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EFFECT OF RANDOM AXIAL CURVATURE OF A THIN-WALLED BEAM ON ITS LOAD-CARRYING CAPACITY

Abstract

The paper deals with the analysis of load-carrying capacity (LCC) of a thin-walled steel beam under compression the axis of which is randomly spatially curved. Open and close thin-walled cross-sections are considered for the beam, respectively. The initial curvature is modelled by a random field. The Latin Hypercube Sampling Method was applied. The load carrying capacity is calculated by geometrically nonlinear solution using ANSYS software. The results are presented both in histograms and in a table. The LCC statistical characteristics of beams with open and closed cross-sections have been compared. A comparison with the LCC according to the standards is carried out as well.

Keywords

Load carrying capacity, initial curvature, random field, limit state, thin-walled member, stress, warping.

Abstract

The paper deals with the analysis of load-carrying capacity (LCC) of a steel beam under compression which axis is randomly spatially curved. Open and close thin-walled cross-sections are considered for the beam, respectively. The initial curvature is modelled by a random field. The Latin Hypercube Sampling Method was applied. The load carrying capacity is calculated by geometrically nonlinear solution by ANSYS software. The results are presented both in histograms and in a table. The LCC statistical characteristics of beams with open and closed cross-sections have been compared. A comparison with the LCC according to the standards is carried out as well.

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1 INTRODUCTION

The initial curvature of beams is modelled mostly in the form of a half-wave of the sine function. For these cases, the solution in explicite form is at the disposal [1,2], and therefore it is not difficult to calculate the load carrying capacity from the response function, if the amplitude of initial curvature e_0 is assigned. The explicite solution is possible under the condition when the deformation of the axis of loaded beam is affine to the initial curvature. However, it is not a rule that the initial axial curvature must have the shape of the half-wave of sine function. In the real state of facts, the general curvature is met much more frequently, and it concerns namely not only the plane one, but

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also the spatial one above all. For such cases, an explicite solution cannot be obtained, because the axis of the deformed beam is not affine to the initial curvature [3].

The present paper deals with an analysis of the load carrying capacity of a steel beam under compression the axis of which is randomly spatially curved. For this beam, thin-walled cross-sections are selected, both close and open. The warping torsion is connected with loading of beams having such cross-section. The warping torsion occurs when warping is prevented. Warping can be limited, e.g., by structural arrangement (bonds preventing the warping), change of twisting moment along the beam axis and/or by the change of shape and dimension of cross-section along the beam axis. The warping torsion can also occur in the case when the beam is loaded by transverse load action, not passing through the shear centre or by excentrically acting longitudinal load action. In these cases, the warping torsion is combined with bending, shearing, compression or tension, and general bending-torsion loading action occurs.

The solution of thin-walled beams was based on the presumption of rigid cross-section, the geometry of which did not change under the influence of load action.The load carrying capacity of the beam under compression was then calculated by means of geometrically nonlinear solution by the ANSYS software.

2 CALCULATION MODEL

A two-hinge beam with length $L = 2.798$ m was considered. Its load carrying capacity was examinated for the value of non-dimensional slenderness $\bar{\lambda} = 1.0$ which is defined, in dependence on the beam length and on the radius of gyration of cross-section, in the standard EUROCODE 3.

For the analysis of load carrying capacity of the beam mentioned, the beam element BEAM 188 with seven degrese of freedom was applied by the ANSYS software (3 degrees of freedom correspond with translations in the *x*, *y*, and *z* directions, other 3 ones, with rotations around the *x*, *y*, and *z* directions, and the 7th degree of freedom corresponds with warping magnitude. The beam scheme is presented in Fig. 1. In node *a,* there are prevented displacements in all directions of all three axes, and also the rotations around axis *x.* Further on, it is considered that both end cross-sections in nodes *a* and *b* cannot warp, so that the model could correspond with real laboratory experiment as much as possible. Subsequently, the model is loaded by displacement in node *b* in direction of axis *x*. In node *a*, the value of reaction in direction *x* is subsequently distracted to determine the load carrying capacity (see Chapter 3).

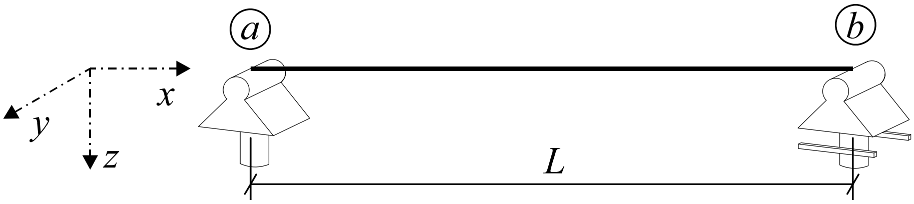

Fig. 1: Scheme of the beam

2.1 Applied cross-sections

For the given problem, two symmetric quadratic thin-walled cross-sections were applied, the first one being close, and the second, open (an incision was done in the middle of one side). The cross-sections are presented in Fig. 2. The beam is modelled so that its axis passes through the centroid of the cross-section.

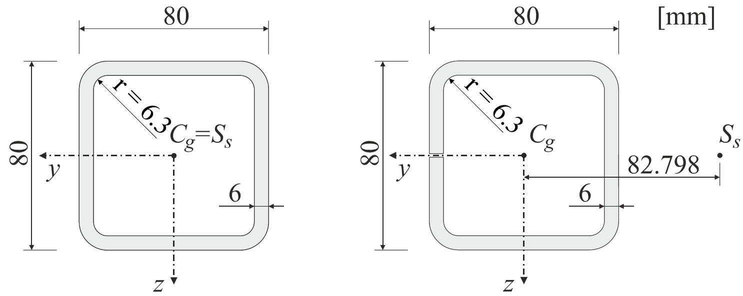

Fig. 2: Cross-sections applied: left - close, right - open (with an incision)

The close cross-section is symmetrical in conformity with both axes, so the position of the centroid C_g corresponds with the position of the shear centre S_s. In case of the open cross-section, the position of shear centre was displaced according to Fig. 2, right side. The cross-section characteristics of both cross-sections are given in Tab. 1.

Tab. 1: Selected cross-section characteristics

Cross-section characteristic	Close cross-section	Open cross-section
Area A [m^2]	$1.6799 \,.\, 10^{-3}$	$1.6787 \,.\, 10^{-3}$
Second moment of area I_y [m^4]	$1.4865 \,.\, 10^{-6}$	$1.4848 \,.\, 10^{-6}$
Second moment of area I_z [m^4]	$1.4865 \,.\, 10^{-6}$	$1.4865 \,.\, 10^{-6}$
Warping constant I_ω [m^6]	$2.8264 \,.\, 10^{-12}$	$6.0957 \,.\, 10^{-9}$

These cross-sections are then applied for each realization of spatial curvature of the beam axis (see Chapter 2.2). Taking into consideration the fact that here, the initial curvature is independent of the cross-section applied, the open cross-section is put into four positions caused by rotation always by 90 °. Then, the load carrying capacity of the given realization is calculated for five cross-sections according to the Fig. 3.

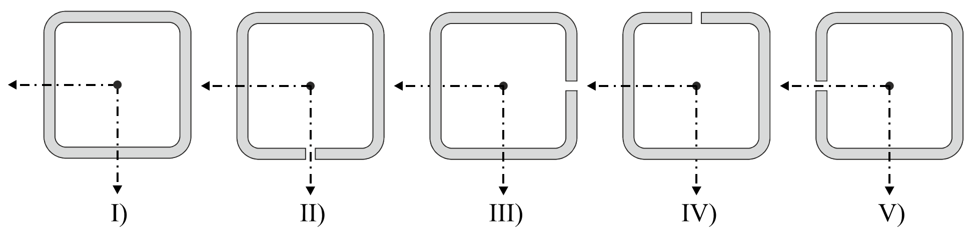

Fig. 3: Cross-sections of the beam – designation

2.2 Random input quantities and random field of initial axial curvature

The Gaussian probability distribution with mean value 297.3 MPa, and standard deviation 16.8 MPa [4] was considered for yield strength f_y, The initial curvature of beam axis was modelled on behalf of eleven nodes interlain by spline – Fig. 4. Each of these nodes had the Gaussian probability distribution with zero mean value and standard deviation 0.0015248 sin($\pi\, x_i$ / 2.798) m, where x_i is the position on the beam axis. The value 0.0015248 was calculated, based on the presumption that, within the tolerance limits ±0.15 % L, there were 95 % realizations of the maximum initial spatial deformation, beam length L being the calculation parameter. The values of coordinates in each of the two planes are mutually correlated by means of the correlation matrix. This represents a random field with the correlation length L_{cor} = 1.44165 m. The correlation is considered only among the values of coordinates of the nodes on one plane. In other words, the curvature on one plane is independent of the curvature on the second plane. Random realizations both of yield strength and initial curvature were simulated applying the Latin Hypercube Sampling method [5,6] implemented in the software Freet.

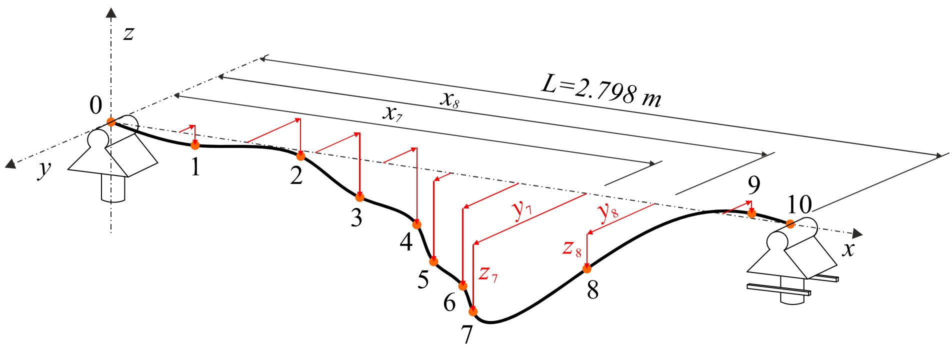

Fig. 4: Spatial curvature definition of the beam axis

The other input quantities – Young's modulus E of steel and geometrical characteristics of the cross-section – were considered to be deterministic ones, and were considered by their average values. In the calculation, there was subsequently sought the force for which the given realization of yield strength would be reached for the given realization of inital curvature of the beam axis. There were simulated 60 random realizations of beam curvatures, and 60 random realizations of yield strength. Each realization of the beam corresponded with just one realization of yield strength. An example of one realization of initial curvature of the beam axis is presented in Fig. 5 and Fig. 6.

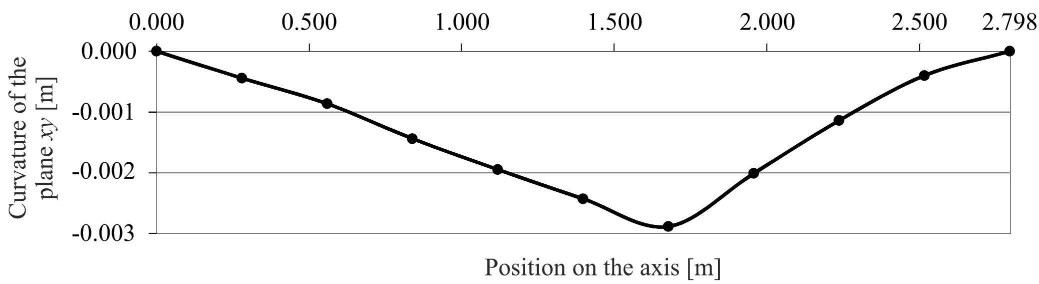

Fig. 5: Curvature of the beam axis on the plane *xy* for one random realization

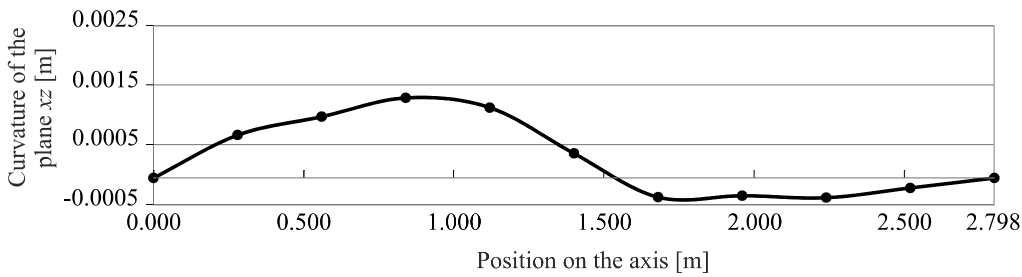

Fig. 6: Curvature of the beam axis on the plane *xz* for one random realization

3 STATE OF STRESS AND LIMIT STATE

Under loading, the beam is in the state of combined stresses. To evaluate the load carrying capacity, it is necessary to know when the state of stress is approaching the limit stress state in the material; in our case, the yield strength f_y is concerned. As the yield criterion, the von Mises (Huber, Hencky) yield criterion was used in the form:

$$f = \bar{\sigma} - \sigma_0 = 0 \tag{1}$$

where:

$\bar{\sigma}$ – is equivalent stress [Pa] which is:

$$\bar{\sigma} = \sqrt{\frac{1}{2}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2\right)\right]} \tag{2}$$

and σ_0 corresponds to f_y. For the solution by the beam model, it is supposed that $\sigma_y = \sigma_z = \tau_{yz} = 0$. Thus, the formula (2) is reduced to the form:

$$\bar{\sigma} = \sqrt{\sigma_x^2 + 3\left(\tau_{xy}^2 + \tau_{zx}^2\right)}\,. \tag{3}$$

Although steel can get into the plastic yielding, and be effective in it, the plastic reserve is very low in a similar case (approximately 3 %). Therefore, the state at the elastic limit, i.e., the state when plastic yielding does not still take place, is considered to bet he limit state in this case. The limit state occurs, if yield stress f_y is reached.

3.1 Stress in open thin-walled cross-sections

A simplifying assumption on the deformation of a thin-walled open profile is the hypothesis that the cross-section will translate as a rigid whole on its plane. This translation is the result of displacements v, w in directions of axes of coordinates y, z, and of rotation ω_x around a certain fixed point. The relative angle of warping torsion will be obtained by the derivation of the rigid body rotation:

$$\frac{\mathrm{d}\omega_x}{\mathrm{d}x} = \Theta\,. \tag{4}$$

The displacement in direction x is given:

$$u = u_0 - \frac{\mathrm{d}v}{\mathrm{d}x}y - \frac{\mathrm{d}w}{\mathrm{d}x}z - \frac{\mathrm{d}\omega_x}{\mathrm{d}x}\omega(s) = u_0 - v'y - w'z - \Theta\omega(s)\,. \tag{5}$$

Thus, the proportional longitudinal deformation can be determined as:

$$\varepsilon_x = \frac{\partial u}{\partial x} = u_0' - v''y - w''z - \Theta'\omega(s)\,. \tag{6}$$

The component σ_x is calculated in the same way as when calculating the linear stress state**.** After substituting the proportional longitudinal deformation according to (6), it is obtained:

$$\sigma_x = E\varepsilon_x = E\left[u_0' - v''y - w''z - \Theta'\omega(s)\right]. \tag{7}$$

Te conditions of equivalence are as follows:

$$\int_A \sigma_x \mathrm{d}A = N_x\,, \tag{8}$$

$$\int_A \sigma_x y \mathrm{d}A = -M_z\,, \tag{9}$$

$$\int_A \sigma_x z \mathrm{d}A = M_y\,, \tag{10}$$

$$\int_A \sigma_x \omega(s) \mathrm{d}A = B\,, \tag{11}$$

where:

B – is the bimoment [Nm^2].

For main sector coordinates and central axes, the conditions of equivalence can be rewritten:

$$N_x = EAu_0', \tag{12}$$

$$-M_z = -EI_z v'' - EI_{yz} w'', \tag{13}$$

$$M_y = -EI_{yz} v'' - EI_y w'', \tag{14}$$

$$B = -EI_\omega \Theta'. \tag{15}$$

The final longitudinal stress in a thin-walled open cross-section can be then expressed:

$$\sigma_x = \frac{N_x}{A} - \frac{M_z}{I_z} y + \frac{M_y}{I_y} z + \frac{B}{I_\omega}\omega. \tag{16}$$

At the warping torsion, secondary tangential stresses which superpose with primary ones corresponding to free warping torsion occur in transverse and longitudinal sections. The primary tangential stress state gets changed linearily along the cross-section thickness, and is given by the expression:

$$^1\tau_{xs} = \frac{2\,^1M_x}{I_t} n \tag{17}$$

where:

^1M_x – is the warping moment correrspoding to the free warping torsion [Nm],

n – is the coordinate measured in the direction of the normal line to the central line [m],

I_t – is the second moment of area in warping torsion [m^4].

The secondary tangential stress $^2\tau_{xs}$ is uniformly divided along the cross-section thickness, and if the axes of coordinates are the main central axes again, the stress is expressed as follows:

$$^2\tau_{xs} = \frac{1}{t}\left(V_y \frac{\overline{S}_z}{I_z} + V_z \frac{\overline{S}_y}{I_y} + {}^2M_x \frac{\overline{S}_\omega}{I_\omega}\right) \tag{18}$$

where :

t – is the cross-section thickness [m],

V_y, V_z – are shear forces [N],

$\overline{S}_y$, $\overline{S}_z$ – are statical moments of incomplete cross-section area [m^3],

$\overline{S}_\omega$ – is the warping statical model of the incomplete cross-section area[m^4],

^2M_x – is bending torsional moment [Nm].

The general tangential stress in a thin-walled open cross-section will be obtained by addition of expressions (17) and (18):

$$\tau_{xs} = {}^1\tau_{xs} + {}^2\tau_{xs} = \frac{2\,^1M_x}{I_t} n + \frac{1}{t}\left(V_y \frac{\overline{S}_z}{I_z} + V_z \frac{\overline{S}_y}{I_y} + {}^2M_x \frac{\overline{S}_\omega}{I_\omega}\right). \tag{19}$$

3.2 Stress in close thin-walled cross-sections

Similarly as in case of beams with open cross-section, it can be assumed that the shape of transverse sections will not get changed during deformation. For warping torsion, it cannot be assumed that warping is proportional to relative warping angle Θ, as it was in case of open cross-sections. However, it can be assumed that the warping of section in the point x is given by the product of unit warping and of warping function $f(x)$. The displacement of the general point of member determined by coordinate x of corresponding section and coordinate s on the central line of thin-walled cross-section can be expressed by the relation:

$$u(x,s) = -f(x)\bar{\omega}(s) \tag{20}$$

where:

s – is the length of central line of the direct section,

$\bar{\omega}(s)$ – is the unit warping taken negatively [m],

$f(x)$ – is the warping function.

Assuming that longitudinal layers of a cross-section do not mutually affect each other the during deformation, the normal stress in transversal sections corresponds to their warping:

$$\sigma_x = E\frac{\partial u}{\partial x} = -E\bar{\omega}(s)f'(x) \tag{21}$$

For the close cross-section, the bimoment is defined by the expression

$$B(x) = -EI_{\bar{\omega}}f'(x) = \int_A \sigma_x \bar{\omega} \mathrm{d}A \tag{22}$$

where:

$I_{\bar{\omega}}$ – is the warping second moment of area [m^6].

For the longitudinal stress at warping torsion, it is obtained by combination of equations (21) and (22)

$$\sigma_x = \frac{B(x)\bar{\omega}(s)}{I_{\bar{\omega}}}. \tag{23}$$

Taking into consideration that also the normal force and bending moments act on the cross-section, the resulting normal stress is given by superposition of all these partial stresses analogously as in case of beams with open cross-section as follows:

$$\sigma_x = \frac{N_x}{A} - \frac{M_z}{I_z}y + \frac{M_y}{I_y}z + \frac{B(x)\bar{\omega}(s)}{I_{\bar{\omega}}}. \tag{24}$$

The following relation holds for tangential stress at St. Venant torsion:

$$^1\tau_{xs} = \frac{^1M_x}{2A_0 t} \tag{25}$$

where:

A_0 – is the area bounded by central line of the cross-section [m^2].

The resulting tangential stress is given again by superposition of the primary and the secondary stress due to warping tension and tangential stress caused by bending.

4 LOAD CARRYING CAPACITY

The load carrying capacity values of all the sixty random realizations of beam are presented by histograms in Fig. 7 to Fig. 11. The Gaussian probability distribution can be interlain, by these histograms, with satisfactory accurateness. The hypothesis on normality of distribution for no cross section is not refused by the Chi-quadrate normality test on the significance level 5 %.

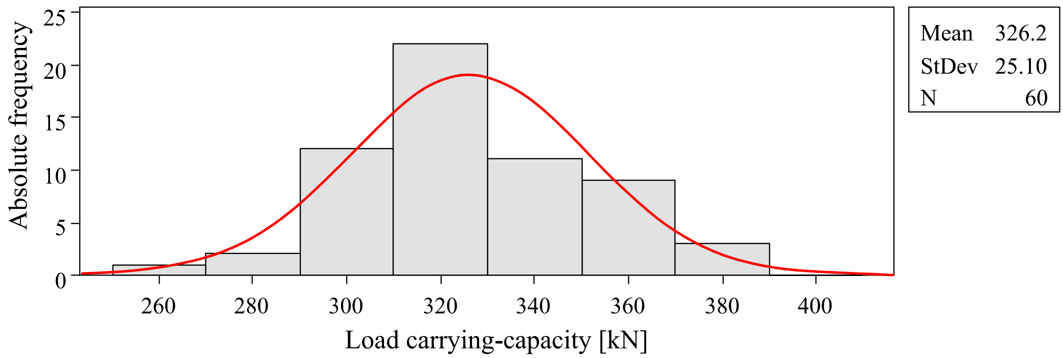

Fig. 7: Histogram of load carrying capacity of beams with cross-section No. I (close cross-section)

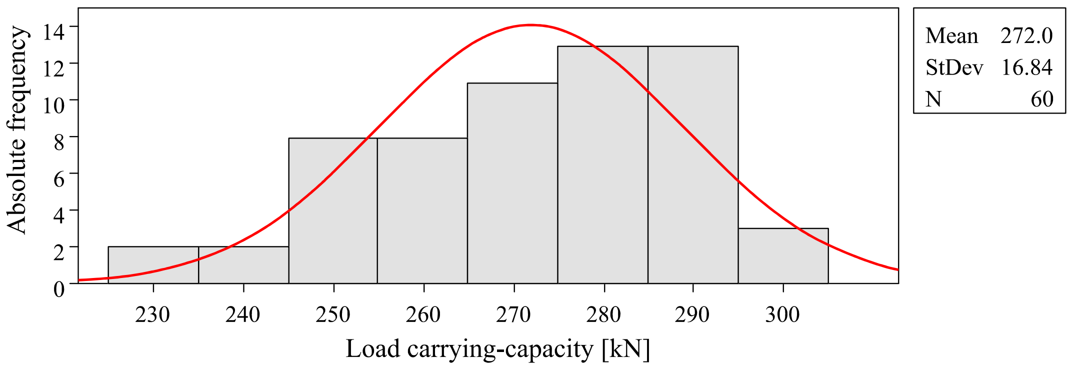

Fig. 8: Histogram of load carrying capacity of beams with cross-section No. II (open cross-section)

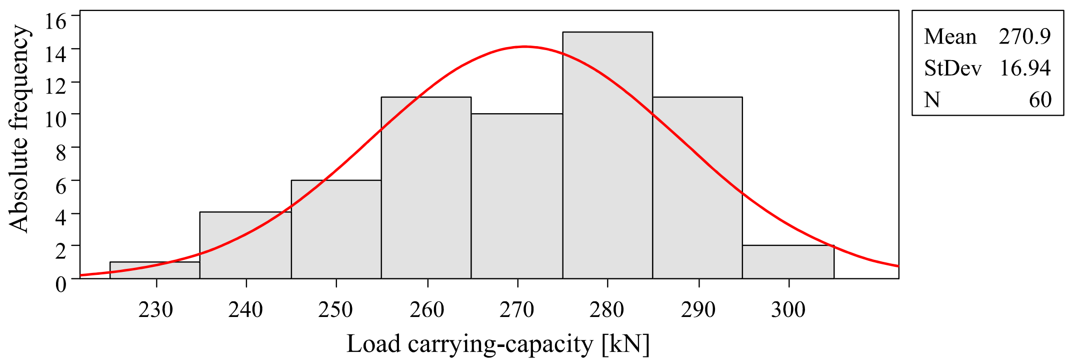

Fig. 9: Histogram of load carrying capacity of beams with cross-section No. III (open cross-section)

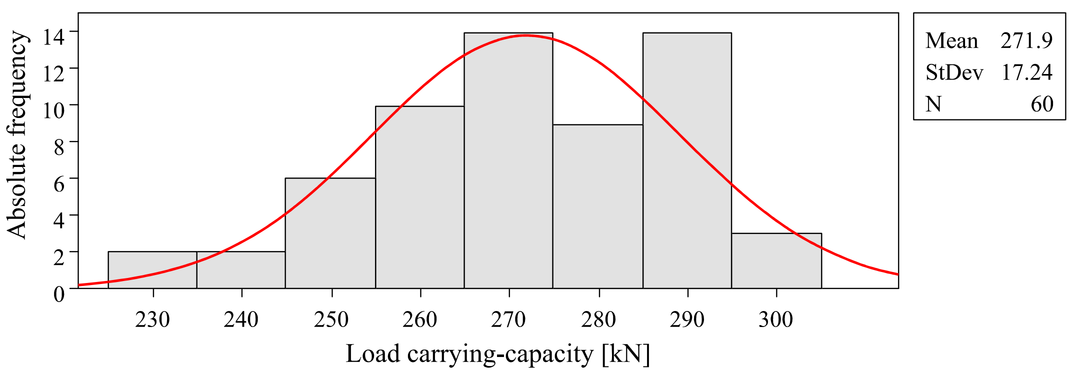

Fig. 10: Histogram of load carrying capacity of beams with cross-section No. IV (open cross-section)

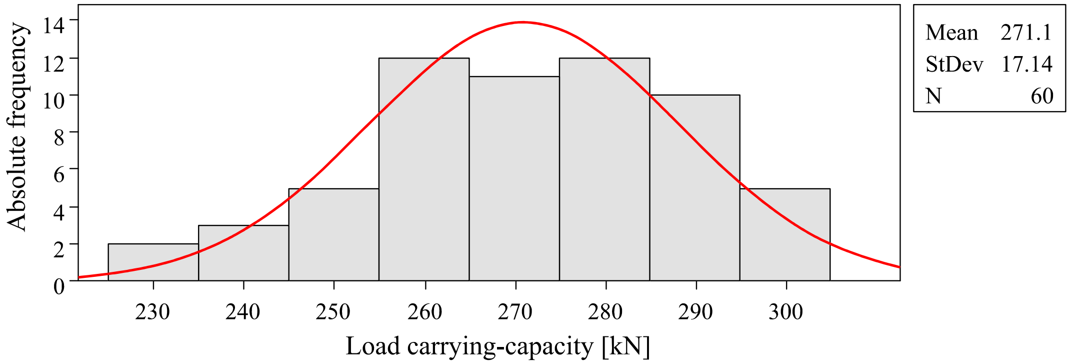

Fig. 11: Histogram of load carrying capacity of beams with cross-section No. V (open cross-section)

The extended statistics of load carrying capacity can be seen in Tab. 2. Based on the standard EN1990, the load carrying capacity is calculated as 0.1% quantile, if the design reliability index $\beta_d = 3.8$. Therefore Tab. 2 also contains the value 0.1% quantile of normal distribution.

Tab. 2: Statistics of load carrying capacity

Cross-section		Mean value [kN]	Standard deviation [kN]	Coefficient of variation [-]	0.1% quantile of normal distribution [kN]
I	(close)	326.20	25.10	7.69	248.63
II	(open)	272.00	16.84	6.19	219.98
III	(open)	270.95	16.94	6.25	218.59
IV	(open)	271.94	17.24	6.34	218.66
V	(open)	271.07	17.14	6.32	218.10

The design value of load carrying capacity of the beam with close cross-section (cross-section I) is – in compliance with EUROCODE 3 – 262.29 kN. EUROCODE 3 does not indicate the design load carrying capacity of beams with open cross-sections (cross-sections II, III, IV, V).

5 CONCLUSION

This paper has presented the histograms and a table of load carrying capacities of beams with initial spatial axial curvature.

The beams with open cross-section have similar mean values and standard deviations of load carrying capacity. Their design load carrying capacity is not given by the standard EUROCODE 3 explicitely. Nevertheless, if the buckling curve *b* is used, the value of load carrying capacity will be 235 kN. This value is higher than all 0.1% quantiles of normal distribution by means of which the values of load carrying capacity of beams with open cross-section were approximated.

The design value of load carrying capacity of beams with close cross-section according to EUROCODE 3 is 262.29 kN. This value is also higher than 0.1% quantile of normal distribution. The design in compliance with EUROCODE 3 can be dangerous. However, it must be verified by other reliability studies according to EN1990.

On the contrary to beams with open cross-sections (cross-sections II, III, IV, V), the beams with close cross-section (cross-section I) have higher mean value and standard deviation of load carrying capacity, although they have quite the same area and second moment of area about both axes and the same realizations of initial axial curvature. Mean values and standard deviations of load carrying capacity of beams with open cross-sections are very close to each other. Therefore it can be said that the position of the incision has not any effect on the resulting load carrying capacity. The decrease of their load carrying capacity is caused by the decrease of torsion stiffness above all. The beams with open cross-sections are additionally stressed by twisting moment and by the bimoment. Warping of the beam with open cross-section represents the main cause of decreasing its load carrying capacity. The beam undergoes warping namely due to the fact that its axis is curved by random curvature of general shape. Let us remark that this problem would not be described satisfactorily in detail for a beam with imperfection having the shape of one half-vawe of the sine function. Random fields are indispensable for the study of this phenomenon. The study of the problems will be continued further on.

ACKNOWLEDGEMENT

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