

---

**Dita VOŘECHOVSKÁ<sup>1</sup>, Miroslav VOŘECHOVSKÝ<sup>2</sup>****ANALYTICAL AND NUMERICAL APPROACHES TO MODELLING  
OF REINFORCEMENT CORROSION IN CONCRETE****Abstract**

Corrosion of reinforcement in concrete is one of the most influencing factors causing the degradation of RC structures. This paper attempts at the application of an analytical and numerical approaches to simulation of concrete cracking due to reinforcement corrosion. At first, a combination with detailed analysis of two analytical models proposed by Liu and Weyers (1998) and Li et al. (2006) is suggested and presented. Four distinct phases of the corrosion process are identified and a detailed guide through the mathematical development is described. Next, numerical computations obtained with nonlinear finite element code are presented. The model features the state-of-the-art in nonlinear fracture mechanics modelling and the heterogeneous structure of concrete is modelled via spatially varying parameters of the constitutive law. Finally, the results of the analytical studies are compared to numerical computations and the paper concludes with the sketch of a real-life numerical example.

**Keywords**

Corrosion, durability, degradation, mathematical modelling.

**1 INTRODUCTION**

Corrosion of reinforcement embedded in concrete is an electrochemical process during which coupled anodic and cathodic reactions take place. Pore water functions as an electrolyte. The detrimental effect of corrosion is due to the fact that a rust product has a volume 2 - 6 times larger than the original steel. Consequently, it causes volume expansion, developing tensile stress in the surrounding concrete. Reinforcement corrosion takes place during the propagation period, and its rate is governed by the availability of water and oxygen on the steel surface. Due to corrosion, the effective area of the steel decreases and rust products grow, causing, at a certain stage, longitudinal cracking, and later, the spalling of concrete cover (delamination). Generally two types of corrosion are distinguished: the uniform (or general) type and the pitting (localized) type of corrosion. The subject of this paper is the uniform type of corrosion.

Modelling of concrete stressed by the rust products developed on the steel reinforcement concerns many authors. Bažant (1979 a,b) developed a physical model for steel corrosion in concrete sea structures formulated as an initial-boundary-value problem. Coupled 3D chemo-hygro-thermo-mechanical model based on microplane model is presented by Ožbolt et al. (2012). The authors apply the model for the calculation of the distribution of radial pressure and the prediction of crack pattern in concrete due to reinforcement corrosion. Böhner et al. (2010) modelled concrete cover cracking

---

<sup>1</sup> Ing. Dita Vořechovská, Ph.D., Institute of Structural Mechanics, Faculty of Civil Engineering, Brno University of Technology, Veveří 95, 602 00 Brno, Czech Republic, phone: (+420) 541 14 73 68, e-mail: vorechovska.d@fce.vutbr.cz.

<sup>2</sup> Doc. Ing. Miroslav Vořechovský, Ph.D., Institute of Structural Mechanics, Faculty of Civil Engineering, Brno University of Technology, Veveří 95, 602 00 Brno, Czech Republic, phone: (+420) 541 14 73 70, e-mail: vorechovsky.m@fce.vutbr.cz.

due to pitting corrosion of the reinforcement using finite element method based on fracture mechanics approach.

In this paper, a combination and detailed analysis of two analytical models proposed by Liu and Weyers (1998) and Li et al. (2006) is suggested and presented. Next, numerical computations obtained with a nonlinear finite element code which is based on the nonlinear fracture mechanics (NLFM) are introduced and subsequently combined with the spatial variation of concrete parameters reflecting the heterogeneous structure of concrete. Finally, the results of the analytical and numerical approaches are compared and a practical example is shown.

## 2 FORMULATION OF THE ANALYTICAL MODEL

The model presented here is a combination of analytical models proposed by Liu and Weyers (1998) and Li et al. (2006). As assumed in these models, concrete with an embedded reinforcing steel bar can be modelled as a thick-wall cylinder (Bažant, 1979 a,b, Pantazopoulou & Papoulia, 2001, Tepfers, 1979). This is shown schematically in Fig. 1.

According to Fig. 1, four different stages of reinforcement corrosion propagation can be identified. Firstly, stage I, when no corrosion is present yet, is illustrated in Fig. 1a. Next, in stage II, the porous zone on the reinforcement-concrete interface is filled by corrosion products and the surrounding concrete starts to be stressed due to corrosion (Fig. 1b). When the tangential stress in concrete exceeds its tensile strength, the crack initiates perpendicularly to the interface (Fig. 1c, stage III). After a certain time the crack propagates through the concrete cover (Fig. 1d) and we are able to measure the crack width on the concrete surface, stage IV.

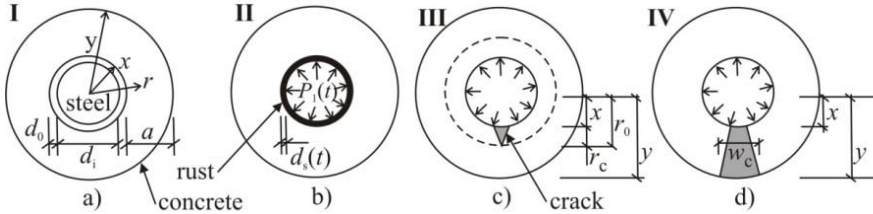


Fig. 1: Scheme of the corrosion induced concrete cracking process; partially adopted from (Li et al., 2006)

**Transition from stage I to stage II:** To determine the time to corrosion initiation  $t_i$ , which is a transition time from stage I to stage II; we may use a wide spectrum of models for concrete carbonation or chloride ingress. The descriptions of those models are not the subject of this paper; see e.g. (Teplý et al., 2007, Vořechovská et al., 2009). In the following text we assume  $t_i = 0$ .

**Transition from stage II to stage III:** A time to crack initiation,  $t_c$  (Fig.1c), which is a time of transition from stage II to stage III, can be estimated according to (Liu & Weyers, 1998) as:

$$t_c = \frac{W_{crit}^2}{2 \times 0.092 \left( \frac{1}{\alpha} \right) \pi d_i i_{corr}} \quad (1)$$

where:

- $d_i$  — is the initial diameter of the reinforcement bar [mm],
- $i_{corr}$  — is the corrosion current density, which is a measure of the corrosion rate [ $\mu\text{A}/\text{cm}^2$ ],
- $\alpha$  — is the coefficient related to the type of corrosion products [-] and

$W_{\text{crit}}$  – is the critical amount of corrosion products that generate the critical tensile stresses [mg/mm] and is defined as (Liu & Weyers, 1998):

$$W_{\text{crit}} = \frac{\rho_{\text{rust}} \pi \left[ d_i (d_{\text{s,crit}} + d_0) + 2d_0 d_{\text{s,crit}} \right]}{1 - \alpha \rho_{\text{rust}} / \rho_{\text{st}}} \quad (2)$$

where:

$\rho_{\text{rust}}$  – is the density of corrosion products [kg/m<sup>3</sup>],

$\rho_{\text{st}}$  – is the density of steel [kg/m<sup>3</sup>],

$d_0$  – is the thickness of the annular layer of concrete pores (i.e. a pore band) at the interface between the bar and the concrete [mm] and

$d_{\text{s,crit}}$  – is the critical thickness of a ring of corrosion products [mm] that is defined as (Liu & Weyers, 1998):

$$d_{\text{s,crit}} = \frac{a f_t}{E_{\text{ef}}} \left( \frac{x^2 + y^2}{x^2 - y^2} + \nu_c \right) \quad (3)$$

where:

$a$  – is the concrete cover [mm],

$f_t$  – is the tensile strength of concrete [MPa],

$E_{\text{ef}}$  – is the effective modulus of elasticity of concrete [GPa],  $E_{\text{ef}} = E_c / (1 + \varphi_{\text{cr}})$ ,

$E_c$  – is the elastic modulus of concrete [GPa],

$\varphi_{\text{cr}}$  – is the creep coefficient of concrete [-],

$\nu_c$  – is the Poisson ratio [-],

$x$  – is defined as  $x = (d_i + 2d_0)/2$  [mm] and

$y$  – is defined as  $y = a + (d_i + 2d_0)/2$  [mm].

The constant 0.092 is assumed according to Bhargava et al., 2005 (Liu & Weyers, 1998 assume 0.098).

**Transition from stage III to stage IV:** Once the time  $t_c$  is reached, the crack starts to develop and we need to determine whether it has already propagated to the surface (Fig. 1d). The crack divides the thick-wall cylinder into 2 co-axial cylinders: inner cracked and outer uncracked ones, as shown in Fig. 1c. For the outer uncracked concrete cylinder, the theory of elasticity still applies. Let us assume that the cracks in the inner cracked concrete cylinder are smeared and uniformly circumferentially distributed, see Fig. 5a (Pantazopoulou & Papoulia, 2001) and that concrete is a quasibrittle material. According to Bažant and Jirásek (2002) and Noll (1972) there exists a residual cracked surface along the radial direction depending on the tangential strain of that point; it is a function of the radial coordinate  $r$ . It is assumed in this model that the residual tangential stiffness is constant along the cracked surface, i.e. in the interval  $[x, r_0]$  and represented by  $\alpha_{\text{stiff}} E_{\text{ef}}$ , where  $\alpha_{\text{stiff}}$  ( $< 1$ ) is the tangential stiffness reduction factor. Li et al. (2005, 2006) present a formula for  $\alpha_{\text{stiff}}$  calculation which is, according to our opinion, not correct. We propose to use a similar formula based on Eq. (7.1.10) in (Bažant & Planas, 1998) in which the (nondimensional) stiffness reduction factor  $\alpha_{\text{stiff}}$  can be dependent on the average tangential strain  $\varepsilon_\theta$  over the cracked surface as follows (Fig. 2):

$$\alpha_{\text{stiff}}(\varepsilon_\theta) = \min \left\{ 1; \frac{f_t}{E_{\text{ef}}} \exp[-\gamma(\varepsilon_\theta - \varepsilon_\theta^s)] \right\} \quad (4)$$

where:

$\varepsilon_0^c$  — denotes the average tangential cracking strain [-] and

$\gamma$  — is a material constant that relates the tensile strength, fracture energy and crack spacing [-]. This variable controls the slope of the descendng part of the diagram demonstrated in Fig. 2.

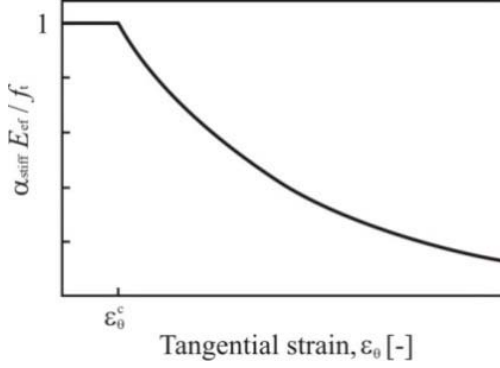


Fig. 2: Plot of the dependence of the residual stiffness  $\alpha_{stiff}$  on the tangential strain  $\varepsilon_0$ , see Eq. (4)

After the introduction of the constitutive relationship between the radial and tangential stresses and strains (Pantazopoulou & Papoulia, 2001), the stress equilibrium (Fenner, 1989), the boundary conditions for the concrete cylinder, and the continuity requirements and their combinations (see Li et al., 2006 for more details), we arrive at the following two implicit equations:

$$F_1(\alpha_{stiff}, r_0) = 0 = f_t - \frac{E_{ef}}{1 - \nu_c^2} \left[ (1 + \nu_c) c_1(r_0) + \frac{(1 - \nu_c) c_2(r_0)}{r_0^2} \right] \quad (5)$$

and

$$F_2(\alpha_{stiff}, r_0) = 0 = \frac{f_t \exp(-\gamma k_2)}{E_{ef} k_1} - \alpha_{stiff} \quad (6)$$

$$k_1 = \frac{\left( r_0^{\sqrt{\alpha_{stiff}}} - x^{\sqrt{\alpha_{stiff}}} \right) \left[ c_3(r_0) + c_4(r_0) / (x r_0)^{\sqrt{\alpha_{stiff}}} \right]}{\sqrt{\alpha_{stiff}} (r_0 - x)}$$

$$k_2 = k_1 - \frac{1}{r_0 - x} \int_x^{r_0} \left( c_1(\xi) + \frac{c_2(\xi)}{\xi^2} \right) d\xi$$

where:

$r_0$  — is the distance between the reinforcing bar centre and a crack tip (Fig. 1c) [mm] and

$c_1(r_0)$ ,  $c_2(r_0)$ ,  $c_3(r_0)$  and  $c_4(r_0)$  — are the coefficients (beyond the scope of this paper, see Li et al. 2006 for details).

If a simultaneous solution to Eqs. (5) and (6),  $r_0$  and  $\alpha_{stiff}$  can be found in the intervals:  $r_0 \in (x, y)$  and  $\alpha_{stiff} \in (0, 1)$ , the crack has not propagated to the surface yet (the process is still in stage III). We have programmed the solution of the problem and based on our experience, the Newton Raphson scheme is successful in solving the set of nonlinear Eqs. (6). A good starting point for the solver is the middle point of the intervals:  $r_0 = (x+y)/2$  and  $\alpha_{stiff} = 0.5$ . In the case that the solution cannot be found in the identified ranges of  $r_0$  and  $\alpha_{stiff}$ , the crack has already penetrated to the surface. By substituting  $r_0 \rightarrow y$  in Eqs. (5) and (6) we obtain a new set of nonlinear equations. Their simultaneous solution leads to finding  $\alpha_{stiff}$  that is needed for the computation of the crack width  $w_c$ :

$$w_c = \frac{4\pi d_s(t)}{(1-\nu_c)(x/y)^{\sqrt{\alpha_{stiff}}} + (1+\nu_c)(y/x)^{\sqrt{\alpha_{stiff}}} - \frac{2\pi y f_t}{E_{ef}}} \quad (7)$$

where:

$d_s(t)$  – is the thickness of a ring of corrosion products (Fig.1b) that can be determined from:

$$d_s = \frac{W_{rust}(t) \left( 1 - \frac{\alpha \rho_{rust}}{\rho_{st}} \right) - \rho_{rust} \pi d_i d_0}{\pi \rho_{rust} (d_i + 2d_0)} \quad (8)$$

where:

$W_{rust}(t)$  – is the mass of corrosion products [mg/mm].

Note that  $d_s$  is not correctly derived in (Li et al., 2006) and that is why our Eq. (8) differs from theirs. Obviously,  $W_{rust}(t)$  increases with time and according to Liu and Weyers (1998) can be determined from:

$$W_{rust}(t) = \sqrt{\frac{2 \times 0.092 \pi a}{\alpha} \int_{t_i}^t i_{corr}(t) dt} \quad (9)$$

The most complicated part of the approach is the solution of functions  $F_1(\alpha_{stiff}, r_0)$  and  $F_2(\alpha_{stiff}, r_0)$ . To help the reader to imagine how the two implicit functions look, their evolution with time is plotted in Fig. 3. For the time of 2 years and a set of realistic inputs the simultaneous solution of  $F_1(\alpha_{stiff}, r_0) = 0$  and  $F_2(\alpha_{stiff}, r_0) = 0$  exists within the above identified ranges of  $\alpha_{stiff}$  and  $r_0$  (stage III). The solution is illustrated by the intersection of two curves in the base, see bottom left in Fig. 3. In other words, if the solution exists, the crack still did not reach the concrete cover surface. In Fig. 3 right, we plot a situation for the time of three years: the two curves do intersect indicating crack propagation to the outer surface. The input data may be find in Table 1 in (Matesová & Vořechovský, 2006).

### 3 NUMERICAL MODEL

In this section an alternative to the above analytical solution is presented, namely numerical solution using finite element method. ATENA program (Červenka & Pukl, 2003) was used for the simulation of a nonlinear response of the corroded reinforcement in concrete. For modelling of a nonlinear behaviour a material constitutive model based on a smeared approach that can successfully describe the discrete crack propagation was applied. In particular, the fracture-plastic model named NLCEM (nonlinear cementitious) available in ATENA program is used. Concrete with reinforcement was modelled as 2D problem and according to the definition of the analytical model; thick-wall cylinder geometry was modelled (see Fig. 1a). The dimensions are:  $a = 30$  mm and  $d_i = 20$  mm. The thickness of the annular layer of concrete pores  $d_0$  was ignored, because this thickness is only important for the time analysis; it delays stress development (this void space is firstly filled by the corrosion products). Expansion of corrosion products was simulated by application of (negative) shrinkage of the reinforcement elements.

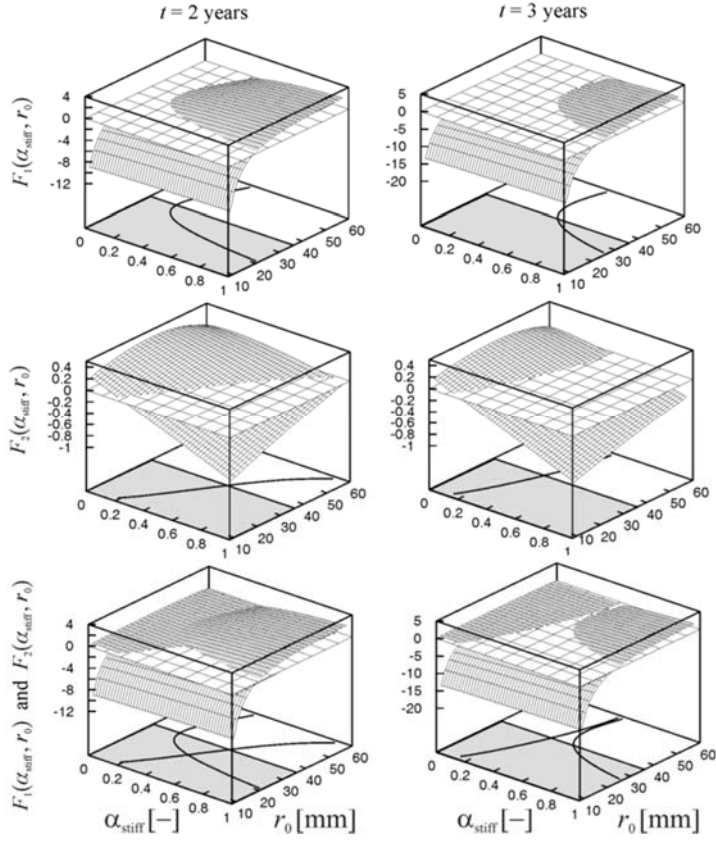


Fig. 3: Evolution of functions  $F_1(\alpha_{\text{stiff}}, r_0)$  and  $F_2(\alpha_{\text{stiff}}, r_0)$  from Eqs. (5) and (6) for exposure times 2 and 3 years ( $t_i = 0$ )

### 3.1 Deterministic model

At first the system was treated as deterministic to study the damage mechanisms and the development of stresses and cracks. Two extreme cases of boundary conditions were studied: without circuit restraint of the outer concrete face (free margins) and with a circuit restraint by applying the fixed hinge supports around the concrete face (see e.g. Fig. 5a, d for illustration of boundary conditions). The second type of boundary conditions is here to represent the case of ring embedded into a stiff surrounding material.

The major input parameters of the concrete constitutive law were: modulus of elasticity  $E = 30.32$  GPa, compressive strength  $f_c = 25.5$  MPa, tensile strength  $f_t = 2.317$  MPa and fracture energy  $G_F = 57.93$  N/m. The stress-opening law used is exponential according to Hordijk (1991). The standard crack band model (Bažant & Oh, 1983) was employed.

Several important variables have been monitored in the numerical calculations: the radial displacement  $d$  at the steel-concrete interface and stresses at three positions of concrete: (1) interface with steel, (2) the middle of concrete layer thickness and (3) outer concrete boundary. Two types of stresses were monitored at the three positions: the radial and tangential stresses ( $\sigma_r$  and  $\sigma_t$ ). Their dependence on the displacement  $d$  is plotted in Fig. 4 for both free and restraint boundary conditions.

For a detailed analysis of stress profiles in concrete presented in Fig. 5, three stages of crack development were chosen coinciding with  $d = 1, 3$  and  $6 \mu\text{m}$  (circled values from Fig. 4). The first stage ( $d = 1 \mu\text{m}$ ) is characterized by crack initiation from the interface for both free and restrained circuit boundary conditions; the peak tangential stress  $\sigma_{t1}$  equals the tensile concrete strength. In the case of free boundary conditions, the radial and tangential stresses differ only in the sign (as can be

easily predicted by a simple analytical computation), while in the case of restraint boundaries, a small pressure can be identified perpendicular to the outer concrete interface. The second and third stages record gradual crack growth; the cracks growth is suppressed by the constraint which is prescribed by the fixed hinge supports – see Fig. 5d-f.

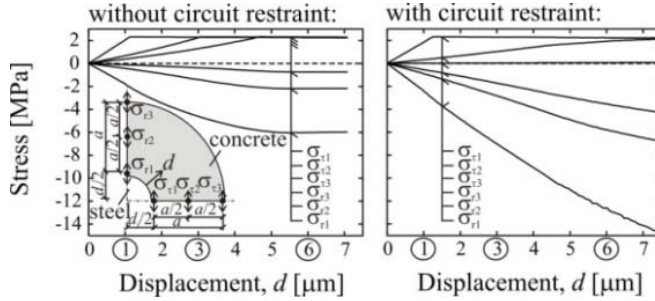


Fig. 4: Deterministic solution of the numerical models; comparison of restrained and non-restrained boundary conditions at the outer concrete boundary

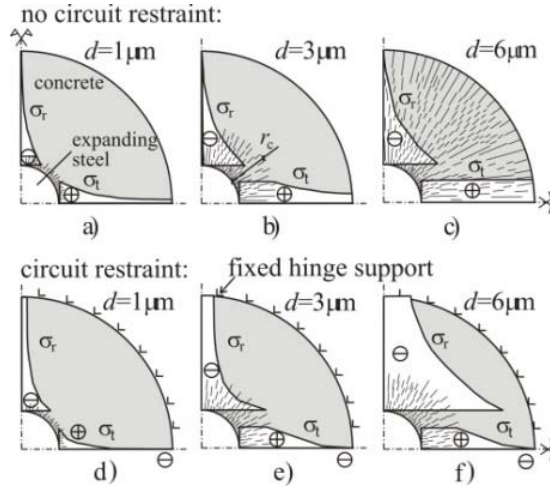


Fig. 5: Tangential and radial stresses development and cracks distribution in concrete cover

### 3.2 Stochastic model

The uniform crack distribution visible in Fig. 5 is not very realistic. In order to simulate the heterogeneous structure of concrete, the spatial variability of material was modelled by modifications of chosen input values of concrete. In reality, concrete does not have uniform properties over its volume and the spatial variability of properties can be suitably modelled by random fields. We remark, that such an approach automatically disturb the rotational stress symmetry and introduce damage initiation similarly to the real situation.

Two parameters of the constitutive law of concrete were selected to be randomized and their effects were studied separately. Namely, the modulus of elasticity  $E$  and the tensile strength  $f_t$  were randomized to trigger fracturing. The applied random fields were normally distributed with coefficient of variation 30% and 20% respectively. The mean value was taken from the deterministic analyses (section 3.1) to obtain consistent results. One of the most important properties of a random field is the autocorrelation structure defined through the autocorrelation function and the autocorrelation length. Briefly, the autocorrelation length is a parameter controlling the rate of spatial variability of the parameter; see (Vořechovský, 2004, 2008) for details on random field modelling.

In our analyses, the correlation lengths of 0.01 m were assumed in both directions together with a squared exponential autocorrelation function (isotropic correlation structure of the field). The autocorrelation length roughly coincides with the maximum aggregate size of concrete. For illustration of the random field and the scale of fluctuation see Fig. 6.

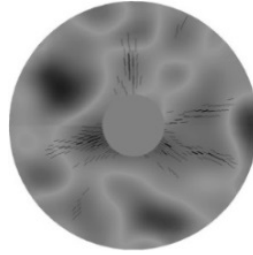


Fig. 6: Realization of a random field of local tensile strength  $f_t$  and cracks developed in the post peak stage of  $\sigma_{t1}$

The results of the stochastic nonlinear calculations are visualized through the dependence of tangential stress  $\sigma_{t1}$  on the radial extension of steel (displacement  $d$ ), see diagrams in Fig. 7. As can be seen, the randomization of the local  $E$  modulus does not affect the crack initiation stress (which still equals the concrete tensile strength) while the randomized strength influences the crack initiation stress while not affecting the initial overall stiffness. We remark, that due to the relatively small dimensions of concrete material with respect to the concrete dimensions, the structure is very ductile (diffused microcracking conveniently modelled as damage) and therefore the mechanism is closer to the parallel coupling of micro-bonds rather than a weakest link principle (see Vořechovský, 2004 for details).

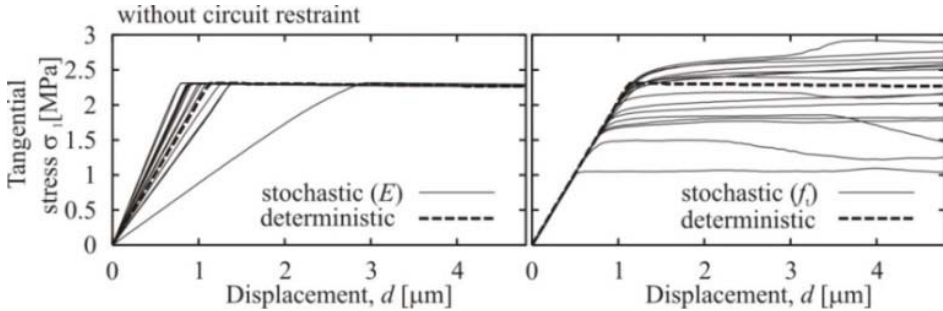


Fig. 7: Results of nonlinear stochastic simulations; comparison of spatial variability applied to modulus of elasticity  $E$  and tensile strength  $f_t$  of concrete

### 3.3 Comparison of analytical and numerical model

The comparison of both models was performed at the deterministic level. The numerical model without circuit boundary conditions was used as it coincides with the formulation of the analytical model. The comparison of both models was done through the crack development quantified by coordinate  $r_c$ , see Figs. 1c and 4b. The parameters identical for both numerical and analytical approaches are:  $d_i = 20$  mm,  $d_0 = 0$  mm,  $a = 30$  mm,  $f_t = 2.317$  MPa,  $E_{ef} = E = 30.32$  GPa and  $\nu_c = 0.2$ . The trend of crack length  $r_c$  in dependence on the displacement  $d$  is plotted in Fig. 8 together with tangential stresses  $\sigma_t$  obtained from numerical calculations. Note, that the displacement  $d$  monitored during numerical calculations does not coincide with the thickness of a ring of corrosion products  $d_s$  that is featured in the analytical approach. The measure of displacement  $d$  does not take into account the loss of steel due to rust production. For the purpose of the comparison of both models,  $d_s$  was recalculated to  $d$  through the steel and rust densities considered as  $\rho_{rust} = 3600$  kg/m<sup>3</sup>



and  $\rho_{st} = 7850 \text{ kg/m}^3$  by realizing that the total weight of both materials must be kept constant. The growth of  $d_s$  (or  $d$ ) is related to the time (see Eqs. 8 and 9); this relation depends mainly on the current density  $i_{corr}$ . A detailed time analysis is beyond the scope of this paper; only the nonlinear trend of  $d$  in time is sketched in the inset of Fig. 8 for  $i_{corr} = 1 \mu\text{A/cm}^2$ . Obviously, in the deterministic case, both models are comparable. However, the numerical model is much more flexible and its predictive capabilities are higher as it can easily accommodate advanced features such as the spatial variability of material parameters or more complex geometries. The latter becomes important when analyzing real-life examples as the one presented in the following section.

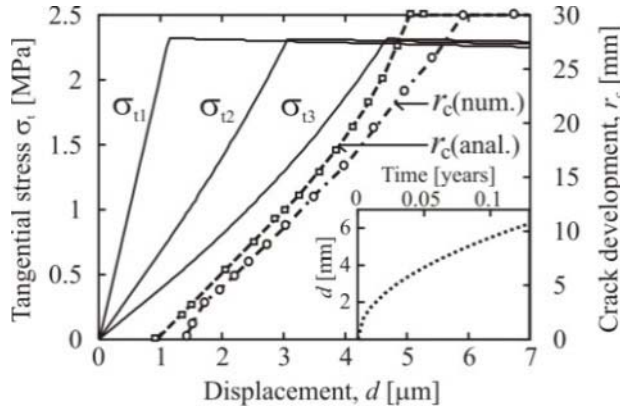


Fig. 8: Comparison of the analytical and numerical models through the crack length  $r_c$  together with tangential stresses obtained from the numerical simulation. The right bottom inset shows the time dependence of  $d$

#### 4 PRACTICAL APPLICATION

The thick wall cylinder geometry around the steel reinforcement is not a usual geometry of a real structure. To illustrate the real danger of steel corrosion in a reinforced concrete girder, we prepared a model of its lower part together with four reinforcing bars (20 mm thick). A uniform type of corrosion and the same corrosion rate for all four reinforcing bars were assumed. The crack development in concrete due to rust products of steel as predicted by the numerical model is sketched in Fig. 9. Note that in reality, opening of crack accelerates the corrosion progress because of easier transport of oxygen and water. The crack patterns agree well with the damage observed in real structures.

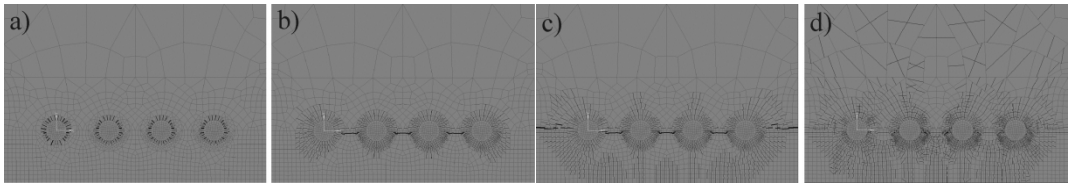


Fig. 9: Crack development due to corrosion in reinforced concrete beam

#### 5 CONCLUSIONS

The application of analytical and numerical approaches to simulation of concrete cracking due to corrosion of reinforcement was presented. At first, a combination with corrections of two existing analytical models by Liu and Weyers (1998) and Li et al. (2006) is proposed. Four distinct phases of the corrosion process are identified and the process is modelled by numerical computations obtained with the nonlinear finite element code. The numerical model features the state-of-the-art in nonlinear fracture mechanics and the heterogeneous structure of concrete is modelled via spatially varying

parameters of the constitutive law. These results are of high importance in durability based design of structures.

## ACKNOWLEDGEMENTS

The financial support from the specific university research project at Brno University of Technology, registered under the number FAST-S-13-2017 and the Project of the Czech Science Foundation number 13-19416J are gratefully acknowledged.

## REFERENCES

- [1] BAŽANT, Z.P. (1979a). Physical Model for Steel Corrosion in Concrete Sea Structures – Theory. *Journal of Structural Division, ASCE*, 105, (ST6), 1137-1153.
- [2] BAŽANT, Z.P. (1979b). Physical Model for Steel Corrosion in Concrete Sea Structures – application. *Journal of Structural Division, ASCE*, 105, (ST6), 1154-1166. Disc. 1980, 1194-1195.
- [3] BAŽANT, Z.P., & OH, B.H. (1983). Crack band theory for fracture of concrete. *Materials and Structures RILEM*, 16, 155-177.
- [4] BAŽANT, Z.P., & JIRÁSEK, M. (2002). Nonlocal Integral Formulations of Plasticity and Damage: Survey and Progress. *Journal of Engineering Mechanics, ASCE*, 128 (11), 1119-1149.
- [5] BAŽANT, Z.P., & PLANAS, J. (1998). *Fracture and Size Effect in Concrete and Other Quasibrittle Materials*, CRC Press.
- [6] BHARGAVA, K., GHOSH, A.K., MORI, Y. & RAMANUJAM, S. (2005). Modeling of time to corrosion-induced cover cracking in reinforced concrete structures. *Cement and Concrete Research*, 35, 2203-2218.
- [7] BOHNER, E., MÜLLER, H.S., & BRÖHL, S. (2010). *Investigations on the mechanism of concrete cover cracking due to reinforcement corrosion*. In: Oh, B.H. et al. (eds.), *Fracture Mechanics of Concrete and Concrete Structures – Assessment, Durability, Monitoring and Retrofitting of Concrete Structures*, Korea Concrete Institute, Seoul.
- [8] ČERVENKA, V., & PUKL, R. (2003). *ATENA Program documentation*, theory guide, Červenka Consulting, Prague.
- [9] FENNER, R.T. (1989). *Mechanics of Solids*. Blackwell Scientific Publications, Oxford.
- [10] HORDIJK, D.A. (1991). *Local Approach to Fatigue of Concrete*. PhD thesis, Delft University of Technology, The Netherlands.
- [11] LI, C.Q., LAWANWISUT, W., ZHENG, J.J., & KIJAWATWORAWET, W. (2005). Crack width due to corroded bar in reinforced concrete structures. *International Journal of Materials and Structural Reliability*, 3(2), 87-94.
- [12] LI, C.Q., MELCHERS, R.E., & ZHENG, J.J. (2006). An analytical model for corrosion induced crack width in reinforced concrete structures. *ACI Structural Journal*, V. 103, No. 4, 479-482.
- [13] LIU, Y., & WEYERS, R.E. (1998). Modeling the time-to-corrosion cracking in chloride contaminated reinforced concrete structures. *ACI Material Journal*, V. 95, No. 6, 675-681.
- [14] MATESOVÁ, D. & VOŘECHOVSKÝ, M. (2006). *Reinforcement corrosion in concrete: analytical approach to modelling*. In: *Pravděpodobnost porušování konstrukcí PPK 2006*, Brno, Czech Republic.
- [15] NOLL, W. (1972). A New Mathematical Theory of Simple Materials. *Arch. Ration. Mech. Anal.*, 48, 1-50.

- [16] OŽBOLT, J., ORŠANIĆ, F., BALABANIĆ, G., & KUŠTER, M. (2012). Modeling damage in concrete caused by corrosion of reinforcement: coupled 3D FE model. *International Journal of Fracture*, V. 178, No. 1-2, 233-244.
- [17] PANTAZOPOULOU, S.J., & PAPOULIA, K.D. (2001). Modeling Cover-Cracking due to Reinforcement Corrosion in RC Structures. *Journal of Engineering Mechanics*, ASCE, 127 (4), 342-351.
- [18] TEPFERS, R. (1979). Cracking of Concrete Cover Along Anchored Deformed Reinforcing Bars. *Magazine of Concrete Research*, 31, (106), 3-12.
- [19] TEPLÝ, B., MATESOVÁ, D., CHROMÁ, M., & ROVNANÍK, P. (2007). *Stochastic degradation models for durability limit state evaluation: SARA – Part VI*. In: 3rd International Conference on Structural Health Monitoring of Intelligent Infrastructure (SHMII-3 2007), Vancouver, Canada.
- [20] VOŘECHOVSKÁ, D., CHROMÁ, M., PODROUŽEK, J., ROVNANÍKOVÁ, P., & TEPLÝ, B. (2009). Modelling of Chloride Concentration Effect on Reinforcement Corrosion. *Computer-Aided Civil and Infrastructure Engineering*, 24, 446-458.
- [21] VOŘECHOVSKÝ, M. (2004). *Stochastic fracture mechanics and size effect*, Brno University of Technology, Brno, Czech Republic.
- [22] VOŘECHOVSKÝ, M. (2008). Simulation of simply cross correlated random fields by series expansion methods. *Structural safety*, 30 (4), 337-363.

#### **Reviewers:**

Ing. Petr Konečný, Ph.D., Department of Structural Mechanics, Faculty of Civil Engineering, VŠB-Technical University of Ostrava.

Ing. Miroslav Sýkora, Ph.D., Department of Structural Reliability, Klokner Institute, Czech Technical University in Prague.