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STOCHASTIC ANALYSIS OF CYLINDRICAL SHELL

Abstract

The paper deals with some chosen aspects of stochastic structural analysis and its application in the engineering practice. The main aim of the study is to apply the generalized stochastic perturbation techniques based on classical Taylor expansion with a single random variable for solution of stochastic problems in structural mechanics. The study is illustrated by numerical results concerning an industrial thin shell structure modeled as a 3-D structure.

Keywords

Stochastic perturbation technique, finite element method, shell structure.

1 INTRODUCTION

In the paper the finite element method has been applied to the analysis of variation of structural parameters due to uncertainties of these parameters. The so-called stochastic finite element method has been used on the basic of the 2nd-order perturbation method [1-5]. This non-statistical approach is numerically much more efficient than a statistical approach, such as Monte Carlo simulation. A major advantage of the statistical finite element approach is that only the first two moments need to be known. Moreover a large number of samples are required in statistical approaches.

2 FORMULATION OF THE PROBLEM

2.1 Second moment perturbation method

The basic concept of second moment perturbation method (SMPM) is descended from the linear transform of a random variable described in term of a powers series expansion [1, 2, 4]. Let us consider a vector $\mathbf{a} = \{a_r\}$, $r = 1, 2, \ldots, \hat{r}$, are assumed to be time-independent random variables, specified by the first two associated central moments – means $\bar{\mathbf{a}} = \{\bar{a}_r\}$ and cross-covariances $Cov(a_r, a_s)$; $r, s = 1, 2, \ldots, \hat{r}$. Expanding the random variables $\{x_i(\mathbf{a})\}$ around the argument means $\{\bar{a}_r\}$ via Taylor series and retaining terms up to second order yields

$$x_i(a_r) = x_i(\bar{a}_r) + \sum_{r=1}^{\hat{r}} \frac{\partial x_i}{\partial a_r}\Big|_{a=\bar{a}} (a_r - \bar{a}_r) + \frac{1}{2} \sum_{r=1}^{\hat{r}} \frac{\partial^2 x_i}{\partial a_r \partial a_s}\Big|_{a=\bar{a}} (a_r - \bar{a}_r)(a_s - \bar{a}_s) \tag{1}$$

The zero, first and mixed second derivatives od $\{x_i\}$ with respect to $\{a_r\}$ at $\{\bar{a}_r\}$ are constant valued. The mean values $\{\bar{x}_i\} = \{E[x_i]\}$, $i = 1, 2, \ldots, \hat{i}$, are expressed as

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$$E[x_i] = x_i(\bar{a}_r) + \sum_{r=1}^{\hat{r}} \frac{\partial x_i}{\partial a_r}|_{a=\bar{a}} E\left[\overbrace{(a_r - \bar{a}_r)}^{=0}\right] + \frac{1}{2}\sum_{r=1}^{\hat{r}} \frac{\partial^2 x_i}{\partial a_r \partial a_s}|_{a=\bar{a}} E[(a_r - \bar{a}_r)(a_s - \bar{a}_s)]$$

$$= x_i(\bar{a}_r) + \frac{1}{2}\sum_{r=1}^{\hat{r}} \frac{\partial^2 x_i}{\partial a_r \partial a_s}|_{a=\bar{a}} Cov(a_r, a_s) \tag{2}$$

or, more concisely

$$\bar{x}_i = x_i(\bar{a}_r) + \frac{1}{2} x_i^{(2)}(\bar{a}_r) \tag{3}$$

where the symbolic symbol

$$(\circ)^{(2)} = \sum_{r=1}^{\hat{r}} \frac{\partial^2(\circ)}{\partial a_r \partial a_s}|_{a=\bar{a}} Cov(a_r, a_s) \tag{4}$$

To determine the cross-covariances $Cov(x_i, x_j)$ we note, by (1) and (3), that the spreads of the random variables $\{x_i\}$ about their means $\{\bar{x}_i\}$ are

$$x_i - \bar{x}_i = \sum_{r=1}^{\hat{r}} \frac{\partial x_i}{\partial a_r}|_{a=\bar{a}} (a_r - \bar{a}_r) + \frac{1}{2}\sum_{r=1}^{\hat{r}} \frac{\partial^2 x_i}{\partial a_r \partial a_s}|_{a=\bar{a}} (a_r - \bar{a}_r)(a_s - \bar{a}_s) - \frac{1}{2} x_i^{(2)} \tag{5}$$

and

$$Cov(x_i, x_j) = E[(x_i - \bar{x}_i)(x_j - \bar{x}_j)]$$

$$\approx \sum_{r,s=1}^{\hat{r}} \left[\frac{\partial x_i \partial \mathbf{x}}{\partial a_r \partial a_s} - \frac{1}{4}\left(\frac{\partial^2 x_i}{\partial a_r \partial a_s} x_j^{(2)}\right)\left(x_i^{(2)} \frac{\partial^2 x_j}{\partial a_r \partial a_s}\right)\right]_{a=\bar{a}} E[(a_r - \bar{a}_r)(a_s - \bar{a}_s)] + \frac{1}{4} x_i^{(2)} x_j^{(2)}$$

$$= \sum_{r,s=1}^{\hat{r}} \frac{\partial x_i \partial x_j}{\partial a_r \partial a_s}|_{a=\bar{a}}\; Cov(a_r - \bar{a}_r)(a_s - \bar{a}_s) - \frac{1}{4}\left(x_i^{(2)} x_j^{(2)} + x_i^{(2)} x_j^{(2)}\right) + \frac{1}{4} x_i^{(2)} x_j^{(2)} \tag{6}$$

or

$$Cov(x_i, x_j) = \sum_{r=1}^{\hat{r}} \frac{\partial x_i \partial x_j}{\partial a_r \partial a_s}|_{a=\bar{a}} Cov(a_r, a_s) - \frac{1}{4} x_i^{(2)} x_j^{(2)}\,. \tag{7}$$

The first two moments (3) and (7) are second-order. In comparison with conventional statistical approaches, Monte Carlo simulation for instance, the drawbacks of the non-statistical SMPM are that (i) random variables $\{x_i\}$ must satisfy the conditions for small fluctuation and for continuity at $\{\bar{a}_r\}$, and (ii) only first two probabilistic moments can be given on output. On the other hand, advantages of SMPM are significant, since (a) the assumption of the normal distribution (even homogeneity) for $\{x_i\}$ is not necessarily needed, (b) only the first two moment for $\{a_r\}$ are required on input, and (c) with the same-order accuracy only $o(\hat{r})$ equation system to be solved in SMPM when compared with $o(\hat{r}^3)$ corresponding systems sampled in Monte Carlo simulation.

2.2 Hierarchical equations

Hierarchical system for the multidegree-of-fredom system describing structural static response with stiffness matrix **K**, displacement vector **q** and load vector **Q** is

$$\mathbf{K}^0 \mathbf{q}^0 = \mathbf{Q}^0 \tag{8}$$

$$\mathbf{K}^0 \mathbf{q}^{,r} = \mathbf{Q}^{,r} - \mathbf{K}^{,r} \mathbf{q}^0 \qquad r = 1, 2, \ldots, \hat{r} \tag{9}$$

$$\mathbf{K}^0 \mathbf{q}^{(2)} = \sum_{r,s=1}^{\hat{r}} \left(\mathbf{Q}^{,rs} - 2\mathbf{K}^{,r} \mathbf{q}^{,s} - \mathbf{K}^{,rs} \mathbf{q}^0\right) Cov(a_r, a_s) \tag{10}$$

where the symbols $(\circ)^0$, $(\circ)^{,r}$ and $(\circ)^{,rs}$ denote the values of the zero, first and mixed second partial derivatives $(\circ)$ with respect to $\{a_r\}$ at $\{\overline{a}_r\}$, respectively.

3 EXAMPLE

In the example a thin shell structure is considered. Fig. 1 shows the half of a cylindrical shell clamped at boundaries under uniformly distributed pressure $p = 15\text{kN/m}^2$. The remaining input data are: radius $R = 2.5\text{m}$, length $L = 12\text{m}$, Young modulus $E = 30\text{MPa}$, Poisson ratio $\nu = 0.2$. The expectation, correlation function and coefficient of variation of the shell thickness are assumed as:

$$E(t) = t_0 = 0.05 \qquad \mathrm{Cov}(t_r, t_s) = \vartheta \exp\left[-\left|x_r - x_0\right|/\lambda\right]\left[-\left|y_r - y_0\right|/\lambda\right]$$

$$\vartheta = 1.5/RL, \qquad \lambda = 2.5/RL, \qquad \alpha = 0.05; 0.10; 0.15.$$

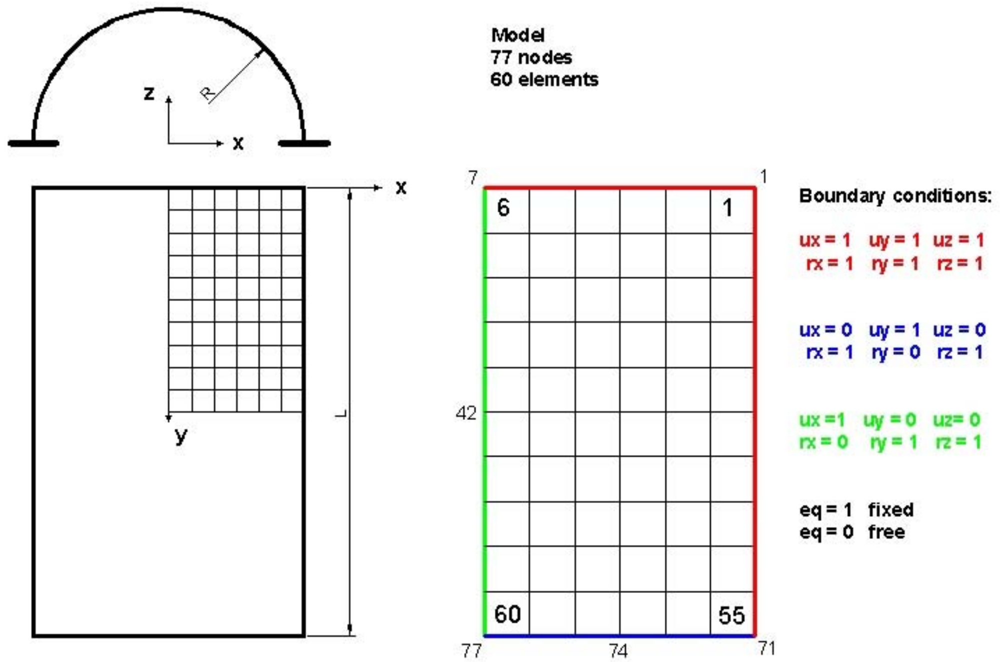

Fig. 1: 60-element shell with mesh grid

Due to symmetry only one-quarter of shell is considered. The finite element mesh include 60 rectangular elements (60 random design variables), and total number of degrees of freedom is 313.

The main motivation behind an application of the generalized perturbation technique is to eliminate the restriction on the input second probabilistic moments to be smaller than 0,15 and impossibility of reliable computations of higher than the second probabilistic moments for the output. Tab. 1 and Fig. 2 give the computed values of expectations and standard deviations for different random thickness shell.

Tab. 1: Expectations and standard deviations displacement q_Z (in symmetry blue line Fig. 1)

Angle	Deterministic	Expectation q_Z			Std. Dev. q_Z		
		$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.15$	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.15$
90°	1.22e-04	1.35e-04	1.74e-04	2.39e-04	3.81e-05	7.63e-05	1.14e-04
75°	1.44e-04	1.58e-04	2.00e-04	2.71e-04	3.86e-05	7.72e-05	1.16e-04
60°	1.84e-04	2.01e-04	2.50e-04	3.32e-04	4.24e-05	8.48e-05	1.27e-04
45°	1.92e-04	2.09e-04	2.59e-04	3.42e-04	4.50e-05	9.00e-05	1.35e-04
30°	1.40e-04	1.53e-04	1.90e-04	2.52e-04	3.61e-05	7.23e-05	1.08e-04
15°	0.56e-04	0.61e-04	0.77e-04	1.02e-04	1.63e-05	3.26e-05	0.49e-04
0°	0	0	0	0	0	0	0

4 CONCLUSIONS

In the stochastic perturbational analysis we deal with one system of the zeroth-order equations, one system of the first-order equations for each of the random variables and one system of the second-order equations. This non-statistical approach does not restrict the analysis to some limits of random fields as in the statistical techniques; it is applicable to both the homogeneous and nonhomogeneous random fields and a normal approximation is not necessarily needed. The restriction of small uncertainties in random variables, being inherent of the mean-point perturbation procedure, is seemingly eliminated by the check-point perturbation scheme in which the point of the system is perturbated around its parameterized variables.

With the transformation from correlated random variables to uncorrelated variables and by using only dominant part of the transformed set, the algorithms worked out are effective even for PC-based stochastic analysis of large-scale systems with acceptable computations cost. Since almost all operations related to random quantities can be carried out by the procedures for deterministic calculations the algorithms developed can be immediately adapted to existing deterministic finite element programs.

REFERENCES

[1] LIU, W.K., BELYTSCHKO, T., MANI, A. *Random field finite elements*, Int. J. Num. Meth. Eng., 1986, vol. 23, issue 10, pp. 1831-1845 (15 p). ISSN 1097-0207

[2] KLEIBER, M., HIEN, T.D. *The Stochastic Finite Element Method*. Wiley, 1992. ISBN 047193626X. 322 p.

[3] GRZYWIŃSKI, M., SŁUŻALEC, A. *Stochastic equations of rigid-thermo-viscoplasticity in metal forming process*, Int. J. Eng. Science, 2002, vol. 40, issue 4, pp. 367-383 (17 p). ISSN 0020-7225

[4] GRZYWIŃSKI, M., HIEN, T.D. Stochastyczna wrażliwość konstrukcji kratowych. In: TARNOWSKI, W., KICZKOWIAK, T. (red.) *Polioptymalizacja i Komputerowe Wspomaganie Projektowania*, 2008, pp. 35-40 (6 p). ISBN 8373651527.

[5] POKORSKA I., A sensitivity analysis of powder forging processes, *Structural and Multidisciplinary Optimization*, 2008, 37, 1, pp. 77-89 (13 p). ISSN 16151488

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