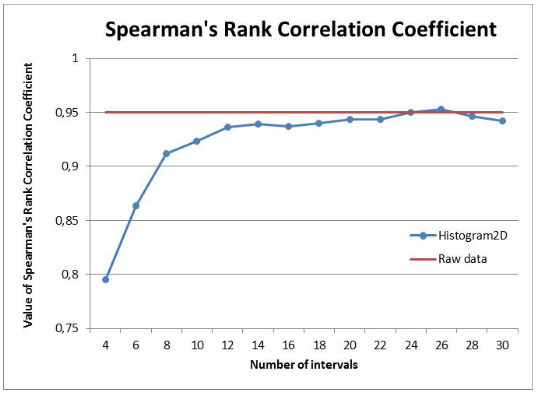

Fig. 8: Spearman's rank correlation coefficient of double histogram vs. the number of intervals

Table 2: The number of classes for double histograms with zero probability vs. the number of intervals chosen during creation of the histograms from the primary data

Number of intervals in a double histogram	*Number of zero-probability intervals*		*Number of intervals in a double histogram*	*Number of zero-probability intervals*	
	Statistically dependent quantities	*Statistically independent quantities*		*Statistically dependent quantities*	*Statistically independent quantities*
$4^2 = 16$	6	0	$18^2 = 324$	288	69
$6^2 = 36$	24	0	$20^2 = 400$	361	112
$8^2 = 64$	48	0	$22^2 = 484$	443	160
$10^2 = 100$	80	0	$24^2 = 576$	531	216
$12^2 = 144$	119	0	$26^2 = 676$	627	258
$14^2 = 196$	166	14	$28^2 = 784$	735	322
$16^2 = 256$	222	46	$30^2 = 900$	847	372

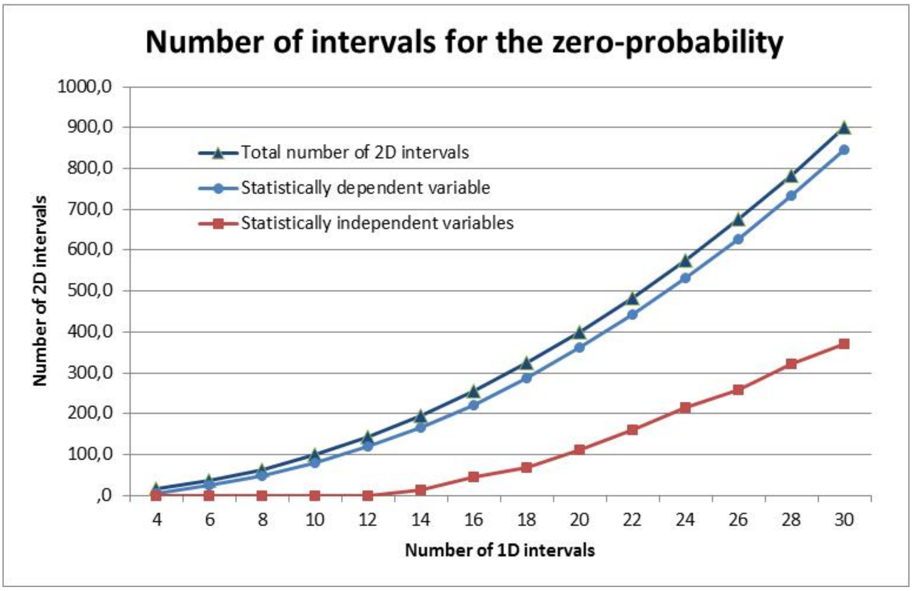

Fig. 9: Number of intervals for the zero-probability in double histogram

4 APPLYING THE COMPUTATIONAL TECHNIQUE WITH THE STATISTICALLY DEPENDENT INPUT QUANTITIES

For the sake of clarity, ProbCalc performed the probabilistic calculation for the reliability assessment of a cross-section in a vertex of an elemental parabolic arch fixed in both ends and loaded in a vertex with a single load. The static scheme is shown in Fig. 10.

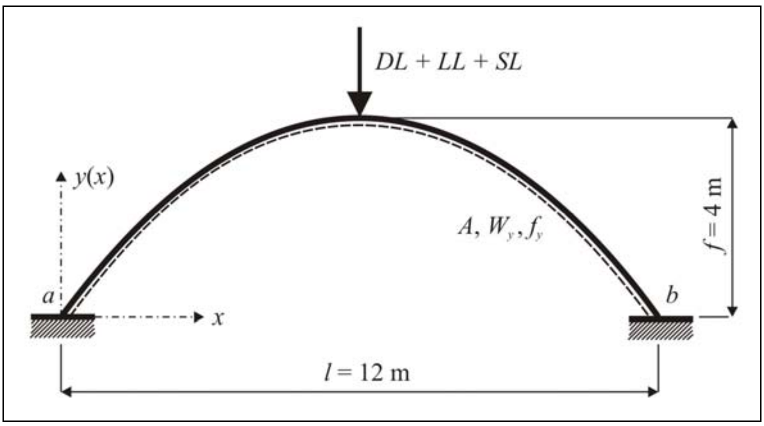

Fig. 10: Static scheme of the elemental structure of a parabolic arch fixed in both ends and loaded with combination of three single loads in a vertex

The center line of the parabolic arch in the coordinate system in Fig. 10 is defined by the following curve and equation:

$$y(x) = \frac{4.f.x}{l^2}.(l-x) \, , \tag{8}$$

where f is the upward deflection and l is the outspan (in this case: $f = 4$ m and $l = 12$ m).

The assessment has been made using the interaction formula:

$$\left(\frac{N_{Ed}}{N_{Rd}}\right)^2 + \frac{M_{Ed}}{M_{Rd}} \le 1 \ , \tag{9}$$

with following variables:

- Normal force in the cross-section:

$$N_{Ed} = -\frac{15.l.F}{64.f} \ , \tag{10}$$

- Bending moment in the cross-section:

$$M_{Ed} = \frac{3}{64}.F.l \ , \tag{11}$$

- Load-carrying capacity in simple compression:

$$N_{Rd} = f_y.A \ , \tag{12}$$

- Bending capacity:

$$M_{Rd} = f_y.W_y \ . \tag{13}$$

The cross-section has been assessed in terms of reliability by calculating the failure probability P_f and by comparing it with the designed probability P_d pursuant to ČSN EN 1990. The failure probability P_f was determined using the reliability function RF:

$$P_f = P(RF < 0) = P\left(1 - \left[\left(\frac{N_{Ed}}{N_{Rd}}\right)^2 + \frac{M_{Ed}}{M_{Rd}}\right] < 0\right) \ . \tag{14}$$

For purposes of the reliability assessment of the structure, following parameters were used: the random variable yield point f_y of steel S235 taken from [16], permanent load (extreme value: 8 kN, histogram DEAD1.dis), short-term load (extreme value 15 kN, histogram LONG1.dis) and long-lasting random (variable) load (extreme value 10 kN, histogram SHORT1.dis) . (The histograms describing the three random variable components of the load were taken from the database of histograms in a light version of Anthill for Windows, ver 2.6 which is available at http://www.sbra-anthill.com/).

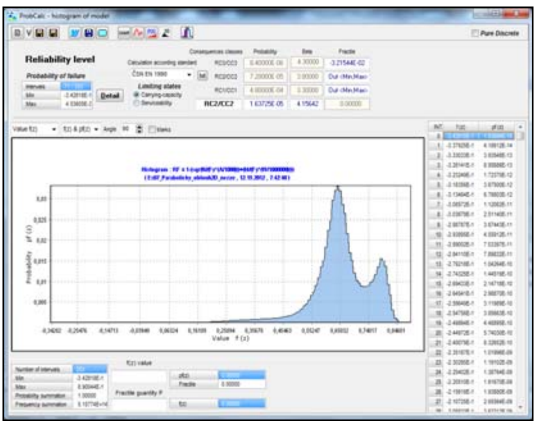

Fig. 11: Histogram - reliability function RF for the probabilistic calculation with statistically independent cross-section parameters of the cross-section area A and cross-section modulus W_y Failure probability $P_f = 1.637.10^{-5}$.

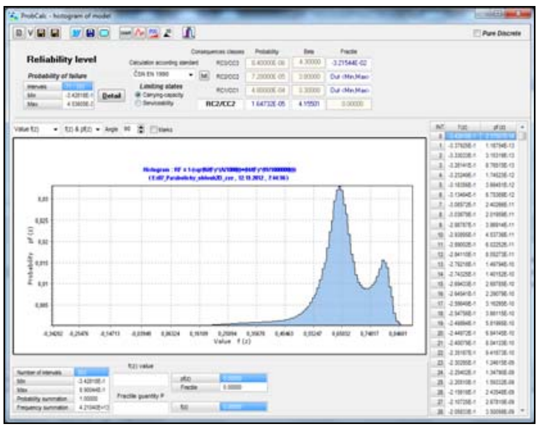

Fig. 12: Histogram - reliability function RF for the probabilistic calculation with statistically dependent cross-section parameters of the cross-section area A and cross-section modulus W_y Failure probability $P_f = 1.647.10^{-5}$.

The probabilistic assessment of the structure was made in the vertex of the arch. First, the both random quantities - the cross-section area A and cross-section modulus W - were regarded at statistically independent (see Fig. 11) and then as statistically dependent (see Fig. 12). A double histogram was used. It consisted of the primary data of the cross-section parameters of the rolled sections IPE 140 in line with data published in [16] and described using the method mentioned in Chapter 2 in HistAn2D [6] in line with chapter 3. Fig. 11 and 12 show the resulting failure probability P_f for both alternatives of the probabilistic calculation.

The designed failure probability P_d (and the reliability index β_d) are differentiated now in ČSN EN 1990 "Eurocode: Basis of structural design" depending on the required reliability, types of ultimate conditions and assumed service life of the structure T_d. Three classes of consequences, CC1 through CC3, and three classes of reliability, RC1 through RC3, are defined in the standard. The standard also lists the designed values of the failure probability P_d and reliability index β_d. In both cases of the probabilistic task, the cross-section of the structure meets requirements of CC2/RR2 with medium consequences in terms of lost lives or with considerable economic/social/environmental consequences. This corresponds to the designed failure probability $P_d = 7.2.10^{-5}$ and reliability index $\beta_d = 3.8$.

The final failure probability P_f in the calculation with the statistically independent cross-section parameters is slightly lower than in the calculation where the cross-section area A and cross-section modulus W are statistically dependent on each other. In general, it is less safe to use the probabilistic calculation where the input quantities are statistically independent random quantities, if it is beyond doubt that the input random variables statistically depend on each other. In that case, the difference in the failure probability is not too significant between the statistically-dependent and statistically-independent cross-section parameters. The reason is that the rolled sections are manufactured with relatively high accuracy and have considerably less dispersion of geometry and cross-section parameters than other random variables that enter the calculation.

5 CONCLUSION

This work describes the algorithm used for creation of multidimensional histograms of statistically dependent variables. This algorithm can be used in calculations based on DOProC techniques – for instance, in ProbCalc. This can describe correlation between the random input variables, e.g. in the cross-section parameters, as explained in this paper, or strength parameters (obtained, for instance, in [1, 12]).

There is still space for investigation of the statistically dependence of the random variables in DOProC. The next objective might be determination of the correlation coefficients for triple histograms or use of parametric distribution of probability for creation of multidimensional histograms. It would be also advisable to determine the relation between this method and linear transformation carried out, for instance, by Choleski decomposition of the correlation matrix or between this method and orthogonal transformation which uses eigen vectors and eigen numbers of the correlation matrix.

FINAL COMMENT

The mentioned DOProC techniques with the statistically dependent input variables have been gradually implemented into ProbCalc. A light version of ProbCalc is available at http://www.fast.vsb.cz/popv. On the same website, it is possible to download light versions of HistAn2D and HistAn3D which describe statistical dependence of two or three random variables.

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