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Kamila KOTRASOVÁ[1]

FLUID IN RECTANGULAR TANK – FREQUENCY ANALYSIS

Abstract

Ground-supported tanks are used to store a variety of liquids. During earthquake activity the liquid exerts impulsive and convective pressures (sloshing) on the walls and bottom of the rectangular tank. This paper provides theoretical background for analytical calculating of circular frequencies and hydrodynamic pressures developed during an earthquake in rectangular container. Analytical results of first natural frequency are compared with experiment.

Keywords

Rectangular tank, fluid, frequency, experiment.

1 INTRODUCTION

Seismic event is certainly one of the most critical external events regarding safety of industrial plants, as demonstrated by recent earthquakes. If industrial facilities store large amount of hazardous materials, accidental scenarios as fire, explosion or toxic dispersion may be triggered, thus possibly involving working people within the installation, population living in close surrounding or in urban area where the industrial installation is located. Liquid storage tanks are considered essential lifeline structures. Large-capacity ground-supported tanks are used to store a variety of liquids, e.g. water for drinking and fire fighting, petroleum, chemicals, and liquefied natural gas. Satisfactory performance of tanks during strong ground shaking is crucial for modern facilities. Tanks that were inadequately designed or detailed have suffered extensive damage during past earthquakes. Knowledge of pressures and forces acting on the walls and bottom of containers during an earthquake and frequency properties of containers and fluid are important for good analysis and design of earthquake resistant structures/facilities – tanks.

2 FLUID IN RECTANGULAR TANK DURING EARTHQUAKE

For tanks, walls of which can be assumed as rigid, a solution of the Laplace equation for horizontal excitation can be obtained in a form, so that the total pressure is again given by the sum of impulsive and convective pressures by use of absolute summation rule:

$$p_{HDw} = p_{HDIw} + p_{HDCw}. \tag{1}$$

Consider a rectangular container as shown in Fig. 1, and at the instant under consideration let the surface of the fluid be horizontal and let the walls of the container have a horizontal acceleration $\ddot{u}_o$ in the x - direction.

Let it be required to find the pressures on the walls of the container due to the acceleration $\ddot{u}_o$. Let the fluid have a depth H, a length $2L$ and a unit thickness, Fig. 1a. It is seen that the action of the fluid is similar to that which would be obtained if the horizontal component of fluid velocity $\dot{u}$ were

[1] Ing. Kamila Kotrasová, Ph.D., Department of structural mechanics, Faculty of Civil Engineering, Technical University of Košice, Vysokoškolská 4, 042 00 Košice, Slovak Republic, phone: (+421) 55 602 4294, e-mail: kamila.kotrasova@tuke.sk.

independent of the y coordinate; that is, imagine the fluids to be constrained by thin, massless, vertical membranes free to move in the x – direction, and let the membranes be originally spaced a distance dx apart.

When the walls of the container are given acceleration, the membranes will be accelerated with the fluid and the fluid will be squeezed vertically with respect to the membranes.

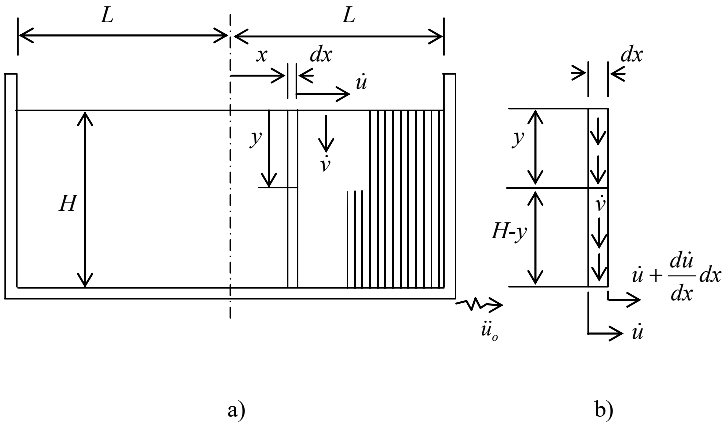

Fig. 1: Rectangular tank is filled with fluid

As shown in Fig. 1b, since the fluid is restrained between two adjacent membranes, the vertical velocity $\dot{v}$ is dependent on the horizontal velocity $\dot{u}$ according to

$$\dot{v} = \left(H - y\right)\frac{\mathrm{d}\dot{u}}{\mathrm{d}x}. \tag{2}$$

This is an equation specifying the constraint on the fluid flow. As the fluid is considered incompressible, it follows that the acceleration $\ddot{v}$ is proportional to the velocity $\dot{v}$ and the acceleration $\ddot{u}$ is proportional to the velocity $\dot{u}$, and the pressure in the fluid between two membranes is given by the standard hydrodynamic equation:

$$\frac{\partial p}{\partial y} = -\rho\ddot{v}, \tag{3}$$

where ρ is density of the fluid.

The acceleration $\ddot{u}_o$ thus produces an increase of hydrodynamic impulsive pressure on one wall and a decrease of pressure on the other wall of

$$p_{HDIw} = \rho\,\ddot{u}_0 H\left(\frac{y}{H} - \frac{1}{2}\left(\frac{y}{H}\right)^2\right)\sqrt{3}\tanh\sqrt{3}\,\frac{L}{H}. \tag{4}$$

The effect of the impulsive pressures is to excite the fluid into oscillations. To examine the fundamental mode of vibration, consider the fluid to be constrained between rigid membranes that are free to rotate as shown in Fig. 2.

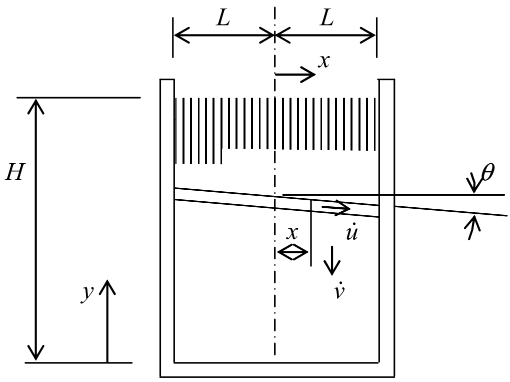

Fig. 2: Rectangular tank are filled with fluid

The constraint is described by the following equations:

$$\dot{u} = \frac{L^2 - x^2}{2} \frac{\mathrm{d}\dot{\theta}}{\mathrm{d}y}, \tag{5}$$

$$\dot{v} = \dot{\theta} z. \tag{6}$$

The pressure in the fluid is given by

$$\frac{\partial p}{\partial x} = -\rho \ddot{u}, \tag{7}$$

$$p = -\rho \frac{L^3}{2} \left(\frac{x}{2} - \frac{1}{3} \left(\frac{x}{L} \right)^3 \right) \frac{\mathrm{d}\ddot{\theta}}{\mathrm{d}x}. \tag{8}$$

The equation of motion of a slice of the fluid is

$$\int_{-l}^{+l} \frac{\partial p}{\partial y} \mathrm{d}y\, x\, \mathrm{d}x = -\rho \frac{\left(2L^3\right)}{12} \ddot{\theta}\, \mathrm{d}y. \tag{9}$$

The solution of this equation, with the boundary conditions appropriate to the problem, is for sinusoidal oscillations

$$\theta = \theta_0 \frac{\sinh \sqrt{\frac{5}{2}} \frac{y}{L}}{\sinh \sqrt{\frac{5}{2}} \frac{H}{L}} \sin \omega t. \tag{10}$$

This specifies the oscillation of the fluid. To determine the natural frequency of vibration, the maximum kinetic energy, W_K, is equated to the maximum potential energy, W_P.

$$W_K = \int_0^h \int_{-l}^{+l} \frac{1}{2} \rho \left(u^2 + v^2\right) \omega^2 \sin^2 \omega t\, \mathrm{d}x \mathrm{d}y, \tag{11}$$

$$W_P = \int_{-l}^{+l} \frac{1}{2} \rho g x^2 \sin \omega t\, \mathrm{d}x. \tag{12}$$

This gives

$$\omega^2 = \frac{g}{L}\sqrt{\frac{5}{2}} \tanh \sqrt{\frac{5}{2}}\frac{H}{L}. \tag{13}$$

The circular frequencies are then for the nth mode

$$\omega_n^2 = \frac{g}{L} n\sqrt{\frac{5}{2}} \tanh n\sqrt{\frac{5}{2}}\frac{H}{L}. \tag{14}$$

The hydrodynamic convective pressures are given by

$$p_{HDCw1} = \left(\rho \frac{L^3}{3}\sqrt{\frac{5}{2}} \frac{\cosh \sqrt{\frac{5}{2}}\frac{x}{L}}{\sinh \sqrt{\frac{5}{2}}\frac{H}{L}} \right) \omega^2 \theta_0 \sin \omega t \cdot \tag{15}$$

3 EXPERIMENTAL ANALYSIS OF FLUID IN RECTANGULAR TANK

The experiment was made with a rectangular tank with inner ground parameters 192 mm x 392 mm and height 242 mm, made of glass. The tank was filled with water by using potassium permanganate; the height of filling of water was 50 mm. The container was excited by horizontal harmonious motion of various frequencies with amplitudes of 5 mm and 10 mm (see Fig. 3).

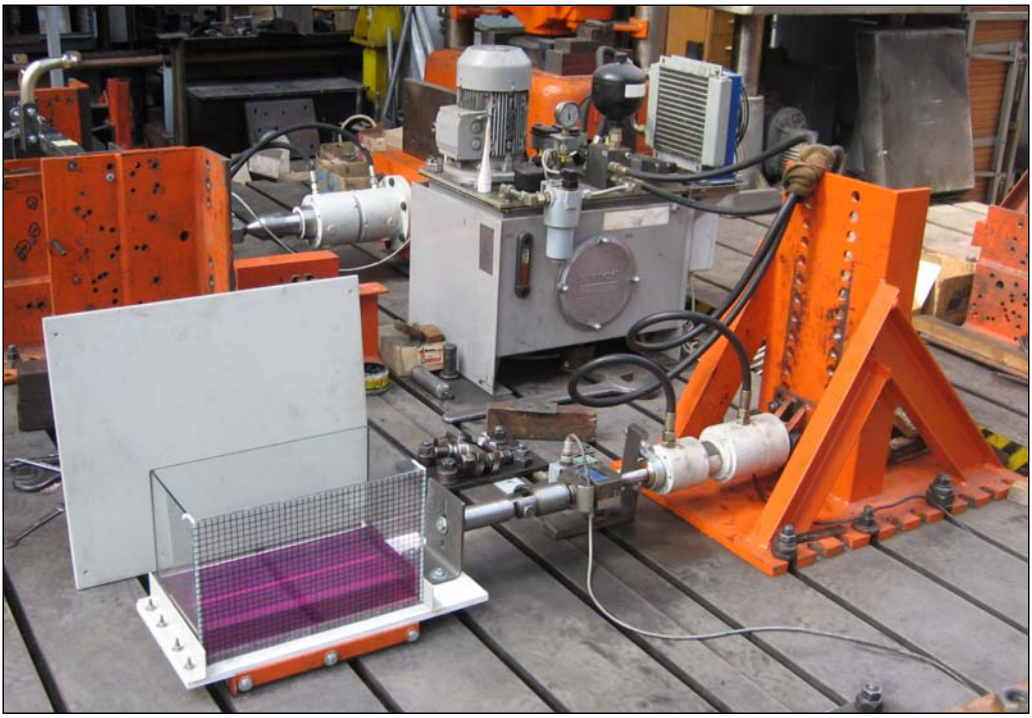

Fig. 3: View of experiment place

4 RESULTS AND CONCLUSION

The first natural frequencies were calculated by analytical solution equation (14). Fig. 4 shows the natural frequencies in [Hz] for realized experiment depend of height of filling.

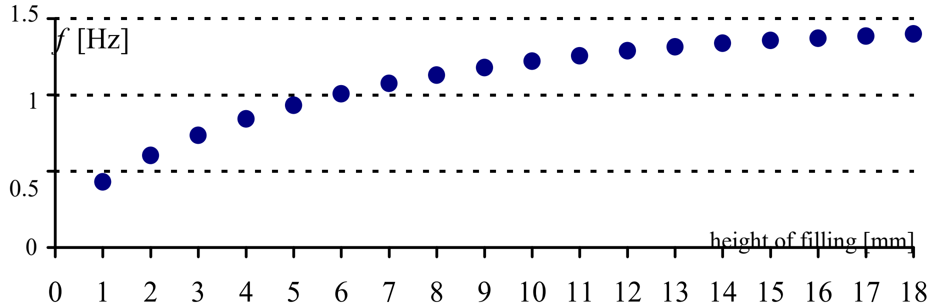

Fig. 4: The first natural frequencies depend of height of filling

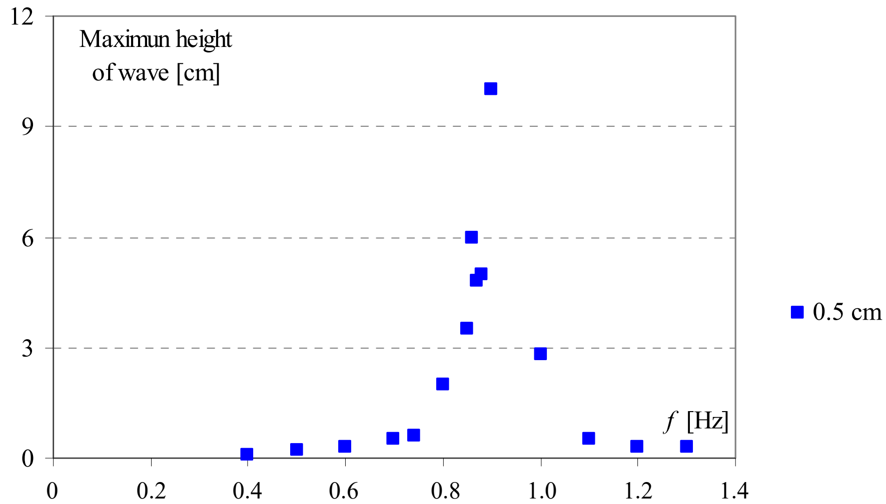

Fig. 5: Maximum heights of wave in [cm]

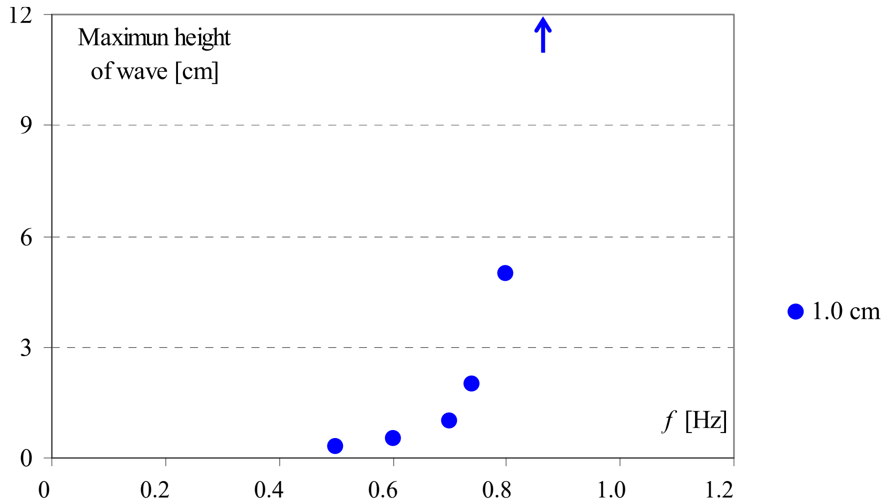

Fig. 6: Maximum heights of wave in [cm]

For a rectangular tank with inner parameters 192 mm x 392 mm and height 242 mm, the tank was filled with water to the height of 50 mm, the natural frequency is given $f_1 = 0.875$ Hz, $f_2 = 1.238$ Hz, and more ..., there were calculated by using of equation (14).

Fig. 5 shows the maximum heights of waves for 50 mm filling of water, by various exciting frequencies with 5 mm amplitude, in dependency from frequencies in [Hz]. Fig. 6 shows the maximum heights of waves for 50 mm filling of water, by various exciting frequencies with 10 mm amplitude, in dependency from frequencies in [Hz]. Fig. 5 shows that the maximum height of wave of water for 50 mm filling of water with 5 mm amplitude is by exciting frequency 0.86 Hz. The maximum height of wave is 60 mm from original free surface of fluid, it is 110 mm from bottom of tank (filling of water is 50 mm). It is corresponding with first natural frequency, which is given $f_1 = 0.875$ Hz by (14). From Fig. 5 is seen, that second natural frequency $f_2 = 1.238$ Hz isn't visible. Fig. 6 shows that sloshing out of water (blue arrow) is by exciting frequency 0.85 Hz for 50 mm filling of water and 5 mm amplitude. It is corresponding with first natural frequency, it is given $f_1 = 0.875$ Hz by (14).

First natural frequency calculated by analytical solution, equation (14), was compared with the experiment, see Tab. 1.

Tab. 1: Comparing of first natural frequencies

	Analytical solution	Experiment	
		5 mm amplitude	10 mm amplitude
First natural frequency in [Hz]	0.8754	0.86	0.85

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Reviewers:

Prof. Ing. Juraj Králik, PhD., Department of Structural Mechanics, Faculty of Civil Engineering, Slovak University of Technology in Bratislava.

Ing. Vladimíra Michalcová, Ph.D., Department of Structural Mechanics, Faculty of Civil Engineering, VŠB-Technical University of Ostrava.