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Maksym GRZYWIŃSKI[1], Iwona POKORSKA[2]

SENSITIVITY ANALYSIS OF CYLINDRICAL SHELL

Abstract

The paper deals with some aspects of structural sensitivity analysis in shell structures. The finite element method has been used for modeling of cylindrical structure. The direct differentiation method has been applied in solution of the problem considered. The examples of sensitivity have been presented concerning displacement sensitivity to variations of thickness in cylindrical shell structural element clamped at boundaries under uniformly distributed pressure. The algorithms presented have been prepared and programmed in the POLSAP code system [1].

Keywords

Sensitivity analysis, shell structure, finite element method.

1 INTRODUCTION

In the sensitivity analysis a variability of chosen functionals is investigated that characterize the behavior of structural system and depend on a number of design variables. All the magnitudes which affect the structural behavior of a system under consideration can serve as the variables, for instance the cross-sectional areas and lengths of particular elements, Young's moduli of material etc. These functionals can in general depend on the current states of displacements and stresses as well as their admissible values called the design constraints. The first paper on the sensitivity analysis was written by Campbell and Zienkiewicz [2] followed over recent years by an increasing number of paper devoted to this subject. Relatively many papers have been published in the aeronautical periodicals (Haftka and Mróz [3], Aurora and Cardoso [4]). The literature on sensitivity analysis is broad. The recent works on this topic are presented in [9] for instance. In sensitivity analysis we can consider both deterministic and stochastic variables. In this paper only deterministic varables are disccused. The sensitivity analysis has found applications in solutions of optimization problems. The goal of this paper is to indicate suitability of analyses in the state-of-the-art structural design and inspection, in particular, of structural systems. In this paper a linear sensitivity analysis of a thin shell structure subjected to static load is presented under the constraints on nodal displacements. The comprehensive aspects of computer implementation are discussed. The algorithms used have been prepared and programmed in the POLSAP system (KLEIBER and HIEN [1]). An example is presented to analyze the sensitivity of a certain concrete structure. The approach is hoped to be useful for the structural designer of real civil systems.

[1] Maksym Grzywiński, Ph.D., Department of Building, Construction and Engineering, Faculty of Civil Engineering, Czestochowa University of Technology, ul. Akademicka 3, 42-200 Czestochowa, Poland, phone: (+48) 343 250 924, e-mail: mgrzywin@bud.pcz.czest.pl.

[2] Iwona Pokorska, Ph.D., Department of Theory of Structures, Faculty of Civil Engineering, Czestochowa University of Technology, ul. Akademicka 3, 42-200 Czestochowa, Poland, phone: (+48) 343 250 920, e-mail: pokorska@bud.pcz.czest.pl.

2 FORMULATION OF THE PROBLEM

In the finite element method model a structure is represented by means of its stiffness matrix, loading vector, nodal displacements vectors and boundary conditions. In the sensitivity analysis is it, in addition, a function characterizing the structural behavior that enters with certain constraints imposed by the designer. These constraints can be expressed in terms of displacements or stresses and thus are related to the limit states of serviceability and load-carrying capacity of structures for given suitable design codes.

The structural response functional ϕ can be expressed as

$$\phi = G[q(h), h] \tag{1}$$

where $h = \{h^e\},\ e = 1,\ldots,E$ is the vector of design variables, $q(h) = \{q_\beta(h)\}, \beta = 1,\ldots,N$ describes the vector nodal displacements. The displacement vector satisfies the equilibrium equations

$$K_{\alpha\beta}(h)\, q_\beta(h) = f_\alpha(h) \tag{2}$$

where $K_{\alpha,\beta}(h)$ and $f_\alpha(h),\ \alpha = 1,\ldots,N$ denote the stiffness matrix and load vector, respectively. Since the stiffness and load are functions of design variables, the displacements are implicit functions of these variables (HAUG et al [4]). The objective of the sensitivity analysis is to determine changes in structural response functional with variations of design parameters, i.e. to find $\dfrac{\partial\phi}{\partial h^e}$ - the sensitivity gradient of functional ϕ. Using the chain rule of differentiation we get

$$\frac{d\phi}{dh} = \frac{\partial G}{\partial h} + \frac{\partial G}{\partial q}\frac{dq}{dh} \tag{3}$$

where $\dfrac{dG}{dh}$ and $\dfrac{\partial G}{\partial q}$ describe the first partial derivatives with respect to the e-th design variable and β-th nodal displacement, respectively. Since G is an explicit function of design variables and displacements, the $\dfrac{dG}{dh}$ and $\dfrac{\partial G}{\partial q}$ are known while $\dfrac{dq}{dh}$ remains to be calculated. Let $K_{\alpha,\beta}(h)$, $f_\alpha(h)$ and consequently $\dfrac{dq}{dh}$ are continuously differentiable with respect to the design variable h. Differentiating both sides of Eq. (2) with respect h^e we get

$$K_{\alpha\beta}\frac{dq_\beta}{dh} = \frac{\partial f_\alpha}{\partial h} - \frac{\partial K_{\alpha\beta}}{\partial h} q_\beta \,. \tag{4}$$

Since the stiffness matrix $K_{\alpha,\beta}(h)$ is nonsingular, Eq. (4) can be solved for $dq_\beta(h)/dh$ which substituted into Eq. (3) gives

$$\frac{dG}{dh} = \frac{\partial G}{\partial h} + \frac{\partial G}{\partial q} K^{-1}_{\alpha\beta}\left(\frac{\partial f_\alpha}{\partial h} - \frac{\partial K_{\alpha\beta}}{\partial h} q_\beta\right). \tag{5}$$

The procedure presented above is called the direct differentiation method.

3 EXAMPLE OF ANALYSIS

3.1 Description of structure

This example the response of a thin shell structure is considered. Fig. 1 shows the half of a cylindrical shell clamped at boundaries under uniformly distributed pressure $p = 15kN/m^2$. The remaining input data are: radius $R = 2{,}5\ m$, length $L = 12$ m, thickness $t = 0{,}10\ m$, Young modulus $E = 30\ MPa$, Poisson ratio $\nu = 0{,}2$.

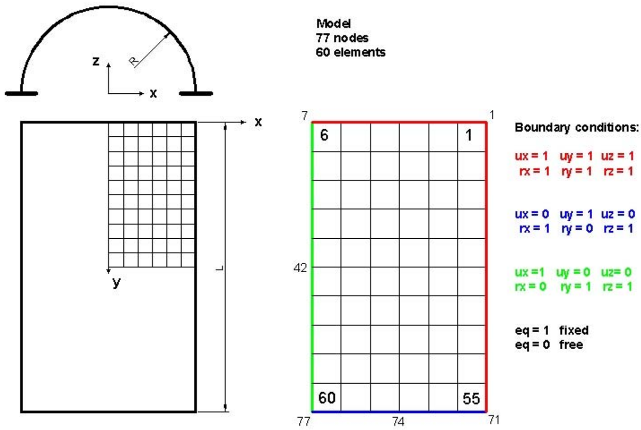

Fig. 1: Finite element model of 60-elements shell structure

3.2 Results

The problem is to choose the most appropriate thickness section of each member of the shell considering the displacement response sensitivity with respect to the thickness. The response functional is assumed as the displacement limit and can be expressed as

$$\frac{dG}{dh} = \frac{|q_\alpha|}{q_\alpha^A} \tag{6}$$

where $|q_\alpha|$ and $q_\alpha^A > 0$ are calculated and allowable of the α-th displacement component. Then the load vector for displacement constraints takes the form

$$\frac{dG}{dh} = sign(g_\alpha)\left(0...0\frac{1}{q_\alpha^A}0...0\right) \tag{7}$$

With the displacement functional defined at node 42 (or 77) in z-direction, $q_\alpha^A = 0{,}01$.

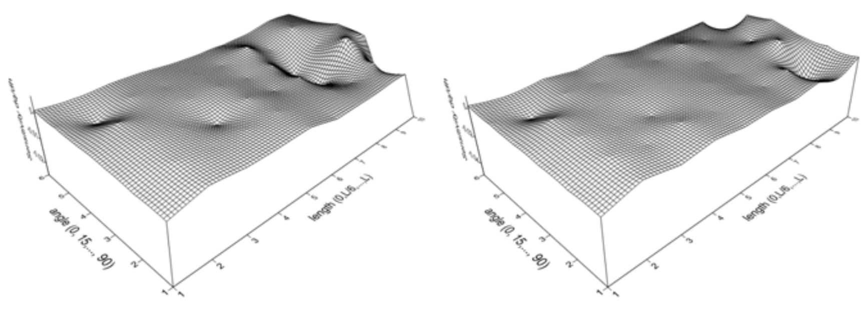

Fig. 2: Displacement sensitivity to variations of thickness at nodes 42 and 77

The displacement sensitivity to variations of thickness is presented in Fig.2. It is seen that for the case of the displacement constraint assumed a unit change in thickness of the element 9 has the largest effect on the vertical displacement at node 42. To decrease displacement effectively at node 42, we have increase the thickness of elements: 1 (-0,0251) and 2 (-0,0246) ; to increase, we have to decrease the thickness of elements: 9 (+0,0068) and 4 (+0,0048).

It is seen that for the case of the displacement constraint assumed a unit change in thickness of the element 4 has the largest effect on the vertical displacement at node 77. To decrease displacement effectively at node 77, we have increase the thickness of elements: 6 (-0,0486) and 8 (-0,0216) ; to increase, we have to decrease the thickness of elements: 4 (+0,0092) and 9 (+0,0060).

4 CONCLUSIONS

Static sensitivity analysis of 3D shell structure is presented in the paper under displacement constraints. The conclusions drawn are directed to structural designer and consulting expert. Thus the classical methods can be supplemented with the sensitivity analysis which has been treated as a tool in problems of structural optimization.

The numerical algorithms developed for sensitivity analysis problems can readily be adapted to existing finite element programs with no considerable modifications required. The numerical results obtained with the code POLSAP show that static sensitivity seems reliable and cost-effective alternatives in research and application environments. The paper can be the basis for subsequent works on design of cylindrical shells.

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Reviewers:

Prof. Ing. Zdeněk Kala, Ph.D., Faculty of Civil Engineering, Brno University of Technology.

Ing. Miroslav Rosmanit, Ph.D., Faculty of Civil Engineering, VŠB-Technical University of Ostrava.