

Ivan BALÁZS¹, Jindřich MELCHER²**GEOMETRICALLY NONLINEAR NUMERICAL ANALYSIS OF BEAMS
OF MONOSYMMETRIC THIN-WALLED CROSS-SECTIONS LOADED PERPENDICULARLY
TO THE PLANE OF SYMMETRY****Abstract**

The paper deals with geometrically nonlinear analysis of steel beams of monosymmetric thin-walled cross-sections loaded perpendicularly to the plane of symmetry eccentrically to the cross-section shear centre. Numerical analysis of selected transversely loaded beams is performed using Dlubal RFEM code based on finite element method. Accuracy of interaction formula for bending with lateral torsional buckling and torsion is studied.

Keywords

Analysis, beam, bending, FEM, numerical model, stability, steel, stress, torsion.

1 INTRODUCTION

Steel beams of monosymmetric thin-walled cross-sections (e. g. channel sections) have been widely used in civil engineering as members of structures (purlins, wall substructure beams). They are usually loaded by normal force, transverse load or theirs combination. Shear centre of such cross-sections is situated outside of the section and is not normally intersected by transverse load vector. It results in torsion of the section. Due to slenderness of the section these beams are prone to stability problems. This paper focuses on beams of channel sections loaded by transverse load perpendicularly to the plane of symmetry eccentrically to the cross-section shear centre causing bending of the beam with possible lateral torsional buckling and torsion.

Theoretical solution of stability problem (lateral torsional buckling) of an ideal beam of arbitrary cross-section loaded by transverse load was derived in [1]. In Czechoslovak literature, this solution was extended for practical assessment of actual beam of at least monosymmetric cross-section loaded in the plane of symmetry in [2]. In [3] formulae for critical moment of beam under transverse load intersecting cross-section shear centre were listed. Formulae for lateral torsional buckling in currently valid standard for design of steel structures [4] apply only for cases when transverse load vector passes through the cross-section shear centre and does not cause additional torsion. For other cases, application of numerical procedures is recommended. There is no interaction formula for bending with lateral torsional buckling and torsion. Behavior of real (imperfect) beam is taken into account by lateral torsional buckling curves with appropriate amplitudes of initial curvature.

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2 NUMERICAL ANALYSIS

2.1 Investigated cases

Steel thin-walled hot-rolled channel section UPE 300 is selected for numerical analysis using Dlubal RFEM code based on finite element method. The steel grade is S355 (yield strength $f_y = 355$ MPa). Parametric study of different beam spans L (from 1 m to 5 m with a step of 1 m) is performed. A simple beam is considered (in bending as well as in torsion).

Three cases of transverse load are considered: concentrated force F at midspan (case A), partial uniformly distributed load $q_{\text{part.}}$ applied on a part of the beam span from the distance $x = L/4$ to $x = 3L/4$ from the support (case B) and uniformly distributed load applied along the entire span of the beam (case C). All the investigated cases are displayed in Fig. 1.

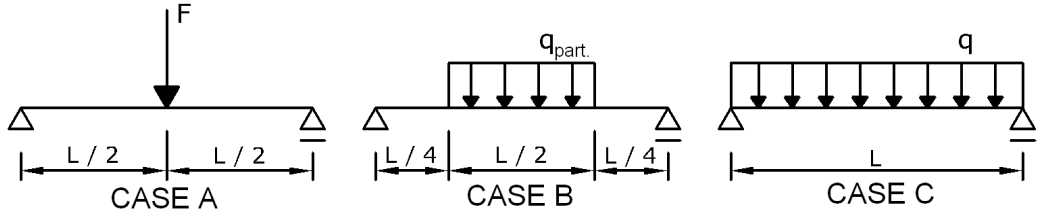


Fig. 1: Investigated load cases

All the transverse loads are applied in vertical direction and their eccentricities e to cross-section shear centre are in all cases equal to 100 mm. Under this condition the loads are applied on the top flange of the beam as it is usual in practice. The cross-section and coordinate system is displayed in Fig. 2. The x -axis is longitudinal axis of the beam. Parametric study of different spans will be performed for all the load cases listed above.

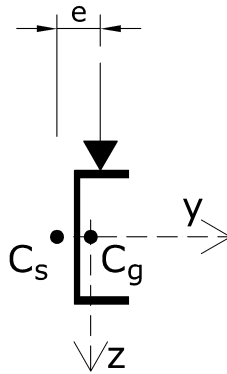


Fig. 2: Cross-section and coordinate system

2.2 Numerical models description

Beams of considered cross-section are numerically analyzed by means of finite element method (FEM) using the RFEM code. Spatial finite elements are utilized. The code allows setting the required size of elements. It is set to 10 mm in all cases. The finite element network is generated automatically by the code. It results in regular network of elements without any sharp angles between elements edges. Sharp angles might have a negative influence on accuracy of results [5]. Dimensions of modeled channel section correspond to considered section UPE 300 and are depicted in Fig. 3.

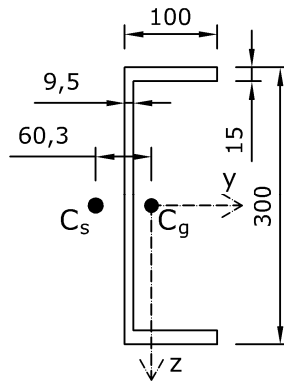


Fig. 3: Dimensions of considered beam cross-section

Simple beam in bending as well as in torsion is considered. It is the so called fork support condition that enables warping to develop while free transverse displacement of the section at supports is prevented. According to the background document [6] of the currently valid standard for design of steel structures [4], it should be considered when dealing with lateral torsional buckling of a beam. An illustrative picture of the fork support condition can be seen in Fig. 4.

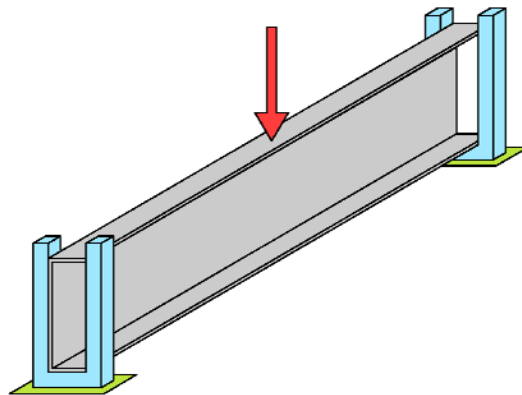


Fig. 4: Fork support condition

In Fig. 5 there is the investigated beam modeled in the RFEM code. Implementation of fork support condition can be seen there as well.

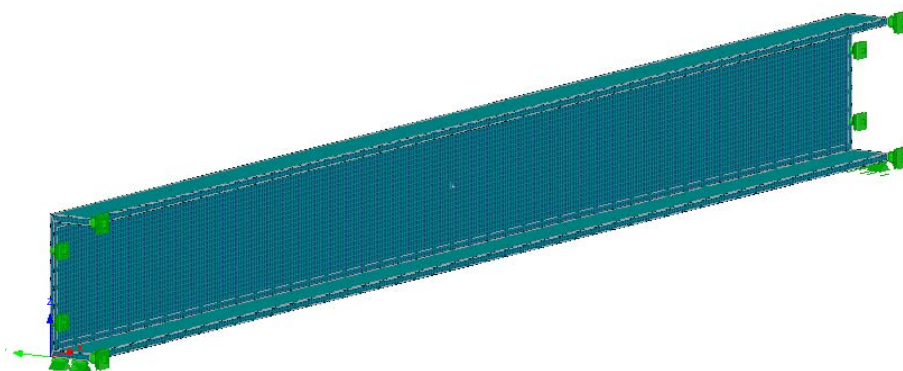


Fig. 5: Model of the beam

Three load cases are investigated. In the case A, a concentrated load is applied at midspan on the top flange of the beam eccentrically to the shear centre. In order to avoid singularities in the numerical analysis, the concentrated load (kN) is converted to equivalent areal circular load (kN/m²). The radius is equal to 10 mm. Since the point of action of a concentrated load is a singularity, the results might not be trustworthy [7]. For the same reason, the uniformly distributed loads (cases B, C) were converted to areal loads of small breadth.

In general, sharp edges of members modeled using spatial finite elements might be also regarded as singularities [7]. In order to quantify their influence on accuracy of results, a model of beam with curved transition from the web to the flange (radius 10 mm) was created. The difference between results was insignificant (about 0,7 MPa in case of σ_x normal stress).

2.3 Numerical analysis process

The numerical analysis comprises several steps. First, the beam geometry, load and supports were modeled and material was assigned. Then, stability analysis using RF-STABILITY plug-in of the RFEM code was performed to obtain the buckling eigenmodes. First eigenmode (see Fig. 6) was utilized as initial curvature of the beam with the amplitude of $e_0 = L / 150$. It conforms to lateral torsional buckling curve d. According to [4], this curve is prescribed for channel sections.

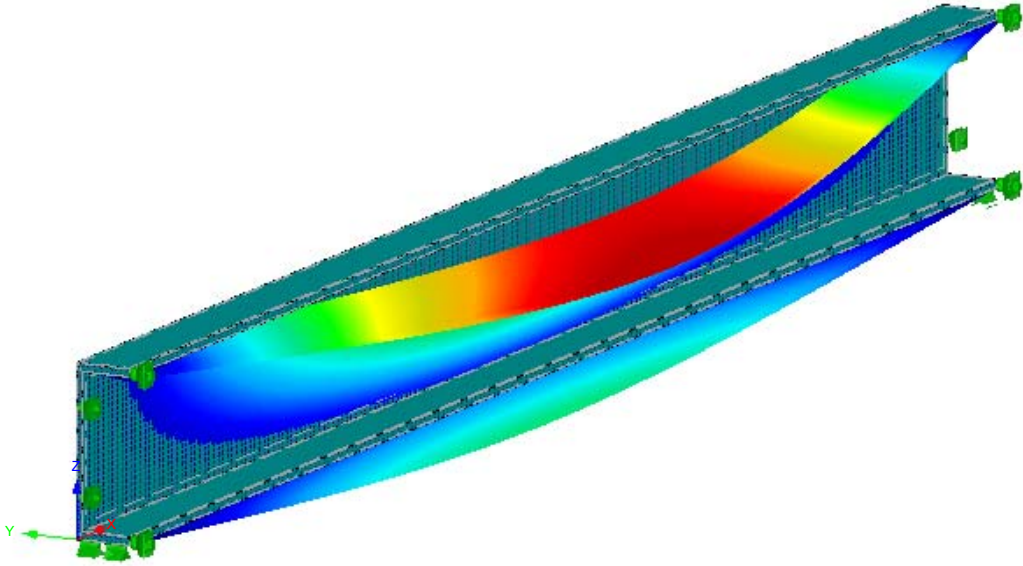


Fig. 6: First eigenmode (stability analysis result)

The geometry of the beam was then modified according to this initial curvature using RF-IMP plug-in of the RFEM code. By initial curvature implementation, transition from ideal to “real” beam (with imperfections) was completed and geometrically nonlinear analysis according to 2nd order theory (GNIA analysis) was ready to be performed. Normal stress σ_x can be then checked according to formula (1), without lateral torsional buckling reduction factor [8]:

$$\sigma_x \leq f_y. \quad (1)$$

In general, amplitude of initial imperfection depends also on residual stresses of the section. There is no data regarding residual stresses of channel sections in literature [9]. In numerical modeling it might be taken into account e. g. by direct implementation of estimated residual stress pattern across the section (based on more common steel beams of double symmetric cross-section) or indirectly by using less favorable lateral torsional buckling curve and thereby greater amplitude of initial curvature. In [9] the amplitude is taken as $e_0 = L / 150$ which conforms to the lateral torsional

buckling curve d. It is also recommended in [4]. In Fig. 7 there is top flange of the imperfect beam. The figure comes from the RFEM code graphical interface.

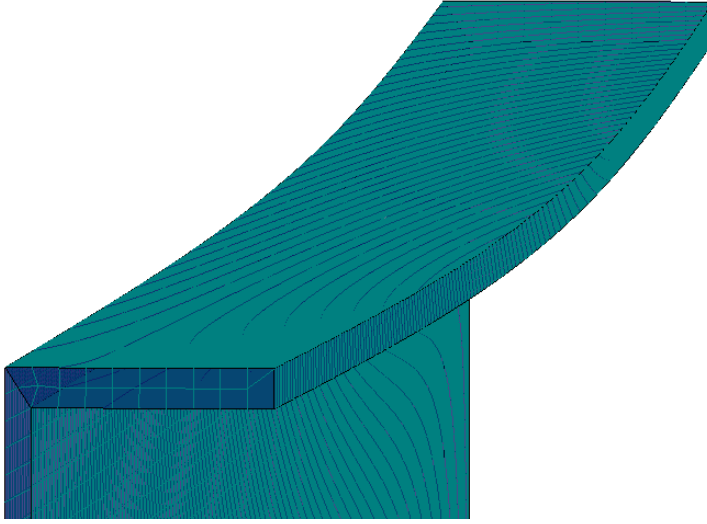


Fig. 7: Initial curvature of the beam

2.4 Numerical analysis results

The goal of the numerical analysis of beams of various spans and under different load cases was to find such load magnitude F_{\max} (case A), $q_{\text{part},\max}$ (case B) or q_{\max} (case C), respectively, which induces the normal stress at midspan to reach the yield strength. Maximum value of bending moment M as well as of bimoment B is expected at midspan. Potentially increased values of stresses at supports modeled as lines were not taken into account [7]. In Table 1 there are results of considered load cases and spans. Magnitude of normal stress was read at point of cross-section depicted in Fig. 8. This figure schematically displays expected normal stress σ_x diagrams caused by bending and torsion and their superposition.

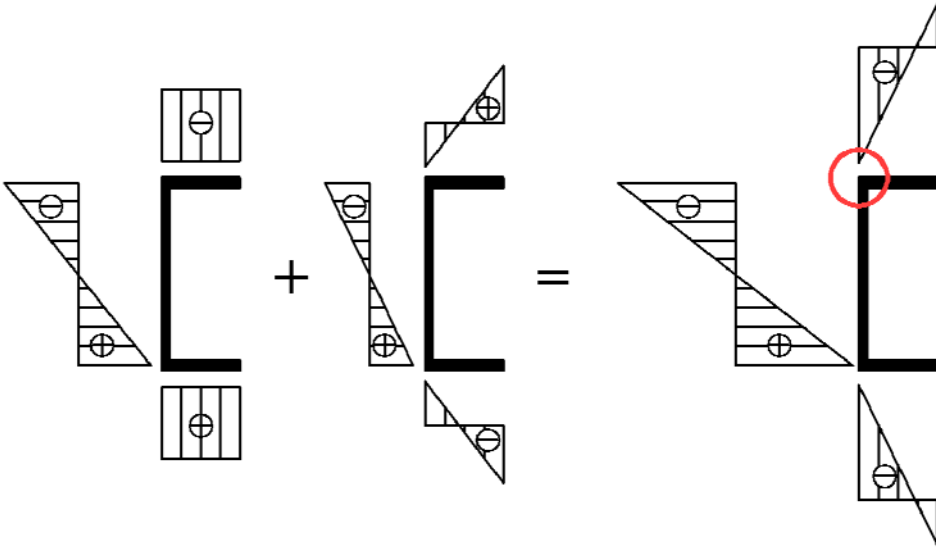


Fig. 8: Stress diagrams and their superposition

Tab. 1: Numerical analysis results

Span	Load cases		
	A	B	C
L (m)	F_{\max} (kN)	$q_{\text{part.,max}}$ (kN/m)	q_{\max} (kN/m)
1	210,0	470,0	340,0
2	101,0	122,0	92,0
3	64,5	53,5	42,0
4	52,5	30,5	24,0
5	49,0	25,5	18,5

Following Fig. 9 displays normal stress σ_x diagram as graphical output of numerical analysis of load case B for normal stress at yield strength. The blue color indicates compression stress. In Fig. 10 there is overall deformation of the beam.

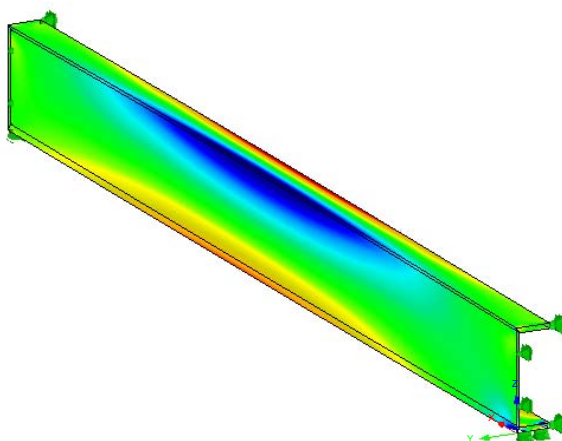


Fig. 9: Normal stress ($L = 3$ m, partial uniformly distributed load)

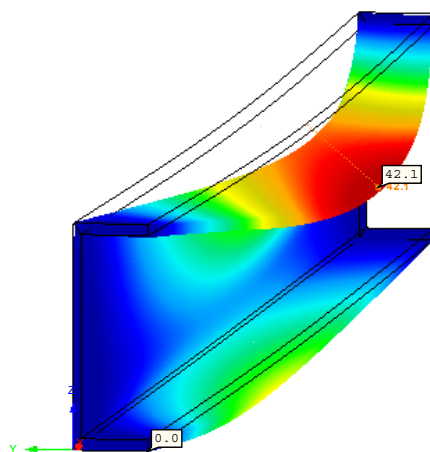


Fig. 10: Deformation ($L = 3$ m, partial uniformly distributed load)

3 ANALYTICAL SOLUTION

For the purpose of evaluation of numerical analysis results a comparative analytical calculation was done. First, general formulae for internal forces caused by bending (bending moment M) and by torsion (bimoment B) are derived.

3.1 Concentrated force F at midspan (case A)

Bending moment at midspan ($x = L / 2$) caused by vertical concentrated force F is given as follows:

$$M = \frac{FL}{4}. \quad (2)$$

General formula for bimoment caused by concentrated force at midspan is given as follows [10]:

$$B = \frac{T \sinh kx}{2k \cosh \frac{kL}{2}} \quad (3)$$

and after modification for section at midspan ($x = L / 2$):

$$B = \frac{T}{2k} \operatorname{tgh} \frac{kL}{2} \quad (4)$$

where

$$T = Fe, \quad (5)$$

$$k = \sqrt{\frac{GI_t}{EI_\omega}}. \quad (6)$$

In above listed formulae:

F – concentrated force [N],

L – beam span [L],

T – torsional moment [Nm],

e – eccentricity to the shear centre [m],

G – shear modulus of steel [Pa],

E – Young's modulus of steel [Pa],

I_t – St. Venant torsional constant [m⁴] and

I_ω – warping constant [m⁶].

3.2 Partial uniformly distributed load q_{part} (case B)

Bending moment at midspan caused by partial uniformly distributed load applied between sections $x = L / 4$ and $x = 3L / 4$ is given as follows:

$$M = \frac{3q_{\text{part}}L^2}{32}. \quad (7)$$

For derivation of formula for bimoment at midspan the so called initial parameters method for solution of differential equation of warping torsion [11] was utilized. For simple beam following boundary conditions apply: angle of torsion at support ν ($x = 0$) is equal to zero (kinematic boundary condition) and bimoment at support B ($x = 0$) is equal to zero (static boundary condition). The beam is thereby free to warp. It results in formula (8):

$$B = \frac{mL^2}{k} \left\{ \frac{1}{2 \cosh \frac{k}{2}} \left[\frac{\sinh k}{4} + \frac{1}{k} \left(\cosh \frac{3k}{4} - \cosh \frac{k}{4} \right) \right] - \frac{\sinh \frac{k}{2}}{4} - \frac{\cosh \frac{k}{4} - 1}{k} \right\} \quad (8)$$

where

$$m = q_{\text{part}} e \quad (9)$$

and k is given as formula (6).

3.3 Uniformly distributed load applied along the entire span q (case C)

For bending moment at midspan determination formula (10) applies:

$$M = \frac{1}{8} q L^2. \quad (10)$$

Bimoment at midspan (general formula for full uniformly distributed load on eccentricity e) is given as follows [10]:

$$B = \frac{m}{k^2} \left[1 - \frac{\cosh k \left(\frac{L}{2} - x \right)}{\cosh \frac{kL}{2}} \right]. \quad (11)$$

After modification for the section at midspan ($x = L / 2$) formula (12) applies:

$$B = \frac{m}{k^2} \left[1 - \frac{1}{\cosh \frac{kL}{2}} \right] \quad (12)$$

where

$$m = qe \quad (13)$$

3.4 Normal stress calculation

Normal stress σ_M caused by bending moment and σ_ω caused by torsion were in all investigated load cases determined using general formulae (14) and (15):

$$\sigma_M = \frac{M}{W_y}, \quad (14)$$

$$\sigma_\omega = \frac{B}{I_\omega} \omega \quad (15)$$

where

M – bending moment at midspan [Nm],

W_y – cross-section modulus [m³],

B – bimoment at midspan [Nm²],

I_ω – warping constant [m⁶] and

ω – normalised warping function at investigated point of cross-section [m²].

3.5 Interaction bending with lateral torsional buckling – torsion

According to the standard [4], verification of a member against lateral torsional buckling should be taken as follows:

$$M_{Ed} \leq M_{b,Rd} \cdot \quad (16)$$

It can be modified to a form with normal stress as follows:

$$\frac{\sigma_M}{\chi_{LT}} \leq f_y, \quad (17)$$

where χ_{LT} indicates reduction factor for lateral torsional buckling determined using routine procedure according to [4].

Having modified the formula (16) to the form with normal stress σ_M (17), adding of warping normal stress σ_ω is possible. A simple linear interaction formula (18) is thereby obtained:

$$\frac{\sigma_M}{\chi_{LT}} + \sigma_\omega \leq f_y. \quad (18)$$

The interaction formula (18) is used to find such load magnitude when the sum of normal stresses caused by bending σ_M and warping normal stress σ_ω gives the total normal stress equal to steel yield strength at investigated section. This process ends when equality between left and right part of the equation (18) is found:

$$\frac{\sigma_M}{\chi_{LT}} + \sigma_\omega = f_y. \quad (19)$$

Results of these calculations are summarized in Table 2. In Table 3 there are some relevant partial results of calculations according to [4]: elastic critical moment for lateral torsional buckling M_{cr} values, non dimensional slenderness for lateral torsional buckling $\bar{\lambda}_{LT}$ and reduction factors for lateral torsional buckling χ_{LT} for lateral torsional buckling curve d. The magnitude of elastic critical moment M_{cr} depends on bending moment diagram which is taken into account by coefficients C_1 , C_2 and C_3 . Bending moment diagrams of load cases B and C are very similar and that is why the same coefficients for cases B and C were utilized.

Tab. 2: Analytical calculation results – solution of equation (19)

Span	Load cases		
	A	B	C
L (m)	F_{max} (kN)	$q_{part.,max}$ (kN/m)	q_{max} (kN/m)
1	165,9	449,0	338,9
2	91,5	101,5	97,3
3	64,7	40,9	46,9
4	49,1	21,2	26,6
5	38,5	12,7	16,5

Tab. 3: Some relevant results of calculation according to [4]

Span	Load cases								
	A			B			C		
L (m)	M_{cr}	$\lambda_{LT} (-)$	$\chi_{LT} (-)$	M_{cr}	$\lambda_{LT} (-)$	$\chi_{LT} (-)$	M_{cr}	$\lambda_{LT} (-)$	$\chi_{LT} (-)$
1	1028,6	0,46	0,81	937,0	0,48	0,79	937,0	0,48	0,79
2	338,0	0,80	0,58	304,3	0,85	0,55	304,3	0,85	0,55
3	201,8	1,04	0,45	179,4	1,10	0,42	179,4	1,10	0,42
4	148,0	1,21	0,37	130,2	1,29	0,34	130,2	1,29	0,34
5	118,9	1,35	0,32	103,7	1,45	0,29	103,7	1,45	0,29

4 COMPARISON OF RESULTS

Charts in Fig. 11, Fig. 12 and Fig. 13 display comparison of numerically and analytically obtained load magnitudes resulting in normal stress equal to yield strength at midspan.

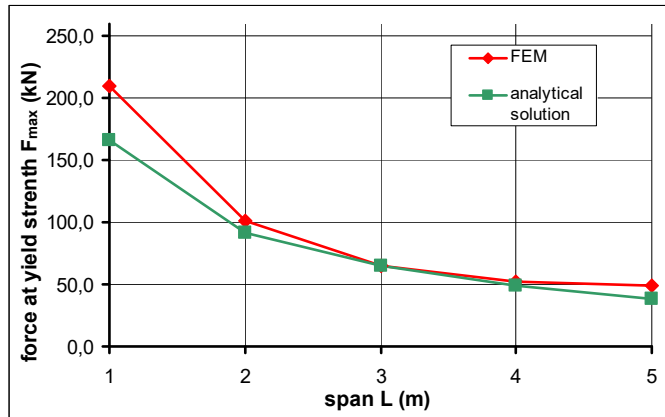


Fig. 11: Comparison (case A – concentrated load)

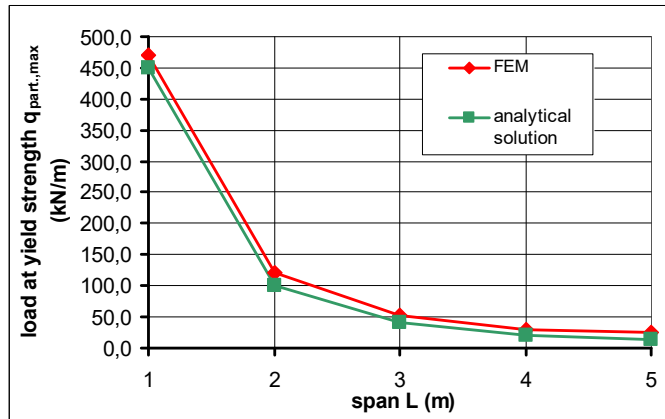


Fig. 12: Comparison (case B – partial uniformly distributed load)

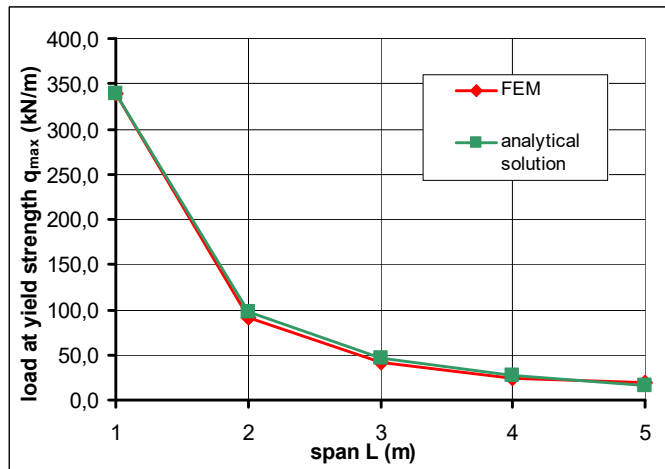


Fig. 13: Comparison (case C – uniformly distributed load)

Comparison of numerically and analytically obtained results for investigated load cases shows that utilization of linear interaction formula (18) gives in most cases lower load magnitudes at yield strength than numerical analysis. Only in case C numerical analysis (FEM) gives slightly lower values for some beam spans than analytical calculation. In Fig. 14 there are differences between numerical and analytical solution for investigated spans L . Positive values indicate that numerical analysis gives higher load magnitude than analytical solution and vice versa.

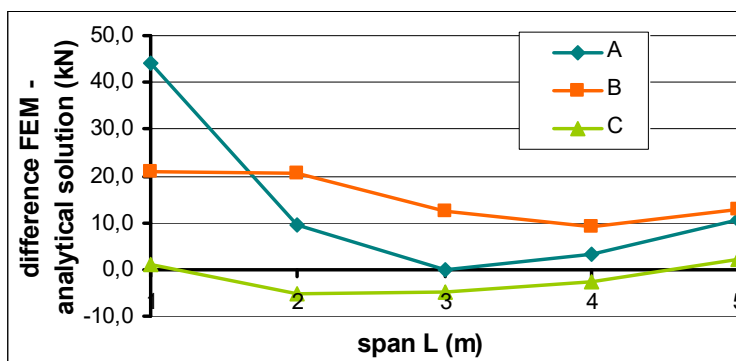


Fig. 14: Differences between numerically and analytically obtained results

5 CONCLUSION

The paper focuses on problem of numerical modeling and geometrically nonlinear analysis of steel thin-walled beams of monosymmetric cross-sections in bending and torsion loaded perpendicularly to the plane of symmetry. Results of numerical analysis using finite element method are compared with analytical calculations using simple linear interaction formula. This formula takes into account bending with lateral torsional buckling as well as torsion.

When evaluating numerical stability analyses certain limitations should be considered. For small spans L of the beam (low values of non dimensional slenderness $\bar{\lambda}_{LT}$, respectively) obtained eigenmode does not have to match the desired eigenmode for lateral torsional buckling due to influence of local buckling of beam web. Lateral torsional buckling is not decisive in such cases. According to [4], its influence might be omitted when $\bar{\lambda}_{LT} \leq 0.4$. The interaction formula (18) does not take the influence of local buckling into account. It probably explains certain difference between numerical and analytical solution for small beam span in case of concentrated load (case A).

This paper deals with analysis of normal stress caused by selected load cases for different beam spans. In a broader context, analysis of deformation could be useful.

It is useful to consider the principle of numerical methods when interpreting their results. Finite element method as numerical method provides approximations of exact results. These approximations might be very accurate. It is influenced, among others, by finite element size. If singularities are avoided finite element method gives usually practically reasonable results.

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