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UPDATING IN THE PROBABILISTIC ASSESSMENT OF EXISTING STRUCTURES

PRAVDĚPODOBNOSTNÍ AKTUALIZACE PŘI HODNOCENÍ EXISTUJÍCÍCH KONSTRUKCÍ

Abstract

Assessment of existing structures should be based on the knowledge about as-built conditions including uncertainties concerning geometry, material properties, loading and environmental conditions. A crucial step of the assessment may be the evaluation of prior information and newly obtained measurements for which Bayesian approach provides a consistent framework. Updating of probabilistic distributions of basic variables, direct updating of failure probability and combination thereof can be applied.

Keywords

Probabilistic updating, existing structures, failure probability.

Abstrakt

Při hodnocení existujících konstrukcí je potřebné uvážit nejistoty související s geometrickými a materiálovými vlastnostmi, zatíženími a vlivy okolního prostředí. Důležitým krokem hodnocení může být využití apriorních znalostí společně s nově získanými informacemi o konstrukci. Bayesovské postupy umožňují aktualizovat pravděpodobnostní rozdělení základních veličin nebo přímo pravděpodobnost poruchy.

Klíčová slova

Pravděpodobnostní aktualizace, existující konstrukce, pravděpodobnost poruchy.

1 INTRODUCTION

Reliability verifications of existing structures cover all aspects of assessing the condition of the structures by inspections, testing, monitoring, and calculations. The most important difference between the assessment of existing structures and the design of a new structure is in the amount of information available about the structure [1].

As a rule, vague prior information, often available in the assessment of existing structures, needs to be supplemented by experimental data and/or by other additional information such as a qualitative assessment on the basis of inspection. In accordance with ISO 13822 for the assessment of existing structures [2] probabilistic updating based on Bayesian methods provide a rational and consistent basis for the inclusion of the new information.

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The present paper is an updated version of a recent contribution [3]. It attempts to clarify the use of the probabilistic updating in the assessment of existing structures. An application of theoretical procedures is illustrated in the example of practical relevance.

2 PRINCIPLES OF UPDATING

When assessing existing structures various types of information may be available. Examples of such information are:

- Survival of a significant overloading,
- Material characteristics from different sources,
- Known geometry, damage and deterioration,
- Outcome of visual inspections,
- Capacity by proof loading,
- Static and dynamic response to controlled loading.

In the assessment of existing structures new information can be taken into account and combined with the prior probabilistic models by updating techniques. This results in the so-called posterior probabilistic models, which may be used for an enhanced assessment of the structure. Prior information is commonly based on experience from assessments of similar structures, long-term material production, findings reported in literature or engineering judgement.

When discussing updating techniques for structural reliability two types of quantitative information should be distinguished:

- Information of the equality type,
- Information of the inequality type.

The information of the equality type represents measured values of some basic or response variables. For example the crack width 3.2 mm has been measured at the stress equal to 200 MPa. Obviously, such measurements are seldom perfect and may suffer from some kind of errors. In a probabilistic evaluation procedure, measurement errors should be modelled as random variables, having means (zero for unbiased estimates), standard deviations and, if necessary some correlation pattern. The standard deviation is a property of the measurement technique, but may also depend on the actual conditions. An important but difficult part of the modelling is the degree of correlation between observations at different places and different points in time.

The information on the inequality type refers to observations when the observed variable is identified to be greater or less than a given limit. For example, a crack may be less than the observation threshold, a limit state is reached or not. Uncertainty in the threshold value should be taken into account. The distribution function for the minimum threshold level is often referred to as the Probability of Detection curve (POD curve). Also here, correlations for the probability of detection in various observations should be known.

Furthermore ISO 13822 [2] distinguishes between two fundamental types of the probabilistic updating (both types are discussed in the following sections):

- Updating of the (multivariate) probability distribution of basic variables,
- Direct updating of the structural failure probability.

A common approach is to update firstly distributions of basic variables (Section 3) and then to analyse reliability considering updated distributions and additional information (Section 4).

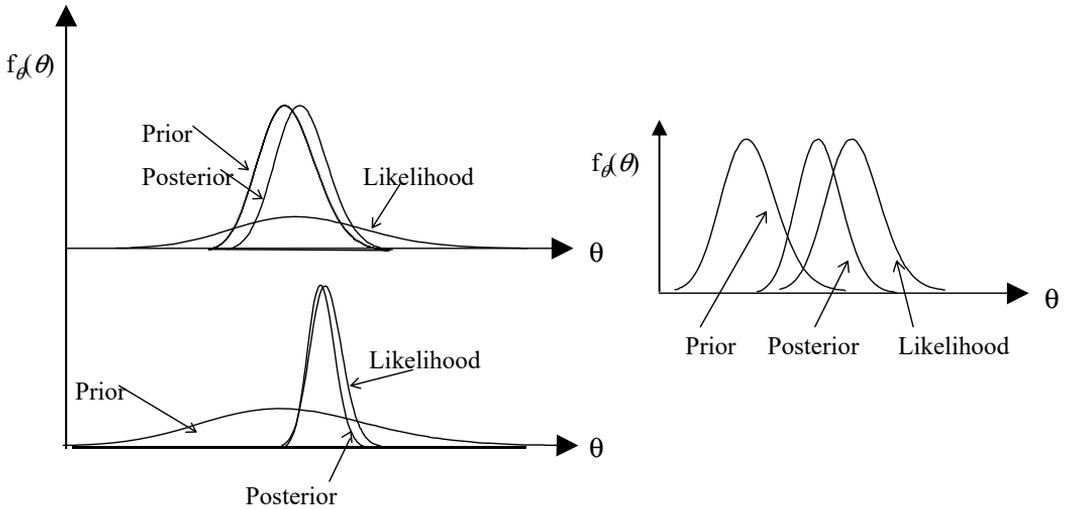


Fig. 1: Illustration of updating of probabilistic models

3 UPDATING OF THE PROBABILITY DISTRIBUTION OF A BASIC VARIABLE

3.1 Basis of Bayesian updating

Updating of the probability distribution of a basic variable is commonly based on Bayesian methods described briefly below. Two events A and B are further considered. The conditional probability $P(A|B)$ of the event A given the event B has occurred with a non-zero probability $P(B)$ is defined as:

$$P(A|B) = P(A \cap B) / P(B) \quad (1)$$

Considering the set of mutually exclusive events B_j , Bayes' rule is given as:

$$P(B_i | A) = \frac{P(B_i) P(A | B_i)}{\sum_j P(B_j) P(A | B_j)} \quad (2)$$

where:

$P(B_i|A)$ – is often referred to as the posterior probability of B_i ,

$P(A|B_i)$ – likelihood,

$P(B_i)$ – prior probability of the event B_i .

Fig. 1 shows corresponding prior and posterior probability density functions together with likelihood functions. In the first case the prior information is strong and the likelihood is weak (small sample size). In the second case the prior information is weak and the likelihood is strong. Finally in the last case the prior information and the likelihood are of comparable strength. It is seen from Fig. 1 that the modelling of both the prior probabilistic models and the likelihood is of utmost importance.

In the updating, characteristics Θ of a random variable X (e.g. mean, standard deviation, skewness, lower bound etc.) are considered as random variables. Prior distributions of these characteristics are then updated using n test results x_1, x_2, \dots, x_n . The variable has the prior probability density function $f^*(x|\Theta)$ dependent on the random parameters Θ and $\Pi^*(\Theta)$ is the prior joint probability density function of the parameters Θ . Note that the symbol ‘ denotes the prior characteristics, symbol “ the posterior characteristics and test results are indicated without the quotation marks. Relationship (2) can be recast as:

$$\Pi''(\theta|x_1, x_2, \dots, x_n) = \frac{\Pi'(\theta) \prod_{i=1}^n f'(x_i|\theta)}{\int_{\Theta} \Pi'(\xi) \prod_{j=1}^n f'(x_j|\xi) d\xi} = C(x_1, x_2, \dots, x_n) \Pi'(\theta) \prod_{i=1}^n f'(x_i|\theta) \quad (3)$$

where:

$\Pi''(\cdot)$ – is the posterior joint probability density function of Θ updated considering the test results,

C – normalizing constant.

Posterior probability density function of the random variable is obtained by integration:

$$f''(x|x_1, x_2, \dots, x_n) = \int_{\Theta} f'(x|\xi) \Pi''(\xi|x_1, x_2, \dots, x_n) d\xi \quad (4)$$

In [4,5] a number of closed form solutions of Equations (3) and (4) can be found for special types of probability distribution functions known as the natural conjugate distributions. These solutions are useful in updating of random variables and cover a number of distribution types important for reliability-based structural assessments. When no analytical solution is available FORM/SORM techniques can be used to assess the posterior distribution [6].

In civil engineering practice Bayesian updating is often directly based on relationships (1) to (4) [2]. Supplementary information can be found elsewhere [4,5,7]. Documents [7,8] assume an extension of relationship (2) described in the following.

3.2 Procedure in accordance with ISO 12491

The procedure accepted here is limited to a normal variable X for which the prior joint probability density function $\Pi'(\mu, \sigma)$ of μ and σ is given as:

$$\Pi'(\mu, \sigma) = C \sigma^{-(1+\nu'+\delta(n'))} \exp\left\{-\frac{1}{2\sigma^2} [\nu'(s')^2 + n'(\mu - m')^2]\right\} \quad (5)$$

where:

$\delta(n') = 0$ for $n' = 0$ and $\delta(n') = 1$ otherwise.

The prior parameters m', s', n', ν' are parameters asymptotically given as:

$$E(\mu) = m', E(\sigma) = s', V(\mu) = \frac{s'}{m'\sqrt{n'}}, V(\sigma) = \frac{1}{\sqrt{2\nu'}} \quad (6)$$

The parameters n' and ν' are independent and may be chosen arbitrarily (it does not hold that $\nu' = n' - 1$). In Equations (6) $E(\cdot)$ denotes the expectation and $V(\cdot)$ the coefficient of variation. Equations (6) can be used to estimate unknown parameters n' and ν' provided the values $V(\mu)$ and $V(\sigma)$ are estimated using experimental data or available experience.

The posterior distribution function $\Pi''(\mu, \sigma)$ of μ and σ is of the same type as the prior distribution function, but with parameters m'', n'', s'', ν'' , given as:

$$\begin{aligned} n'' &= n' + n; & \nu'' &= \nu' + \nu + \delta(n'') \\ m''n'' &= n'm' + nm; & \nu''(s'')^2 + n''(m'')^2 &= \nu'(s')^2 + n'(m')^2 + \nu s^2 + nm^2 \end{aligned} \quad (7)$$

where:

m – is the sample mean,

s – sample standard deviation,

n – size of the sample,

$\nu = n - 1$ number of degrees of freedom.

If no prior information is available, then $n' = v' = 0$ and the characteristics m'', n'', s'', v'' equal the sample characteristics m, n, s, v .

4 DIRECT UPDATING OF THE STRUCTURAL FAILURE PROBABILITY

The failure probability, related to the period from the assessment to the end of a working life t_D , can be obtained from a general probabilistic relationship:

$$p_f(t_D) = P\{\min Z[\mathbf{X}(\tau)] < 0 : 0 < \tau < t_D\} = P\{F(t_D)\} \quad (8)$$

where:

$Z(\cdot)$ – denotes the limit state function(s),

$\mathbf{X}(\cdot)$ – vector of basic variables including model uncertainties, resistance, permanent and variable actions,

$F(t_D)$ – failure in the interval $(0, t_D)$.

When additional new information I related to structural conditions is available, the failure probability may be updated according to ISO 13822 [2] as follows:

$$p_f''(t_D|I) = P\{F(t_D) \cap I\} / P(I) \quad (9)$$

If the information is of an inequality type (e.g. $Z(\cdot) \geq 0$ when no damage or failure has been observed after some loading) standard methods for a system reliability analysis can be used to evaluate the probability $P\{F(t_D) \cap I\}$ [9]. The procedure proposed in [10] can be applied in the case of the information of an equality type.

The information should be selected to maximise correlation between the events $\{F\}$ and $\{I\}$. Strong correlation improves the posterior estimate of failure probability while a weak correlation yields nearly the same estimates as based on Equation (8) [11]. With reference to Section 2 examples of such information are survival of a significant overloading, capacity by proof loading and static and dynamic response to controlled loading.

In the case of survived overloading the satisfactory past performance of a structure during a period t_A till the time of assessment may be included in the reliability analysis considering the conditional failure probability $p_f''(t_D|t_A)$. This is a probability that the structure will fail during t_D given that it has survived the period t_A . This probability can be estimated in several ways.

When the load to which the structure has been subjected during t_A is known with negligible uncertainties, the resistance or a joint distribution of time-invariant variables may be truncated (a lower bound is set to the value of load). Using the bounded distribution, the conditional (updated, posterior) probability $p_f''(t_D|t_A)$ can be estimated. This approach, similar to the updating for proof load testing [4], is illustrated elsewhere [12]. More generally, the updated failure probability can be determined using the following relationship:

$$p_f''(t_D|t_A) = \frac{P\{F(t_D) \cap \bar{F}(t_A)\}}{P\{\bar{F}(t_A)\}} = \frac{P\{F(t_D)\} - P\{F(t_D) \cap F(t_A)\}}{1 - P\{F(t_A)\}} \quad (10)$$

where:

\bar{F} – denotes a complementary event to the failure.

The updated probability can be determined by standard techniques for reliability analysis (FORM/SORM, importance sampling) as shown in a numerical example.

Finally it should be mentioned that individual random variables may also be updated by inspections of events involving the outcomes of several random variables. This should nevertheless be done with a care. It is important to recognise that all the random variables may contribute to a result of the inspection. For instance when a crack length in a selected cross-section of the structure is measured, this result is influenced by loads, material and geometrical properties. Consequently all these variables become correlated through the result of the measurement even if they have been independent before the inspection.

5 NUMERICAL EXAMPLE

Selected techniques of the updating are applied in the example of reliability assessment of a structural member of the building constructed in 1960s as a part of a textile mill. The building is to be used as an office building. An anticipated working life is $t_D = 50$ years. The reliability assessment is focused on a simply supported steel beam exposed to bending moment due to permanent and imposed loads. Axial and shear forces need not to be taken into account, the beam is laterally restrained. For the sake of clarity the reliability assessment is considerably simplified to illustrate general steps of the probabilistic verification rather than to describe specific details.

Initially reliability of the member is verified by the partial factor method. Characteristics of the resistance and permanent action are specified considering results of on-site surveys and original design documentation. During the previous use of the structure, degradation has resulted in a minor loss of the steel section. In the assessment the actual steel section characteristics are considered and no further degradation is expected during the remaining working life. Characteristic value of the imposed load is determined in accordance with EN 1991-1-1 [13].

Using the load combination rule (6.10) given in EN 1990 [14] and yield strength obtained from tests (see below), the deterministic verification reveals that reliability of the member is insufficient as the actual resistance is approximately by 40 % lower than required by Eurocodes.

5.1 Updating of the yield strength of steel

Six specimens have been taken from unloaded structural members to verify the yield strength of structural steel. The tests provided the following results in MPa; $\mathbf{x}^T = \{290.7; 287.5; 298.8; 302.3; 294.6; 297.2\}$. The measurement error is assumed negligible.

Using the method of moments [5] the point estimates of the sample characteristics are:

$$m = \sum_i x_i / n = 295.2 \text{ MPa}, s = \sqrt{[\sum_i (x_i - m)^2 / (n - 1)]} = 5.43 \text{ MPa}, \text{ and } V = s / m = 0.0184 \quad (11)$$

where:

$$n = 6 - \text{ sample size } (i = 1 \dots 6).$$

The sample coefficient of variation is unrealistically low. This is attributed to the fact that the tested steel likely originates from a single production batch and batch-to-batch variability of yield strength is not captured in the sample. Considering that and also due to the small sample size, statistical characteristics obtained from the sample are deemed not to be representative for the steel of the whole structure. That is why the test results are combined with available prior information to obtain more realistic model of the yield strength.

Extensive statistical evaluation of properties of structural steels [15] indicated that the yield strength of steel produced in 1960s can be described by a two-parameter lognormal distribution, with the mean $m' = 299$ MPa, standard deviation $s' = 28.3$ MPa and coefficient of variation $V' = 0.094$.

It is, however, emphasised that prior information should be applied with an uttermost caution. Materials produced by various manufacturers may have considerably different mechanical characteristics. When suitability of the prior information for an assessed material is doubtful it is advised to obtain more data by destructive or well validated non-destructive tests.

An auxiliary variable $Y = \ln|X|$ is further introduced (X denotes the yield strength here). The variable Y is normally distributed with the mean and standard deviation:

$$\begin{aligned} \text{prior information: } m_y &= \ln(m') - 0.5 \ln[1 + (V')^2] = 5.69; s_y &= \sqrt{\ln[1 + (V')^2]} = 0.0945 \\ \text{tests: } m_y &= 5.69; s_y &= 0.0184 \end{aligned} \quad (12)$$

Prior information on structural steel may be relatively strong and the corresponding hypothetical sample size is $n' \approx 50$ [7]. For concrete compressive strength ISO 2394 [16] indicates a prior number of degrees of freedom for the prior standard deviation $\nu' = 5$ while JCSS Probabilistic Model Code [7] suggests $\nu' = 10$. Conservatively $\nu' = 5$ is accepted in this study for structural steel.

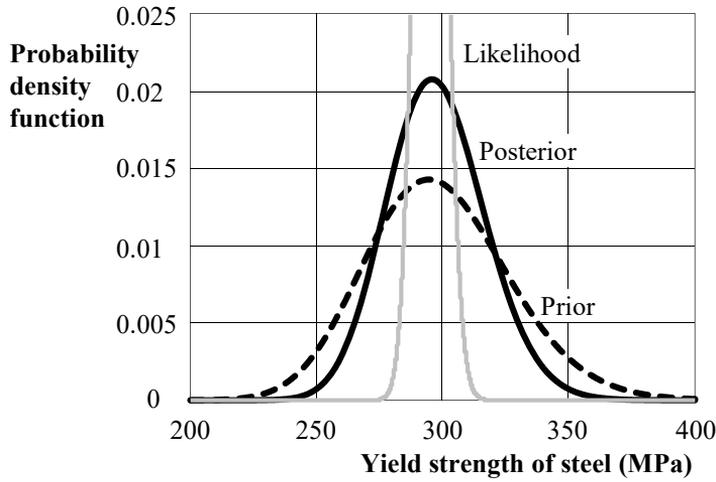


Fig. 2: Prior and posterior probability density functions with the likelihood

The characteristics of Y are assumed to have the prior distribution function in accordance with Equation (5). The posterior distribution function $\Pi''(\cdot)$ has the updated parameters m'' , s'' , n'' and ν'' given in Equation (7):

$$\begin{aligned} n'' &= n + n' = 56; & \nu'' &= \nu + \nu' + \delta(n') = 10; & m_y'' &= (m_y n + m_y' n') / n'' = 5.69; \\ s_y''^2 &= (\nu s_y^2 + \nu' s_y'^2 + n m_y^2 + n' m_y'^2 - n'' m_y''^2) / \nu'' = 0.0042 \end{aligned} \quad (13)$$

The updated characteristics of the yield strength are obtained as follows:

$$V'' = \sqrt{[\exp(s_y''^2) - 1]} = 0.065; \quad m'' = \exp[m_y'' + 0.5 \ln(1 + V''^2)] = 297.6 \text{ MPa} \quad (14)$$

The prior and posterior probability density functions with the likelihood are plotted in Fig. 2. It follows that the posterior distribution has a lower variance than the prior distribution.

The characteristic value can be obtained in accordance with EN 1990 [14]:

$$x_k = \exp[m_y'' + q_t(0.05, \nu'') \sqrt{(1 + 1/n'') s_y''^2}] \approx 264 \text{ MPa} \quad (15)$$

where:

q_t - denotes the p -fractile of t -distribution for a given number of degrees of freedom.

5.2 Probabilistic reliability analysis

Probabilistic reliability analysis is based on the limit state function $Z(\cdot)$ for the member exposed to bending (notation and probabilistic models of the basic variables \mathbf{X} given in Tab. 1):

$$Z(\mathbf{X}, t_D) = K_{RR} - K_E [G + Q_D] \quad (16)$$

The probabilistic models are based on recommendations of JCSS Probabilistic Model Code [7] and additional findings published elsewhere [17]. For convenience all the basic variables in Tab. 1 are normalised by $L^2 / 8$ (L is a span of the member).

It is noted that the accepted mean value of the model uncertainty for a flexural resistance $\mu_{KR} = 1.0$ differs from the mean value reported in [18] where $\mu_{KR} \approx 1.15$ is obtained by the statistical evaluation of test results. For assumptions made in the design of new structures, actual resistance is positively influenced by tolerance specifications in dimensions of a rolled sections and the mean of the model uncertainty increases. However, the assessment of the existing beam is based on actual dimensions and this positive effect vanishes.

Using the FORM method, the reliability verification is firstly based on Equation (8) (updated resistance model only). The reliability index $\beta \approx 1.3$ is too low compared to the target reliability level $\beta_t = 3.1$ indicated in ISO 2394 [16] for moderate consequences of failure and moderate costs of safety measures.

Tab. 1: Models for basic variables

Variable	Sym.	Unit	Distribution	x_k	μ_X / x_k	V_X
Flexural resistance (updated)	R	kN/m	Lognormal	4.32	1.13	0.065
Permanent load effect	G	kN/m	Normal	2.0	1	0.05
Imposed load effect (50-y. maxima)	Q_{ID}	kN/m	Gumbel	3.0	0.6	0.35
Effect of the survived load	S	kN/m	Normal	3.3	1	0.05
Resistance uncertainty	K_R	-	Lognormal	1	1	0.05
Load effect uncertainty	K_E	-	Lognormal	1	1	0.1

x_k = characteristic value; μ_X = mean; V_X = coefficient of variation.

Secondly, the reliability is updated considering the satisfactory past performance to improve this estimate. It is known from previous performance of the structure that the member has survived the load S equal to 1.1-times the characteristic value of the imposed load. Uncertainties in the survived load effect are described by the normal distribution with the mean equal to the observed value and coefficient of variation 0.05. Given the survival of the load S , the updated reliability index $\beta'(t_D|S) \approx 2.7$ follows from the conditional failure probability based on Equation (9):

$$p_f^{\rightarrow}(t_D|S) = P\{[K_R R - K_E(G + Q_{ID}) < 0] \cap [K_R R - K_E(G + S) > 0]\} / P\{K_R R - K_E(G + S) > 0\} \quad (17)$$

Note that the present conditions of the beam are assumed to be the same as those at the time of exposure to the load S . It is emphasised that information on previous loads should be always considered carefully and related to relevant uncertainty.

The predicted reliability, $\beta'(t_D|S) \approx 2.7$, is still rather low. In general five options can now be considered:

1. To improve information on variables significantly affecting structural reliability by inspections or tests.
2. To upgrade the member,
3. To propose an adequate limit on the imposed action,
4. To accept a shorter remaining working (such as 10 years) and after that re-assess the beam,
5. To derive optimum target reliability following the principles provided by ISO 2394 [16].

Note that the third option may be applicable for industrial structures rather than office buildings. Frequently limits on vehicle weight are applied on road bridges. When the fourth option is accepted the updated reliability index $\beta'(10 \text{ y.}|S) \approx 3.15$ is obtained from Equation (17) using 10-year maxima of the imposed load. The fifth option is thoroughly discussed in [19,20] where optimisation of the total costs related to a structure including potential failure consequences and human safety criteria are considered.

5.3 Parametric study

To generalise findings of the probabilistic analysis, a parametric study is conducted for the load ratio given as the fraction of the characteristic variable action over the total characteristic load:

$$\chi = Q_k / (G_k + Q_k) \quad (18)$$

The load ratio χ may vary within the interval from nearly 0 (underground structures, foundations) up to nearly 1 (local effects on bridges, crane girders). For steel members in buildings the load ratio is expected within the range from 0.4 up to 0.8.

For a given load ratio and characteristic imposed load, the characteristic permanent load is obtained from Equation (18). Characteristic resistance is then derived from the load combination rule (6.10) and multiplied by the coefficient ξ which represents the ratio between the actual characteristic resistance of an existing structural member and characteristic resistance according to the Eurocodes. To be consistent with the previous analysis $\xi = 0.6$ is considered (i.e. actual resistance is by 40 % less than that required by Eurocodes).

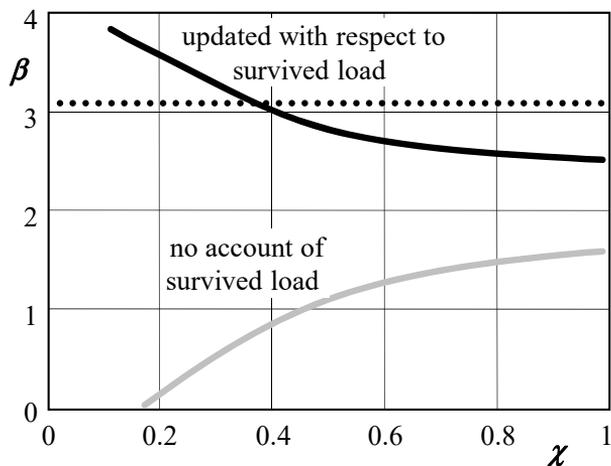


Fig. 3: Variation of β with χ for $\xi = 0.6$

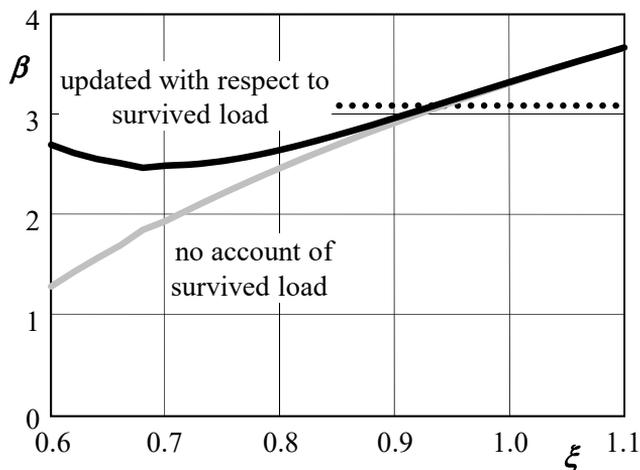


Fig. 4: Variation of β with ξ for $\chi = 0.6$

Variation of the reliability index with the load ratio is shown in Fig. 3 for $\xi = 0.6$. Fig. 3 demonstrates that effect of the updating increases with a decreasing load ratio when variable actions (not updated variables) become less significant. Variation of the reliability index with the ratio ξ is shown in Fig. 4 for $\chi = 0.6$ (the middle value of the expected range). It appears that the updating improves reliability estimates only for low ratios ξ (i.e. for low reliability levels, say $\beta < 2.0$). Moreover, it has been shown that the updating with respect to a survived load is less efficient when the intensity of this load decreases below the characteristic variable load [21].

6 CONCLUDING REMARKS

Reliability verifications of existing structures should be backed up by inspection including collection of appropriate data. Assessments based on simplified conservative procedures used for structural design may lead to expensive repairs and waste of resources.

Probabilistic methods can thus be applied to better describe uncertainties and take into account results of inspections and tests as well as satisfactory past performance by an updating. Two

fundamental types of the probabilistic updating include updating of probability distributions of basic variables and direct updating of the structural failure probability.

Numerical example reveals that:

- It may be misleading to develop a model for material property from a limited number of tests and consideration of prior information is advisable. On the contrary the prior information should be used cautiously since erroneous results may be obtained when the prior information is obtained from non-homogeneous sample having different material properties.
- The effect of the updating considering a survived load increases when variable actions become less significant. The updating then improves reliability estimates particularly for low reliability levels ($\beta < 2.0$).

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REFERENCES

- [1] FABER, M.H. Reliability based assessment of existing structures. *Progress in Structural Engineering and Materials*. 2000, Vol. 2, Nr. 2, pp. 247-253. ISSN 1528-2716.
- [2] ISO 13822. *Bases for design of structures - Assessment of existing structures*, 2003. 35 pp.
- [3] HOLICKÝ, M., SÝKORA, M. & JUNG, K. Reliability updating in the assessment of existing structures. *In Proc. Modelování v mechanice 2012*. Ostrava : VŠB-Technical University of Ostrava, Faculty of Civil Engineering, 2012, pp. 11. ISBN 978-80-248-2694-3.
- [4] DIAMANTIDIS, D. *Probabilistic Assessment of Existing Structures*. Joint Committee on Structural Safety, RILEM Publications S.A.R.L., 2001.
- [5] ANG, A.H.S. & TANG, W.H. *Probabilistic Concepts in Engineering Emphasis on Applications to Civil and Environmental Engineering*. USA : John Wiley & Sons, 2007. ISBN 978-0-471-72064-5.
- [6] DITLEVSEN, O. & MADSEN, H.O. *Structural Reliability Methods*. Chichester (England) : John Wiley & Sons, 1996. 372 pp. ISBN 0471960861.
- [7] JCSS. *JCSS Probabilistic Model Code*. Zurich : Joint Committee on Structural Safety, 2001.
- [8] ISO 12491. *Statistical methods for quality control of building materials and components*, 1997. 30 pp.
- [9] MELCHERS, R.E. *Structural Reliability Analysis and Prediction*. Chichester, England : John Wiley & Sons Ltd., 2001. 437 pp. ISBN 0 471 98324 1.
- [10] STRAUB, D. Reliability updating with equality information. *Prob.Eng.Mech.* 2011, Vol. 26, Nr. 2, pp. 254-258. ISSN 0266-8920.
- [11] ELLINGWOOD, B.R. Reliability-based condition assessment and LRFD for existing structures. *Struct.Saf.* 1996, Vol. 18, Nr. 2-3, pp. 67-80. ISSN 0167-4730.
- [12] SÝKORA, M., HOLICKÝ, M., JUNG, K., MARKOVÁ, J., KVAAL, K. & THUIS, T. Reliability Assessment of Industrial Heritage Structures and Application to a Light-Weight Steel Roof. *In Proc. ICSA2010*. Leiden : CRC Press/Balkema, 2010, pp. 605-612. ISBN 978-0-415-49249-2.
- [13] EN 1991-1-1. *Eurocode 1: Actions on structures - Part 1-1: General actions; Densities, self-weight, imposed loads for buildings*, 2002. 43 pp.
- [14] EN 1990. *Eurocode - Basis of structural design*. Brussels : CEN, 2002. 87 pp.

- [15] MRÁZIK, A. *Teoria spolahlivosti ocelovych konstrukcii (in Slovak - Reliability theory of steel structures)*. Bratislava : VEDA, 1987. 360 pp.
- [16] ISO 2394. *General principles on reliability for structures*, 1998. 73 pp.
- [17] HOLICKÝ, M. & SÝKORA, M. Conventional probabilistic models for calibration of codes. *In Proc. ICASP11*. Leiden : CRC Press, 2011, pp. 969-976. ISBN 978-0-415-66986-3.
- [18] EUROCODE 3 EDITORIAL GROUP. *Background Documentations to Eurocode No. 3 Design of Steel Structures Part 1 – General Rules and Rules for Buildings, Background Document for Chapter 5 of Eurocode 3, Document 5.01*. 1989.
- [19] STEENBERGEN, R.D.J.M. & VROUWENVELDER, A.C.W.M. Safety philosophy for existing structures and partial factors for traffic loads on bridges. *Heron*. 2010, Vol. 55, Nr. 2, pp. 123-139.
- [20] SYKORA, M. & HOLICKY, M. Target reliability levels for the assessment of existing structures - case study. *In Proc. IALCCE 2012*. Leiden : CRC Press/Balkema, 2012, pp. 813-820. ISBN 978-0-415-62126-7.
- [21] FABER, M.H., VAL, D.V. & STEWART, M.G. Proof load testing for bridge assessment and upgrading. *Engineering Structures*. 2000, Vol. 22, Nr. 12, pp. 1677-1689.

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