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ASSESSMENT OF THE MODEL UNCERTAINTY IN SHEAR RESISTANCE OF REINFORCED CONCRETE BEAMS WITHOUT SHEAR REINFORCEMENT

HODNOCENÍ MODELOVÉ NEJISTOTY PRO SMYKOVOU ODOLNOST ŽELEZOBETONOVÝCH NOSNÍKŮ BEZ SMYKOVÉ VÝZTUŽE

Abstract

The paper is focused on the model uncertainty related to shear resistance of reinforced concrete beams without special shear reinforcement considering available test results. Variation of the model uncertainty with basic variables is analysed and significant variables are identified for the section-oriented formula provided in EN 1992-1-1. Proposed probabilistic description of the model uncertainty consists of the lognormal distribution having the coefficient of variation of 0.15 and the mean value varying from 0.9 to 1.05 for beams with light to heavy longitudinal reinforcement.

Keywords

Model uncertainty, shear resistance, reinforced concrete structures.

Abstrakt

Článek se zaměřuje na stanovení modelové nejistoty smykové odolnosti železobetonových nosníků bez smykové výztuže s využitím dostupných experimentálních měření. Analyzuje se vliv základních veličin na modelovou nejistotu pro smykovou odolnost stanovenou podle EN 1992-1-1. Ukazuje se, že modelovou nejistotu lze popsat lognormálním rozdělením s variačním koeficientem 0.15 a průměrnou hodnotou pohybující se mezi 0.9 a 1.05 pro nosníky s nízkým až vysokým stupněm podélného vyztužení.

Klíčová slova

Modelová nejistota, smyková odolnost, železobetonové konstrukce.

1 INTRODUCTION

Previous studies [1-4] indicated that structural resistances can be predicted by appropriate modelling of material properties, geometry variables and uncertainties associated with an applied model. The effect of variability of materials and geometry is relatively well understood and has been extensively investigated. However, improvements in the description of model uncertainties are still needed [4].

For reinforced concrete structures flexural resistances are predicted with reasonable accuracy while accurate prediction of the shear resistances is difficult due to the uncertainties in the shear transfer mechanism, particularly after initiation of cracks [5]. The submitted study is therefore aimed

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at the model uncertainties of the shear resistance of beams without special shear reinforcement such as stirrups or inclined bars (hereafter referred to as "shear reinforcement" to simplify the text).

Although members without shear reinforcement where shear failure is likely are not common in practice, adequate reliability needs to be assured since their failure is brittle and sudden. For members with shear reinforcement sudden failure is prevented by the reinforcement [6]. The shear behaviour of reinforced concrete members with shear reinforcement is considerably different from that without such reinforcement. However, even in the case of designing a member with shear reinforcement, it is essential to accurately assess the shear strength of a member without shear reinforcement. This is because shear design provisions usually require assessment of the shear capacity without shear reinforcement in order to check whether additional reinforcement is necessary or not [5].

EN 1992-1-1 [7] allows members without shear reinforcement to be used for slabs and members of minor importance such as lintels with a span length less than 2 m. The relationship for shear resistance of such members in the Code has been calibrated against extensive database of shear tests [8]. Background documents of the CEN/TC 250 Horizontal Group – Bridges indicate that EN 1992-1-1 [7] requires shear reinforcement in some cases where it was not previously needed. Analysis of the EN resistance model for structural members without shear reinforcement thus seems to be desired.

The present paper is an extension of a recent contribution [9]. In the paper probabilistic description of the model uncertainties is investigated. The model uncertainty factor is derived using the design value method to facilitate operational applications of the partial factor methods.

2 DEFINITIONS OF THE MODEL UNCERTAINTIES

According to [10] the model uncertainty is generally a random variable accounting for effects neglected in the models and simplifications in the mathematical relations. The model uncertainties can be related to:

- Resistance models (based on structural mechanics, constitutive laws),
- Models for action effects (assessment of load effects and their combinations).

This study is fully focused on the uncertainties related to resistance models of reinforced concrete structures. It is assumed that the uncertainty of actions can be treated separately.

The uncertainty of a resistance model θ should cover the following aspects (if relevant):

- Simplifications of known physical principles in an applied model (θ_S),
- Approximations inherent to numerical methods and influence of different interpretations of complex software tools (θ_A).

The uncertainty θ_S is related to the selection of a resistance model (e.g. application of the Finite Element Methods (FEM) compared with simplified engineering formulas) and possibly shortcomings of the whole profession (imprecision of the most suitable available model). In many cases this type of uncertainty can hardly be reduced. On the contrary the uncertainty θ_A is often reducible (or can be eliminated) using finer mesh in FE computations and particularly by quality control (independent checks, consultations with experts on modelling etc.).

Commonly variability of material properties and possibly related statistical uncertainty are included in relevant models for material properties. In general the model uncertainty can be obtained from comparisons of physical tests and model results. A great care should be taken to define correctly test conditions and evaluate test results. It should be always assured that a specimen fails in an investigated failure mode. For instance when the model uncertainty in shear is investigated, beams failed in bending should be excluded from the assessment. Accuracy of tests (related to the test method and execution of an individual test) is commonly accounted for by a measurement error ε.

Relationships among tests, models, related uncertainties θ_i and the measurement error ε are indicated in Fig. 1 with examples of influences to be considered.

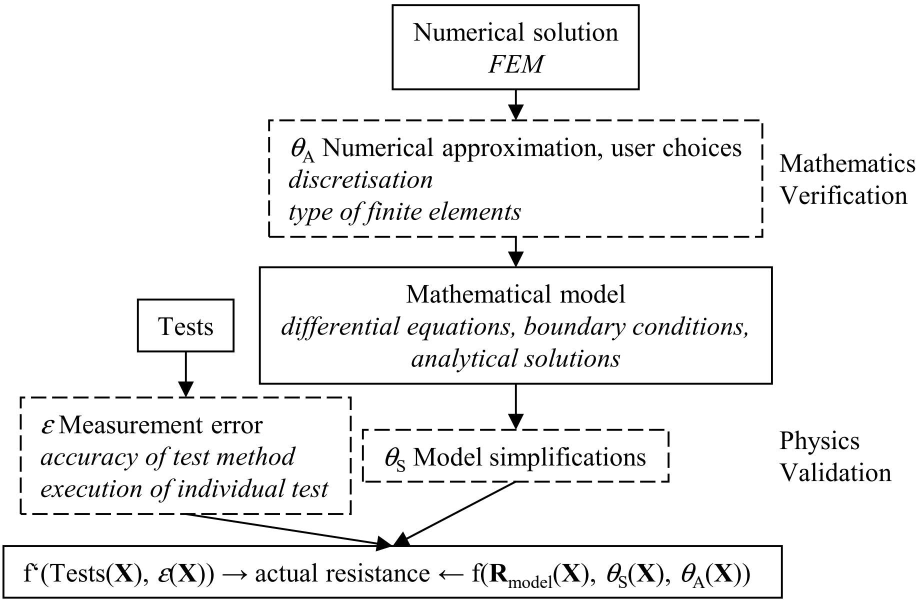


Fig. 1: Relationships among tests, models, related uncertainties and the measurement error

In Fig. 1 terminology used in [11,12] is accepted and thus:

- Validation denotes the process of determining if a mathematical model of a physical event represents the actual physical event with sufficient accuracy ("validation deals with physics"),
- Verification is the process of determining if a computational model obtained by discretizing a mathematical model of a physical event and the code implementing the computational model can be used to represent the mathematical model of the event with sufficient accuracy ("verification deals with mathematics").

It is emphasised that the presented overview of factors affecting the model uncertainty is simplified; advanced general concept of the model uncertainty is provided in [13].

The present study is focused on the assessment of model uncertainties related to a simple model for shear resistance provided in EN 1992-1-1 [7] for which the model uncertainty θ_A is assumed to be irrelevant. Moreover, variability of the measurement error ε is assumed to be negligible. For convenience of the notation the model uncertainty due to model simplifications is hereafter referred to as θ.

In the JCSS Probabilistic Model Code [10] the following definitions of the model uncertainty θ based on different relationships between the response of a structure (actual resistance) R and a model resistance $\mathrm{R_{model}}$ (estimate of the resistance based on a numerical model or analytical expression) are proposed:

$$R = \theta \, \mathrm{R_{model}}(\mathbf{X}) \tag{1}$$

or

$$R = \theta + \mathrm{R_{model}}(\mathbf{X}) \tag{2}$$

or a combination of both; $\mathbf{X}^{\mathrm{T}} = (X_1,\ldots, X_m)$ is the vector of basic variables X_i. In this paper the model uncertainty is assumed to be a random variable θ. However, in more advanced analyses it may be represented by functions of several auxiliary random variables $\boldsymbol{\theta}$ and basic variables $\mathbf{X}$ involved in the model resistance [5].

It is difficult to specify general conditions under which Eq. (1) or (2) becomes preferable since the choice always depends on task-specific conditions. Current practice indicates that the multiplicative definition in Eq. (1) is widely applied to the model uncertainties while the additive relationship in Eq. (2) is used to account for systematic measurement errors.

From a purely statistical point of view the multiplicative relationship is more appropriate when the structural resistance R and the model resistance $\mathrm{R}_{\mathrm{model}}(\cdot)$ are described by lognormal distributions since the model uncertainty θ is likewise lognormally distributed and its statistical characteristics can be readily derived. Similarly, the additive formula becomes preferable when normal distributions are relevant. It is worth noting that Eq. (1) can be transformed to Eq. (2) using the logarithmic transformation:

$$\ln R = \ln \theta + \ln[\mathrm{R}_{\mathrm{model}}(X_1,\ldots, X_m)] \tag{3}$$

The model uncertainty θ in general depends on basic variables **X**. Influence of individual variables on θ can be assessed by a regression analysis as described e.g. in [14]. It is also indicated that the model describes well the essential dependency between R and **X** only if the model uncertainty:

- Has either a suitably small coefficient of variation (how small is the question of the practical importance of the accuracy of the model) or
- Is statistically independent of the basic variables **X**.

The model uncertainty should be always clearly associated with an assumed resistance model. It may also be important to define ranges of the input parameters **X** for which the accepted model uncertainty is valid. Such intervals should be established on the basis of:

- admissible ranges of **X** for the model under investigation (for instance limits on reinforcement ratio) and
- simplifications in modelling of θ (for instance when θ is considered independent of X_i, but only for some restricted interval of the basic variable).

3 UNCERTAINTIES RELATED TO THE MODEL PROVIDED IN EN 1992-1-1

3.1 Model in EN 1992-1-1

In this section uncertainties related to the basic resistance model provided in EN 1992-1-1 [7] for beams without shear reinforcement are considered:

$$\mathrm{R}_{\mathrm{model}}(\mathbf{X}) = \max[0.18k(100\rho_l f_c)^{1/3} b_w d; \quad 0.035k^{3/2} f_c^{1/2} b_w d] \tag{4}$$

where:

$k = \min[1 + \sqrt{(200\text{ mm} / d)}; 2.0]$

Notation of the basic variables is provided in Tab. 1. No axial compressive force is considered and neither the partial factor γ_C nor the characteristic value of f_c is applied in Eq. (4).

3.2 Database of experimental results

Researchers at the University of Stellenbosch collected a database of 184 tests of beams without shear reinforcement [15]. Overview of the experimental data is given in Tab. 1. The database covers a wide range of beams with low to medium concrete strengths; and small, ordinary and large effective depths. Lightly, moderately and heavily reinforced beams are included. The shear span-to-depth ratio a/d exceeds 2.9 for all the beams to exclude deep beam and shear bond failures [16].

It is worth noting that the database contains:

- three specimens with the longitudinal reinforcement of yield strength f_y = 999 MPa and
- five specimens with the longitudinal reinforcement of f_y = 1780 MPa.

Tab. 1: Scatter of variables included in the database and parameters describing their influence on θ

Variable	Min.	Max.	ρ	R^2
b_w (mm) – smallest width of a cross-section in the tensile area	100	1000	-0.30	0.09
h (mm) – height	125	1250	-0.29	0.08
d (mm) – effective depth	110	1200	-0.30	0.09
a/d – shear span-to-depth ratio	2.95	8.03	-0.23	0.05
agg (mm) – aggregate size	6.4	38	-0.22	0.05
f_c (MPa) – concrete compressive strength	14.7	45.7	0.23	0.05
A_{sl} (mm^2) – area of the tensile reinforcement	199	7000	-0.12	0.01
$\rho_l = 100 \times A_{sl} / (b_w d) \leq 2$ (%) – longitudinal reinforcement ratio	0.42	4.73	0.45	0.20
f_y (MPa) – yield strength	276	1780*	-0.26	0.07
V_u (kN) – shear force at failure	19.5	392	-	-
$\rho_l f_y$ (MPa)	2.24	16.62	0.43	0.18

* 800 MPa after exclusion of specimens with high f_y (see below)

Histogram of yield strengths in the whole database is shown in Fig. 2. Annex C of EN 1992-1-1 [7] states that the design rules of Eurocode are valid when reinforcing steel of the characteristic yield strength f_{yk} between 400 to 600 MPa is used. Therefore, particularly values of f_y mentioned above seem to be high and the eight specimens are excluded from the database. Since the yield strength is not included in Eq. (4), the other specimens for which f_y is less significantly beyond the limits remain included in the database for a statistical evaluation of the model uncertainty.

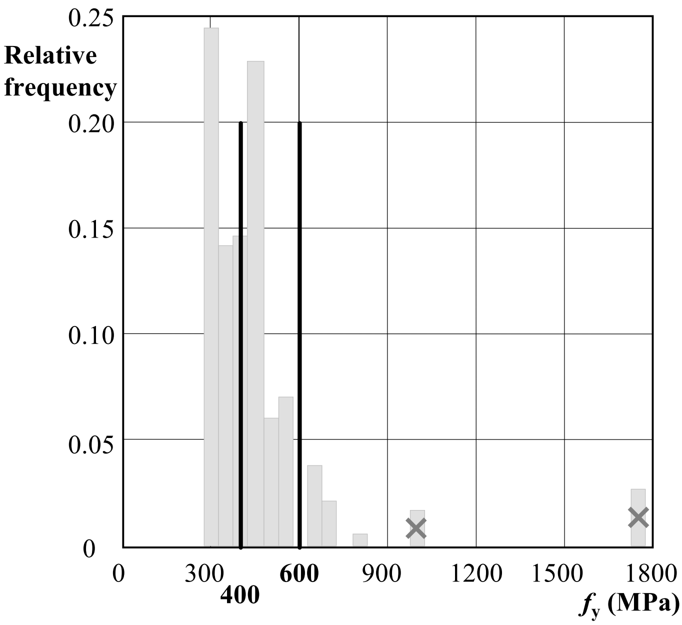


Fig. 2: Histogram of f_y for the whole database

3.3 Statistical evaluation of the model uncertainty

For each experiment the model resistance is assessed from Eq. (4) and the model uncertainty from Eq. (1). Note that the first term in Eq. (4) is decisive for all the specimens. Obtained sample statistical characteristics of θ for the whole database (mean m, coefficient of variation v, skewness w) are given in Tab. 2.

Tab. 2: Sample characteristics of the model uncertainty

Description of the sample	m	v	w
Whole database, n = 176	1.00	0.14	0.72
Whole database without the outliers, n = 174	1.00	0.13	0.15
Lightly reinforced beams ($0.14 \leq \rho_l \leq 1$ %), n = 27	0.89	0.13	0.60
Moderately reinforced beams ($1 < \rho_l \leq 2$ %), n = 63	0.98	0.12	-0.15
Heavily reinforced beams ($2 < \rho_l \leq 4.73$ %), n = 86	1.05	0.13	1.21
Heavily reinforced beams without the outliers, n = 84	1.04	0.11	0.45

The ratio of skewness and coefficient of variation indicates that a two-parameter lognormal distribution (having the skewness $w = 3v + v^3$) is an appropriate probabilistic model for θ which is in agreement with common assumptions [10].

Previous study [6] reported influence of d and ρ_l on the model uncertainty. A simple sensitivity analysis as proposed in [15] is conducted for the present database. Trends in θ with a basic variable are assessed using:

- The correlation coefficient ρ (correlation between θ and X_i), and
- The coefficient of determination R^2, a measure of the linear relationship between θ and X_i [17].

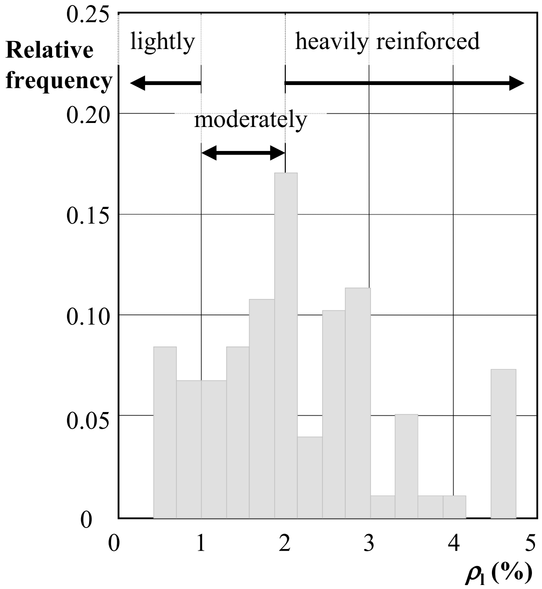
Results are provided in Tab. 1. A combination of strong ρ (say, $|\rho| > 0.5$) and strong R^2 (say, $R^2 > 0.5$) indicates a significant linear relationship between θ and X whereas strong correlation with relatively weak R^2 suggests a non-linear relationship. The results in Tab. 1 reveal weak to moderately weak correlations of all the shear parameters with θ; the most influential parameter is ρ_l ($\rho = 0.45$; $R^2 = 0.20$).

A multiple linear regression with all the shear parameters yields $R^2 = 0.49$ and somewhat improves the model of θ. However, the model uncertainty as a function of 10 variables is impractical. Therefore, the influence of the longitudinal reinforcement ratio on θ is considered hereafter only. Fig. 3 shows the histogram of ρ_l for the whole database; limits for lightly, moderately and heavily reinforced beams are accepted from [5]. It appears that the database contains a sufficient number of the test results for each amount of reinforcement. Sample sizes are n = 27, 63 and 86 for lightly, moderately and heavily reinforced beams, respectively.

Fig. 4 shows variation of the model uncertainty with ρ_l. The model uncertainty clearly increases with an increasing reinforcement ratio and its differentiation with respect to ρ_l is thus proposed. Sample characteristics of θ for the different levels of reinforcement are provided in Tab. 2. It follows that the mean depends on ρ_l while the coefficient of variation can be considered independent of ρ_l. The skewness for the three data groups significantly differs.

Statistical testing of outliers is conducted to exclude measurements obtained under significantly different conditions or affected by errors. For each data group Grubb's test at a significance level of 0.05 [17] is performed. Two outliers are identified and excluded from the data for the heavily reinforced beams (see Fig. 4). Sample characteristics of the model uncertainty derived from the whole database and from data available for heavily reinforced beams without the outliers are provided in Tab 2. The exclusion of the outliers leads to a remarkable reduction of the sample skewness while the means and coefficients of variation are affected insignificantly.

Note that the characteristics of the model uncertainty can be alternatively assessed using the procedure for statistical determination of resistance models given in Annex D of EN 1990 [18]. It is foreseen that application of this procedure would yield similar results as obtained in this study.



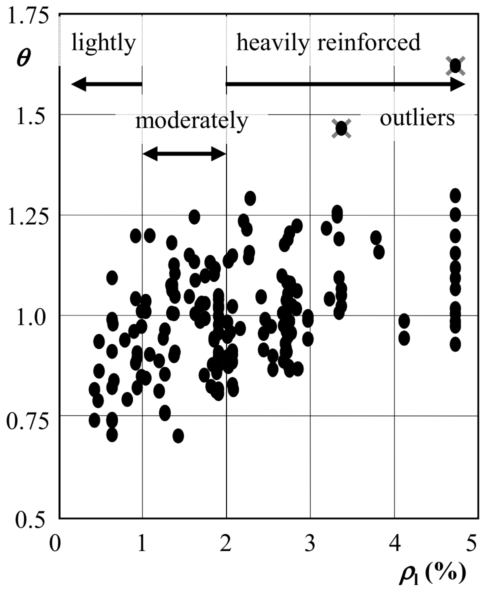


Fig. 3: Histogram of ρ_l for the whole database Fig. 4: Variation of θ with ρ_l for the whole database

4 MODEL UNCERTAINTY FACTOR FOR DETERMINISTIC RELIABILITY VERIFICATIONS

For deterministic reliability verifications EN 1990 [18] introduces the partial factor γ_{Rd} to describe the uncertainty associated with the resistance model ("design value of the model uncertainty"). Fig. 5 illustrates the relationship between the probabilistic distribution of θ and factor γ_{Rd}. As an example the lognormal distribution (mean $\mu_\theta = 1$ and coefficient of variation $V_\theta = 0.15$) and the relevant model uncertainty factor $\gamma_{Rd} = 1.20$ obtained from Eq. (6) (see text below) are shown.

The model uncertainty factor γ_{Rd} for reinforced concrete structures can be obtained as a product of [19]:

$$\gamma_{Rd} = \gamma_{Rd1}\ \gamma_{Rd2} \tag{5}$$

where:

γ_{Rd1} - denotes the partial factor accounting for model uncertainty,

γ_{Rd2} - partial factor accounting for geometrical uncertainties.

EN 1992-1-1 [7] provides no specific recommendations concerning model uncertainties. EN 1992-2 [20] introduces the global safety format for a nonlinear analysis with the recommended model uncertainty factor of 1.06. However, it has been shown [4] that such a factor is rather low and should be increased in most cases depending on relevant failure mode (bending, shear, compression).

$\gamma_{Rd1} = 1.05$ for concrete strength and $\gamma_{Rd1} = 1.025$ for reinforcement may be assumed in common cases [19]. However, larger model uncertainty needs to be considered for punching shear in the case when concrete crushing is governing. A value of $\gamma_{Rd2} = 1.05$ may be assumed for geometrical uncertainties of the concrete section size or reinforcement position. When relevant measurements of an existing structure indicate insignificant variability of geometrical properties, $\gamma_{Rd2} = 1.0$ may be considered.

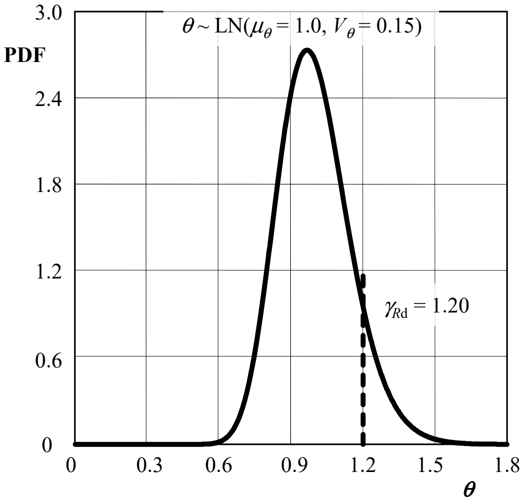


Fig. 5: PDF of θ and γ_{Rd}

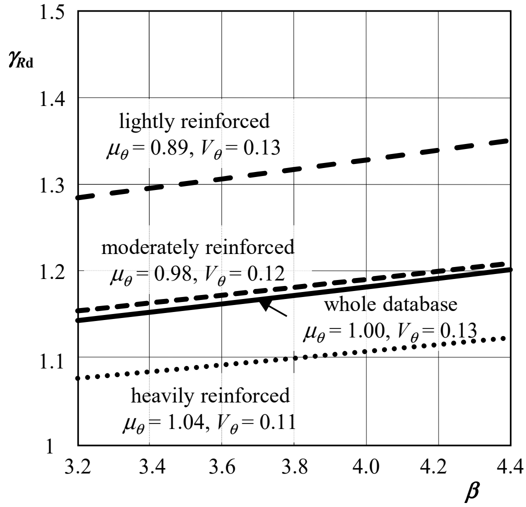


Fig. 6: Variation of γ_{Rd} with β for α_R = 0.32

Alternatively, the partial factor γ_{Rd} can be obtained from the following relationship based on a lognormal distribution:

$$\gamma_{Rd} = 1 / [\mu_\theta \exp(-\alpha_R \beta V_\theta)] \tag{6}$$

where:

α_R - denotes the FORM sensitivity factor

β - target reliability index according to EN 1990 [18].

Fig. 5 illustrates relationship between probability density function (PDF) of θ and the model uncertainty factor γ_{Rd}.

Considering the statistical characteristics of the model uncertainty given in Tab. 2, variation of the partial factor γ_{Rd} obtained from Eq. (6) with the target reliability β for $\alpha_R = 0.4 \times 0.8 = 0.32$ ("non-dominant resistance variable") is indicated in Fig. 6.

It follows from Fig. 6 that the model uncertainty factor γ_{Rd} increases with an increasing target reliability index β. For the considered range of β from 3.2 to 4.4 the model uncertainty varies approximately within the following intervals:

- 1.30-1.35 for lightly reinforced members ($\gamma_{Rd} \approx 1.3$ may be commonly accepted),
- 1.15-1.20 for moderately reinforced members ($\gamma_{Rd} \approx 1.2$ as a first approximation),
- 1.10-1.15 for heavily reinforced members ($\gamma_{Rd} \approx 1.1$ as a first approximation).

However, this differentiation will somewhat complicate applications of the partial factor method and its implementation into codes of practice needs to be carefully considered.

The selection of α_R = 0.32 deserves additional comments. Leading and accompanying actions (with associated factors α_E = -0.7 and $\alpha_E = -0.4 \times 0.7 = -0.28$, respectively) are distinguished in Annex C of EN 1990 [18] while α_R = 0.8 is recommended for resistance variables under conditions specified in the Eurocode. When the model uncertainty factor γ_{Rd} and material factor γ_m are assessed separately considering α_R = 0.8, overly conservative designs may be obtained. Therefore, CEB bulletin [21] and also present working materials of *fib* assume that the model uncertainty is not a leading resistance variable and the sensitivity factor is thus reduced to $\alpha_R = 0.4 \times 0.8 = 0.32$.

5 CONCLUDING REMARKS

Description of model uncertainties is a crucial problem in reliability verifications of reinforced concrete structures. The present study is focused on the model uncertainties in shear resistance of beams without shear reinforcement; the following concluding remarks are drawn:

- The model uncertainty should be always clearly associated with an assumed resistance model and related to specified ranges of basic variables.
- Longitudinal reinforcement ratio influences the mean of the model uncertainty and its differentiation for lightly, moderately and heavily reinforced beams is advisable.
- Uncertainties related to the section-oriented model provided in EN 1992-1-1 can be described by the lognormal distribution with a coefficient of variation of about 0.15 and the mean values of 0.9, 1.0 and 1.05 for beams with light, moderate and heavy longitudinal reinforcement, respectively.
- For reliability verifications based on the partial factor methods the model uncertainty factors of 1.3, 1.2 and 1.1 can be accepted for beams with light, moderate and heavy longitudinal reinforcement, respectively.

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