
Miroslav SÝKORA¹, Milan HOLICKÝ², Jan KREJSA³**MODEL UNCERTAINTY FOR SHEAR RESISTANCE OF REINFORCED CONCRETE BEAMS WITH SHEAR REINFORCEMENT ACCORDING TO EN 1992-1-1****Abstract**

The submitted contribution is focused on the model uncertainty related to shear resistance of reinforced concrete beams with special shear reinforcement considering available test results. Variation of the model uncertainty with basic variables is analysed and significant variables are identified for the section-oriented formula provided in EN 1992-1-1. Proposed probabilistic description of the model uncertainty consists of the lognormal distribution having the coefficient of variation of about 0.25 and the mean significantly varying with the strength of shear reinforcement.

Keywords

Model uncertainty, shear resistance, reinforced concrete.

1 INTRODUCTION

Previous studies [1–4] indicated that structural resistances can be predicted by appropriate modelling of material properties, geometry variables and uncertainties associated with an applied model. The effect of variability of materials and geometry is relatively well understood and has been extensively investigated. However, improvements in the description of model uncertainties are still needed [4].

For reinforced concrete structures flexural resistances are predicted with a reasonable accuracy while accurate prediction of the shear resistances is difficult due to the uncertainties in the shear transfer mechanism, particularly after initiation of cracks [5]. Recently the model uncertainties of the shear resistance of beams without a special shear reinforcement have been analysed in several studies; an overview is provided in [6].

The presented study is focused on the model uncertainties of the shear resistance of beams with a special shear reinforcement (such as stirrups or inclined bars, hereafter referred to as “shear reinforcement” to simplify the text). Simple engineering relationship for shear resistance provided in EN 1992-1-1 [7] is considered. Obtained results are critically compared with available experimental data.

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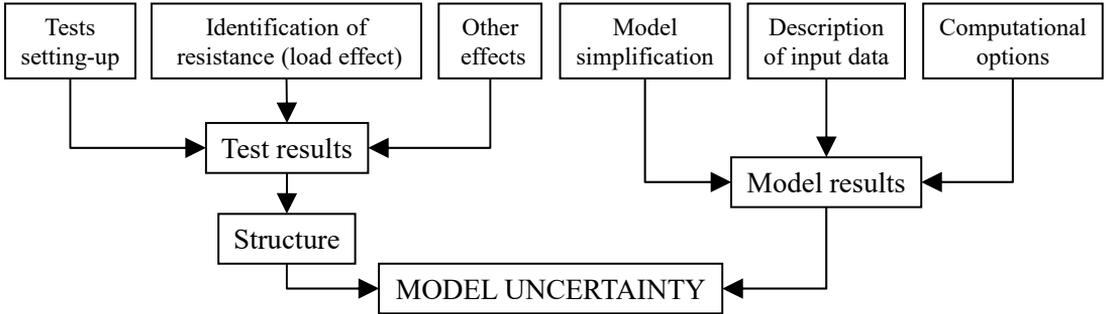


Fig. 1: General concept of the model uncertainty

2 MODEL UNCERTAINTY

According to [8] the model uncertainty is generally a random variable accounting for effects neglected in the models and simplifications in the mathematical relations. Model uncertainties can be related to:

- Resistance models (based on simplified or complex models such as the Finite Element analysis),
- Models for action effects (assessment of load effects and their combinations).

The model uncertainty can be obtained from comparisons of physical tests and model results. Actual structural conditions not covered by tests should be taken into account if needed. Obviously the model uncertainty should be associated with a computational model under consideration. General concept of the model uncertainty applicable to both resistance and load effect models is indicated in Fig. 1. Significance of factors affecting tests, model results and actual structural conditions depends substantially on the analysed structural member or load effect.

Only resistance models are addressed hereafter. Overview of factors affecting the uncertainty related to resistance models is given elsewhere [9].

In this study the model uncertainty θ is considered to be a random variable. The multiplicative relationship for θ is assumed in accordance with [8]:

$$R = \theta R_{\text{model}}(\mathbf{X}) \quad (1)$$

where:

R – denotes the response of a structure – actual resistance estimated from test results and structural conditions;

R_{model} – model resistance – estimate of the resistance based on a model; and

$\mathbf{X}^T = (X_1, \dots, X_m)$ – vector of basic variables X_i .

Note that an additive relationship or combination of the multiplicative and additive formulas may be used to assess the model uncertainty [8]. In more advanced analyses the model uncertainty may be represented by functions of several auxiliary random variables θ and variables X as considered e.g. in [5].

Assuming lognormal distribution with the origin at zero (hereafter simply “lognormal distribution”) for R and $R_{\text{model}}(\cdot)$, the model uncertainty given by relationship (1) is also lognormal. Its characteristics can be assessed using the method provided in Annex D of EN 1990 [10]. When few experimental data are available, Bayesian approaches can be used to combine these data with expert judgements.

The model uncertainty θ in general depends on basic variables X . Influence of individual variables on θ can be assessed by a regression analysis as described e.g. by [11]. It is also indicated that the model describes well the essential dependency of R on X only if the model uncertainty:

- Has either a suitably small coefficient of variation (how small is the question of the practical importance of the accuracy of the model) or
- Is statistically independent of the basic variables (X_1, \dots, X_m) .

It may also be important to define ranges of the input parameters X for which the accepted model uncertainty is valid. Such intervals should be established on the basis of:

- Admissible ranges of \mathbf{X} for the model (for instance limits on reinforcement ratio) and
- Simplifications in modelling of θ (for instance when θ is considered independent of X_i for a specified interval of the basic variable).

3 UNCERTAINTIES RELATED TO THE MODEL PROVIDED IN EN 1992-1-1

3.1 Model in EN 1992-1-1

The model uncertainty should always be clearly associated with an assumed resistance model. In this section uncertainties related to the basic resistance model provided in EN 1992-1-1 [7] for beams with stirrups are considered:

$$R_{\text{model}}(\mathbf{X}) = \min_{1 \leq \cot \zeta \leq 2.5} [\rho_w b_w z f_{yw} \cot \zeta; \alpha_{cw} b_w z \nu_1 f_c / (\cot \zeta + \tan \zeta)] \quad (2)$$

where:

ν_1 – denotes the strength reduction factor for concrete cracked in shear, $\nu_1 = 0.6$ for $f_c \geq 60$ MPa or $\nu_1 = \max[0.5; 0.9 - f_c / 200 \text{ MPa}]$ otherwise.

For the angle between concrete compression struts and the main tension chord, the symbol ζ is introduced here instead of θ used in EN 1992-1-1 [7] to avoid confusion with the symbol for model uncertainty. Notation of the basic variables is provided in Tab. 1. No axial compressive force is considered and actual concrete strengths instead of the characteristic value are applied in equation (2).

3.2 Database of experimental results

Researchers at the University of Stellenbosch collected a database of 222 tests of beams with shear reinforcement [12]. For 22 tests information on ρ_w and f_{yw} is missing and these test results are hereafter not considered. Overview of the experimental data is given in Tab. 1. The database covers a wide range of beams with low to high concrete strengths, shear reinforcement ratio, and effective depths. Beams with light, moderate and heavy longitudinal reinforcement are included.

It is worth noting that the database contains seven specimens with the longitudinal reinforcement of yield strength $f_y = 820$ MPa. Histogram of yield strengths in the whole database is shown in Fig. 2. Annex C of EN 1992-1-1 [7] indicates that the design rules of Eurocode are valid for reinforcement with the characteristic yield strength f_{yk} between 400 to 600 MPa. Therefore, the values of 820 MPa seem to be exceedingly high and these seven specimens are excluded from the database. Since the yield strength is not included in equation (2), the other specimens for which f_y is less significantly beyond the limits remain in the database for a statistical evaluation of the model uncertainty.

Tab. 1: Scatter of variables included in the database and parameters describing their influence on θ

Variable	Min.	Max.	ρ exp. (lin.)	R^2 exp. (lin.)
b_w (mm) – smallest width of a cross-section in the tensile area	76	457	0.14 (0.11)	0.02 (0.01)
d (mm) – effective depth	95	1200	-0.01 (-0.04)	0 (0)
s (mm) – stirrup spacing	48	600	0.03 (0.01)	0 (0)
$\rho_1 = A_{sl}/(b_w d) \leq 2$ (%) – longitudinal reinforcement ratio	0.5	4.54	0.07 (0.08)	0 (0.01)
$\rho_w = A_{sw}/b_{ws}$ (%) – shear reinforcement ratio	0.07	1.19	-0.69 (-0.60)	0.48 (0.37)
f_c (MPa) – concrete compressive strength	12.8	125	0.16 (0.14)	0.02 (0.02)
f_{yw} (MPa) – yield strength of stirrups	182	820	0.09 (0.05)	0.01 (0)
$\rho_w f_{yw}$ (MPa) – strength of shear reinforcement	0.21	2.62	-0.75 (-0.68)	0.56 (0.46)
a/d – shear span-to-depth ratio	2.49	5.05	0.12 (0.11)	0.02 (0.01)
V_{fail} (kN) – shear force at failure	15.6	1172.4	-0.02 (-0.04)	0 (0)

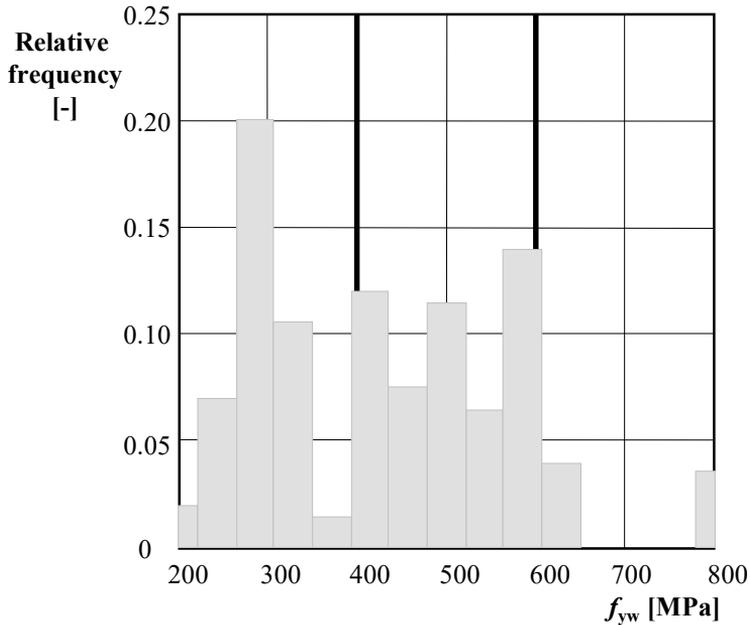


Fig. 2: Histogram of f_y for the whole database

Tab. 2: Sample characteristics of the model uncertainty

Description of the sample	m	v
Whole database, $n = 200$	1.32	0.34
Lightly reinforced beams ($\rho_w f_{yw} \leq 1$ MPa), $n = 147$	1.56	0.26
Moderately reinforced beams ($1 \text{ MPa} < \rho_w f_{yw} \leq 2$ MPa), $n = 45$	1.25	0.23
Heavily reinforced beams ($2 \text{ MPa} < \rho_w f_{yw}$), $n = 8$	0.76	0.20

3.3 Statistical evaluation of the model uncertainty

For each experiment the model resistance is assessed from equation (2) and the model uncertainty is evaluated from equation (1). Note that the first term in equation (2) is decisive for all the specimens. Sample characteristics of θ (mean m and coefficient of variation v), estimated for the whole database by the Annex D of EN 1990 [10], are given in Tab. 2. Limits for lightly, moderately and heavily reinforced beams are accepted from [13]. A lognormal distribution is accepted in accordance with [8].

To verify influence of basic variables (Tab. 1) on the model uncertainty, a simple sensitivity analysis as proposed in [12] is conducted for the present database. Trends in θ with a basic variable are assessed using:

- The correlation coefficient ρ (correlation between θ and X_i), and
- The coefficient of determination R^2 , a measure of the linear relationship between θ and X_i [14].

A combination of strong ρ (say, $|\rho| > 0.5$) and strong R^2 (say, $R^2 > 0.5$) indicates a significant linear relationship between θ and X whereas strong correlation with relatively weak R^2 suggests a non-linear relationship.

Regression analysis is based on a linear or exponential model described by the following relationships:

$$\text{linear: } \theta(\rho_w f_{yw}) = b_0 + b_1 \rho_w f_{yw} \quad (3)$$

$$\text{exponential: } \theta(\rho_w f_{yw}) = \exp(b_0 + b_1 \rho_w f_{yw}) \quad (4)$$

where:

b_0 and b_1 – denote regression parameters determined by the Least square method.

The results provided in Tab. 1 reveal strong correlations between θ and ρ_w or $\rho_w f_{yw}$ while weak correlations appear for the other shear parameters. The most influential parameter is strength of shear reinforcement $\rho_w f_{yw}$ ($\rho = -0.68$ and $R^2 = 0.46$ for linear regression; $\rho = -0.75$ and $R^2 = 0.56$ for exponential regression) as already recognised in [12,13]. For most of the shear parameters the exponential regression is a more appropriate model.

A multiple linear regression with all the shear parameters yields $R^2 = 0.68$ and somewhat improves the model of θ . However, the model uncertainty as a function of eight variables is impractical. Therefore, the influence of the longitudinal reinforcement ratio on θ is considered hereafter only. Fig. 3 shows the histogram of the strength of the shear reinforcement for the whole database. It appears that the database contains a sufficient number of the test results for light and medium reinforced beams while a limited amount of data is available for heavily reinforced beams (sample sizes are $n = 147, 45$ and 8 , respectively).

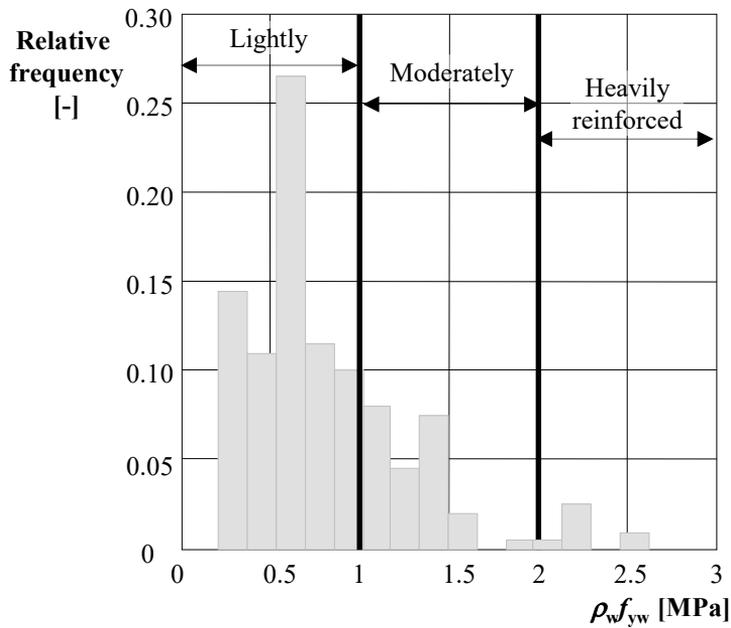


Fig. 3: Histogram of $\rho_w f_{yw}$ for the whole database

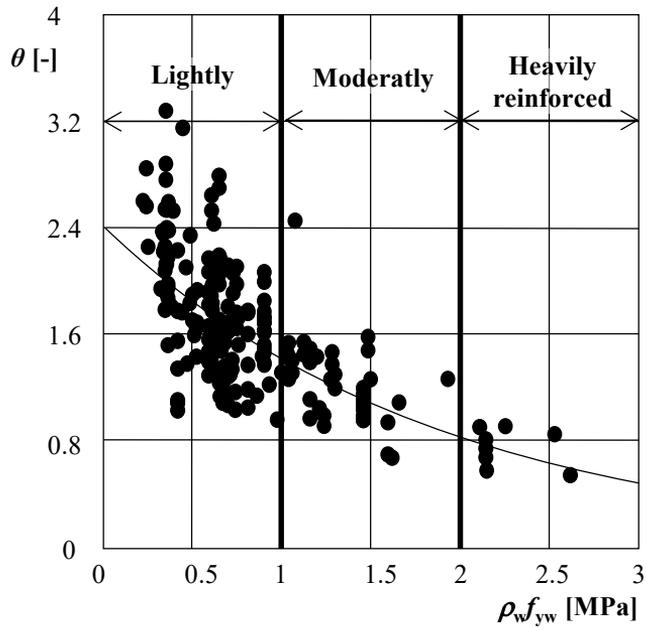


Fig. 4: Variation of θ with $\rho_w f_{yw}$ for the whole database

Fig. 4 shows variation of the model uncertainty with the strength of shear reinforcement based on the exponential regression. The model uncertainty clearly decreases with an increasing strength and its differentiation with respect to $\rho_w f_{yw}$ is thus proposed. Similar observations have been already made in [15] with an argument that the truss model in EN 1992-1-1 [7] may be unconservative for highly reinforced concrete members ($\rho_w f_y > 2$ MPa).

Sample characteristics of θ for light to heavy reinforced beams are provided in Tab. 2. It follows that the mean depends on the strength of shear reinforcement while the effect on the coefficient of variation is less significant.

Statistical testing of outliers is conducted to exclude measurements obtained under significantly different conditions or affected by errors. For each data group Grubb's test at a significance level of 0.05 [14] is performed; none of the 200 samples was excluded.

Based on the results given in Tab. 2 the following stochastic characteristics of θ may be accepted as a first approximation for the shear resistance of the members with shear reinforcement:

- lightly reinforced beams ($\rho_w f_{yw} \leq 1$ MPa): $\mu_\theta \approx 1.56$; $V_\theta \approx 0.26$,
- moderately reinforced beams (1 MPa $< \rho_w f_{yw} \leq 2$ MPa): $\mu_\theta \approx 1.25$; $V_\theta \approx 0.23$,
- heavily reinforced beams (2 MPa $< \rho_w f_{yw}$): $\mu_\theta \approx 0.76$; $V_\theta \approx 0.20$ (note that particularly these values are indicative since they are based on eight test results only).

4 MODEL UNCERTAINTY FACTOR FOR DETERMINISTIC RELIABILITY VERIFICATIONS

For deterministic reliability verifications, EN 1990 [10] introduces the partial factor γ_{Rd} to describe the uncertainty associated with the resistance model (“design value of the model uncertainty”). Fig. 5 illustrates the relationship between the probabilistic distribution of θ and factor γ_{Rd} . As an example the lognormal distribution (mean $\mu_\theta = 1.25$ and coefficient of variation $V_\theta = 0.25$) and the relevant model uncertainty factor $\gamma_{Rd} = 1.08$ (for $\beta = 3.8$) obtained from equation (6) (see the text below) are shown.

The model uncertainty factor γ_{Rd} for reinforced concrete structures can be obtained as a product of [16]:

$$\gamma_{Rd} = \gamma_{Rd1} \gamma_{Rd2} \tag{5}$$

where:

γ_{Rd1} – denotes the partial factor accounting for model uncertainty and

γ_{Rd2} – partial factor accounting for geometrical uncertainties.

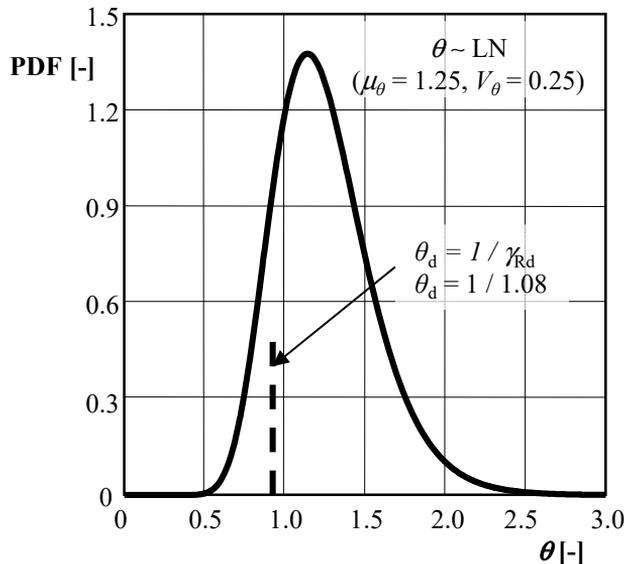


Fig. 5: Probability density function of θ and the model uncertainty factor γ_{Rd}

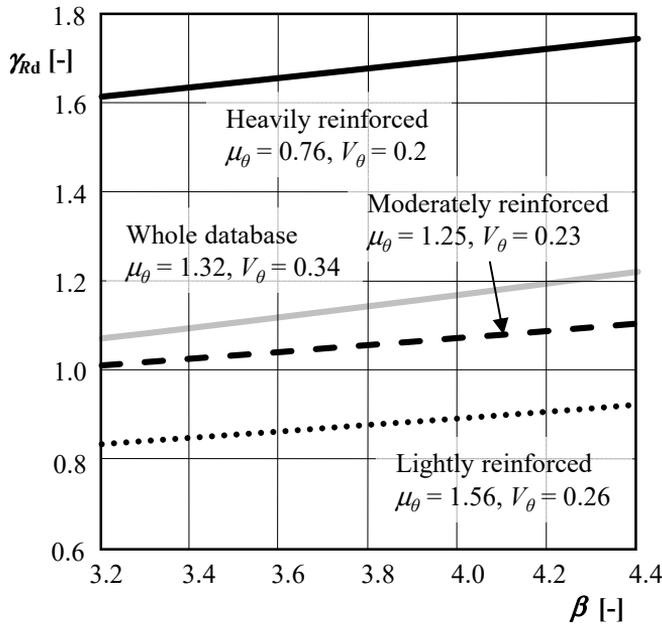


Fig. 6: Variation of the partial factor γ_{Rd} with β for $\alpha_R = 0.32$

EN 1992-1-1 [7] provides no specific recommendations concerning model uncertainties. EN 1992-2 [17] introduces the global safety format for a nonlinear analysis with the recommended model uncertainty factor of 1.06. However, it has been shown [4] that such a factor is rather low and should be increased in most cases depending on relevant failure mode (bending, shear, compression).

$\gamma_{Rd1} = 1.05$ for concrete strength and $\gamma_{Rd1} = 1.025$ for reinforcement may be assumed in common cases [16]. However, larger model uncertainty needs to be considered for punching shear in the case when concrete crushing is governing. A value of $\gamma_{Rd2} = 1.05$ may be assumed for geometrical uncertainties of the concrete section size or reinforcement position. When relevant measurements of an existing structure indicate insignificant variability of geometrical properties, $\gamma_{Rd2} = 1.0$ may be considered.

Alternatively, the partial factor γ_{Rd} can be obtained from the following relationship based on a lognormal distribution:

$$\gamma_{Rd} = 1/[\mu_\theta \exp(-\alpha_R \beta V_\theta)] \quad (6)$$

where:

- α_R – denotes the FORM sensitivity factor and
- β – target reliability index according to EN 1990 [10].

Considering the statistical characteristics of the model uncertainty given in Tab. 2, variation of the partial factor γ_{Rd} obtained from equation (6) with the target reliability β for $\alpha_R = 0.4 \times 0.8 = 0.32$ (“non-dominant resistance variable”) is indicated in Fig. 6.

It follows from Fig. 6 that the model uncertainty factor γ_{Rd} increases with an increasing target reliability index β . For the considered range of β from 3.2 to 4.4 the model uncertainty varies approximately within the following intervals:

- 0.84–0.93 for lightly reinforced members ($\gamma_{Rd} \approx 0.9$ as a first approximation),
- 1.02–1.10 for moderately reinforced members ($\gamma_{Rd} \approx 1.1$ may be commonly accepted),
- 1.61–1.75 for heavily reinforced members ($\gamma_{Rd} \approx 1.7$ as a first approximation).

The selection of $\alpha_R = 0.32$ deserves additional comments. Leading and accompanying actions (with associated factors $\alpha_E = -0.7$ and $\alpha_E = -0.4 \times 0.7 = -0.28$, respectively) are distinguished in Annex C of EN 1990 [10] while $\alpha_R = 0.8$ is recommended for resistance variables under the conditions specified in the Eurocode. When the model uncertainty factor γ_{Rd} and material factor γ_m are assessed separately considering $\alpha_R = 0.8$, overly conservative designs may be obtained. Therefore, CEB bulletin [18] and ISO 2394 [19] considered the model uncertainty as a non-dominant resistance variable and accepted the reduction $\alpha_R = 0.4 \times 0.8 = 0.32$. Note that the value α_R significantly affects the partial factor γ_{Rd} [9].

5 CONCLUDING REMARKS

It appears that description of uncertainties related to resistance and load effect models can be a crucial problem of reliability analyses. The present paper is particularly focused on the model uncertainties in shear resistance of beams with shear reinforcement; the following concluding remarks are drawn:

- The model uncertainty should be related to test uncertainties, to actual structural conditions and computational model under consideration.
- In common cases actual resistance can be estimated as a product of the model uncertainty and resistance obtained by the model.
- Uncertainties related to models provided in EN 1992-1-1 [7] can be described by the following statistical characteristics and partial factors:
 - lightly reinforced beams ($\rho_{wf_{yw}} \leq 1$ MPa): $\mu_\theta \approx 1.6$; $V_\theta \approx 0.25$ and $\gamma_{Rd} \approx 0.9$,
 - moderately reinforced beams (1 MPa $< \rho_{wf_{yw}} \leq 2$ MPa): $\mu_\theta \approx 1.25$; $V_\theta \approx 0.25$ and $\gamma_{Rd} \approx 1.1$,
 - heavily reinforced beams (2 MPa $< \rho_{wf_{yw}}$): $\mu_\theta \approx 0.8$; $V_\theta \approx 0.2$ and $\gamma_{Rd} \approx 1.7$ (particularly these values are indicative since they are based on eight test results only).

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