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Martin PSOTNÝ[1]

NONLINEAR ANALYSIS OF BUCKLING & POSTBUCKLING

Abstract

The stability analysis of slender web loaded in compression was presented. To solve this problem, a specialized computer program based on FEM was created. The nonlinear finite element method equations were derived from the variational principle of minimum of potential energy. To obtain the nonlinear equilibrium paths, the Newton-Raphson iteration algorithm was used. Corresponding levels of the total potential energy were defined. The peculiarities of the effects of the initial imperfections were investigated. Special attention was focused on the influence of imperfections on the post-critical buckling mode. The stable and unstable paths of the nonlinear solution were separated. Obtained results were compared with those gained using ANSYS system.

Keywords

Stability, postbuckling, geometric nonlinear theory, initial imperfection, finite element method, Newton-Raphson method, arc-length method.

1 INTRODUCTION

The snap-through effect means a sudden modal change in the buckling surface of a slender web. Even in the case when the snap-through of the slender web does not mean the collapse of the structure, we consider it to be a negative phenomenon. In the presented paper we try to explain the behaviour of the snap-through of the slender web loaded in compression [1]. The geometrically nonlinear theory represents a basis for the reliable description of the postbuckling behaviour of the slender web. The result of the numerical solution represents a lot of the load versus displacement paths. Except the presentation of the different load-displacement paths the level of the total potential energy has been evaluated as well.

2 THEORY

Let us assume a rectangular slender web simply supported along the edges (Fig. 1) with the thickness t. The displacements of the point of the neutral surface are denoted $\boldsymbol{q} = [u, v, w]^T$ and the related load vector is $\boldsymbol{p} = [p_x, 0, 0]^T$.

We assume the so called von Kármán theory, when the out of plane (plate) displacements (w) are much bigger as in-plane (web) displacements (u,v). Taking into account the non-linear terms one gets the strains

$$\varepsilon = \varepsilon_{Lm} + \varepsilon_{Nm} - z * \boldsymbol{k} \tag{1}$$

where $\varepsilon_{Lm} = \left[u_{,x}, v_{,y}, u_{,y} + v_{,x}\right]^T$, $\varepsilon_{Nm} = \frac{1}{2}\left[w_{,x}^2, w_{,y}^2, 2w_{,x} * w_{,y}\right]^T$, $\boldsymbol{k} = \left[w_{,\boldsymbol{xx}}, w_{,yy}, 2w_{,xy}\right]^T$,

the indexes denote the partial derivations.

[1] Doc. Ing. Martin Psotný, PhD., Department of Structural Mechanics, Faculty of Civil Engineering, Slovak University of Technology, Radlinského 11, 813 68 Bratislava, Slovakia, phone: (+421) 259 274 652, e-mail: martin.psotny@stuba.sk.

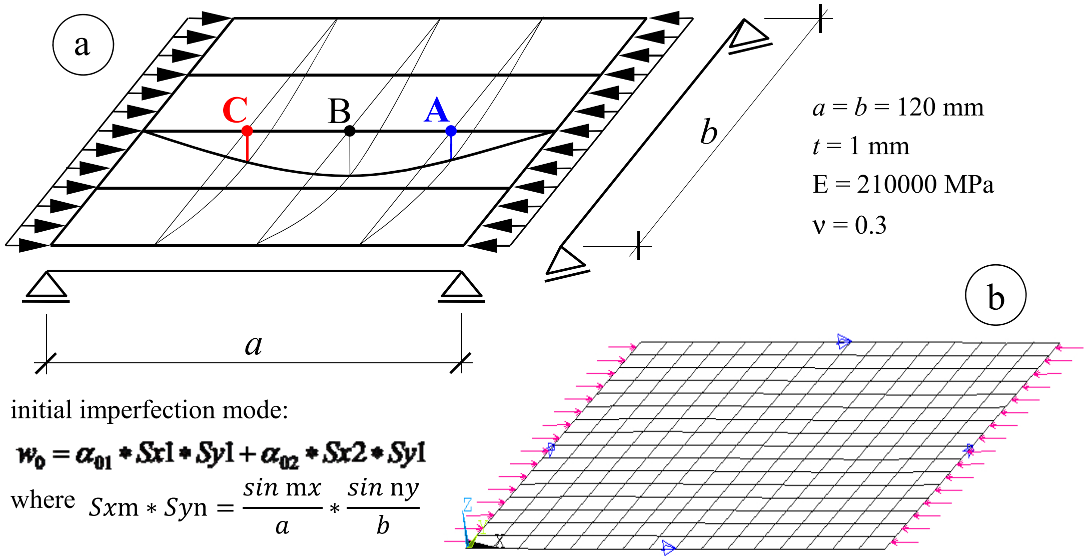

Fig. 1: Slender web: a) Notations of the quantities, b) FEM model – SHELL 143

The initial displacements will be assumed as the out of plane displacements only and so it yields

$$\boldsymbol{\varepsilon}_0 = \boldsymbol{\varepsilon}_{0Nm} - z * \boldsymbol{k}_0 \, . \tag{2}$$

Restricting to the isotropic elastic material and to the constant distribution of the residual stresses $(\sigma_{xw}, \sigma_{yw}, \tau_w)$ over the thickness, the total potential energy can be expressed as

$$U = \int_A \frac{1}{2} (\boldsymbol{\varepsilon}_m - \boldsymbol{\varepsilon}_{0m})^T t \boldsymbol{D} (\boldsymbol{\varepsilon}_m - \boldsymbol{\varepsilon}_{0m}) dA + \int_A \frac{1}{2} (\boldsymbol{k} - \boldsymbol{k}_0)^T \frac{t^3}{12} \boldsymbol{D} (\boldsymbol{k} - \boldsymbol{k}_0) dA - \int_A \boldsymbol{q}^T \boldsymbol{p} \, dA \, , \tag{3}$$

where $\boldsymbol{\varepsilon}_m, \boldsymbol{k}$ are strains and curvatures of the neutral surface,

$\boldsymbol{\varepsilon}_{0m}, \boldsymbol{k}_0$ are initial strains and curvatures,

$\boldsymbol{q}, \boldsymbol{p}$ are displacements of the point of the neutral surface, related load vector.

The system of conditional equations can be obtained from the condition of the minimum of the increment of the total potential energy [6]

$$\delta \Delta U = 0 \, . \tag{4}$$

This system can be written as:

$$\boldsymbol{K}_{inc} \, \Delta \boldsymbol{a} + \boldsymbol{F}_{int} - \boldsymbol{F}_{ext} - \Delta \boldsymbol{F}_{ext} = 0 \, , \tag{5}$$

where $\boldsymbol{K}_{inc} = \left[\begin{array}{c:c} \boldsymbol{K}_{incD} & \boldsymbol{K}_{incDS} \\ \hdashline \boldsymbol{K}_{incSD} & \boldsymbol{K}_{incS} \end{array} \right]$ is the incremental stiffness matrix,

$\boldsymbol{F}_{int} = \left\{ \begin{array}{c} \boldsymbol{F}_{intD} \\ \hdashline \boldsymbol{F}_{intS} \end{array} \right\}$ is the vector of the internal forces,

$\boldsymbol{F}_{ext} = \left\{ \begin{array}{c} \boldsymbol{F}_{extD} \\ \hdashline \boldsymbol{F}_{extS} \end{array} \right\}$ is the vector of the external load of the web,

$$\Delta \boldsymbol{F}_{ext} = \begin{Bmatrix} \Delta \boldsymbol{F}_{extD} \\ \hline \Delta \boldsymbol{F}_{extS} \end{Bmatrix}$$ is the increment of the external load of the web,

$$\boldsymbol{q} = \boldsymbol{B} * \boldsymbol{\alpha} = \left[\begin{array}{c|c} \boldsymbol{B}_D & \\ \hline & \boldsymbol{B}_S \end{array}\right] \begin{Bmatrix} \boldsymbol{\alpha}_D \\ \hline \boldsymbol{\alpha}_S \end{Bmatrix}, \quad \Delta \boldsymbol{q} = \boldsymbol{B} * \Delta \boldsymbol{\alpha} .$$

For more details see [3].

In the case of the structure in equilibrium $\boldsymbol{F}_{int} - \boldsymbol{F}_{ext} = 0$, we can do the incremental step $\boldsymbol{K}_{inc} \Delta \boldsymbol{\alpha} = \Delta \boldsymbol{F}_{ext} \Rightarrow \Delta \boldsymbol{\alpha} = \boldsymbol{K}_{inc}^{-1} \Delta \boldsymbol{F}_{ext}$ and $\boldsymbol{\alpha}^{i+1} = \boldsymbol{\alpha}^{i} + \Delta \boldsymbol{\alpha}$.

The Newton-Raphson iteration can be arranged in the following way: we suppose that $\boldsymbol{\alpha}^i$ does not represent the exact solution and the residua are $\boldsymbol{F}_{int}^{i} - \boldsymbol{F}_{ext}^{i} = \boldsymbol{r}^{i}$. The corrected parameters are $\boldsymbol{\alpha}^{i+1} = \boldsymbol{\alpha}^{i} + \Delta \boldsymbol{\alpha}^{i}$, where $\Delta \boldsymbol{\alpha}^{i} = -\boldsymbol{K}_{inc}^{-1} \boldsymbol{r}^{i}$.

We have used the identity of the incremental stiffness matrix with the Jacobbian of the system of the nonlinear algebraic equation $\boldsymbol{J} \equiv \boldsymbol{K}_{inc}$.

To be able to evaluate the different paths of the solution, the pivot term of the Newton-Raphson iteration has to be changed during the solution.

For the stable path of solution the determinant of the incremental stiffness matrix must be positive det $\boldsymbol{K}_{inc} > 0$, all the principal minors must by positive as well and the load must be taken as the pivot term.

3 FEM NONLINEAR ANALYSIS

The FEM computer program using a 48 DOF element [5] has been used for analysis. FEM model consists of 8x8 finite elements. Full Newton-Raphson procedure, in which the stiffness matrix is updated at every equilibrium iteration, has been applied. The fundamental path of the solution starts from the zero load level and from the initial displacement. It means that the nodal displacement parameters of the initial displacements and the small value of the load parameter have been taken as the first approximation for the iterative process. To obtain other paths of the solution we have used random combinations of the parameters as the first approximation. Interactive change of the pivot member during calculation is necessary for obtaining required number of L-D paths, subsequently it was possible to separate the stable and unstable paths of solution.

Obtained results were compared with results of the analysis using ANSYS system, where 16x16 elements model was created (Fig. 1b). Element type SHELL143 (4 nodes, 6 DOF at each node) was used. The arc-length method was chosen for analysis, the reference arc-length radius is calculated from the load increment. Only fundamental path of nonlinear solution has been presented. Shape of the web in postbuckling has been also displayed.

4 ILLUSTRATIVE EXAMPLES

Illustrative examples of compressed steel web from Fig. 1 are presented as load – displacement paths for different amplitudes of initial geometrical imperfection [2] mentioned in this Figure. From Figs. 2 and 3 it is obvious that two almost identical modes of initial imperfection at the beginning of the process offer two different solutions in postbuckling.

These presented nonlinear solutions of the postbuckling behaviour of the slender web are divided into two parts. On the left side, there is load versus nodal displacement parameters relationship, on the right side the relevant level of the total potential energy is drawn. (Unloaded web represents a zero total potential energy level.)

Due to the mode of the initial imperfection the nodal displacements denoted **A**, **C** have been taken as the reference nodes (see Fig. 1a). The thick line represents the stable path and the thin line represents the unstable path of the solution. More details about the solution of the equilibrium paths were mentioned in [3], [4].

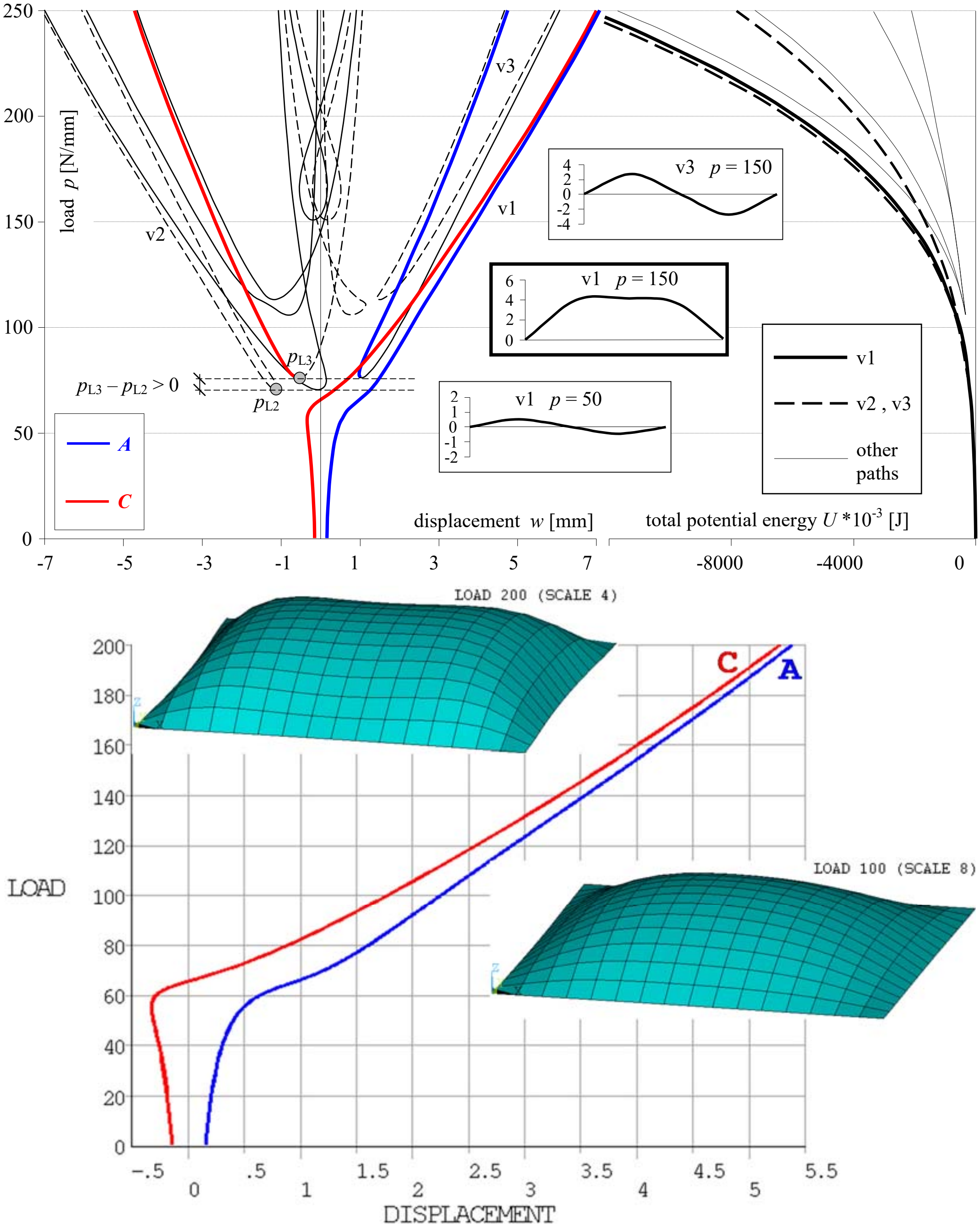

Fig. 2: The postbuckling of the slender web with the initial displacement $w_0 = 0.01 * \sin\frac{\pi x}{a} * \sin\frac{\pi y}{b} + 0.15 * \sin\frac{2\pi x}{a} * \sin\frac{\pi y}{b}$

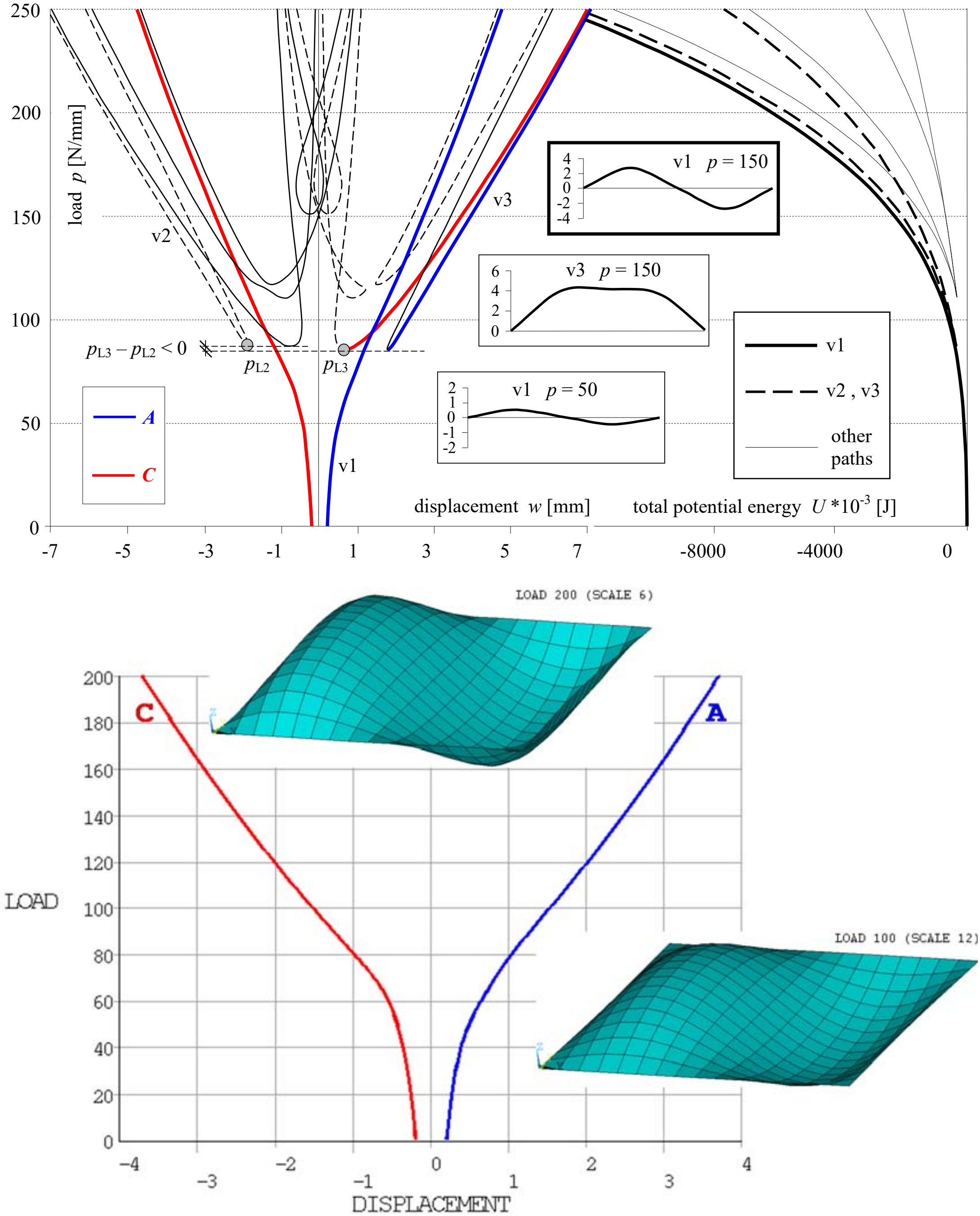

Fig. 3: The postbuckling of the slender web with the initial displacement $w_0 = 0.01 * \sin\frac{\pi x}{a} * \sin\frac{\pi y}{b} + 0.2 * \sin\frac{2\pi x}{a} * \sin\frac{\pi y}{b}$

The aim of this paper was to try to give an answer to the problem of the threat of collapse of the slender web loaded in compression in the second mode of buckling. Fig. 2 shows the solution for the initial displacement parameters $\alpha_{01} = 0.01$ and $\alpha_{02} = 0.15$. We can see that the fundamental path is in the postbuckling phase in mode 1 (v1 – the thick line). The lowest value of the total potential energy is related to the path v3 (mode 2). The energy barrier protects the snap from the path v1 to the path v3. When we increase effect of the mode 2 in the mode of the initial displacement ($\alpha_{01} = 0.01$ and $\alpha_{02} = 0.2$) the postbuckling mode of the slender web is the mode 2 (Fig. 3).

Let us find the connection between the load – displacement path and corresponding level of the total potential energy. From Fig. 2 and 3 one can see, that relative position of limit points in L – D diagram mentions on magnitude of energetic barrier. The increase of the parameter α_{02} is related to decrease of parameter p_{L3}. This is a value of load at limit point of the lowest energy path. If p_{L3} is the lowest limit point in L – D diagram, energetic barrier is eliminated and solution will continue in postbuckling phase in the most convenient way, i.e. in the lowest energy path.

5 CONCLUSIONS

The influence of the value of the amplitude and the mode of the initial geometrical imperfections for the postbuckling behaviour of the slender web is presented. As the important result we can note, that the level of the total potential energy of the fundamental stable path can be higher than the total potential energy of the secondary stable path. This is the assumption for the change in the buckling mode of the slender web.

The evaluation of the level of the total potential energy for all paths of the non-linear solution is a small contribution in the investigation of the post buckling behaviour of the slender web. To be able to give a full answer for the mechanism of the snap-through effect, more in-depth research will be required.

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REFERENCES

[1] BLOOM, F. – COFFIN, D. *Handbook of Thin Plate Buckling and Postbuckling*. Chapman&Hall/CRC, Boca Raton, 2001, 770 p. ISBN 1-58488-222-0.

[2] KALA, Z. – KALA, J. – ŠKALOUD, M. – TEPLÝ, B. Sensitivity Analysis of the Effect of Initial Imperfections on the Stress State in the Crack-Prone Areas of Breathing Webs. *Proc. of the Fourth Int. Conf. on Thin-walled Structures,* Loughborough (England, UK), 2004, pp. 499-506. ISBN 0 7503 1006-5.

[3] PSOTNÝ, M. – RAVINGER, J. Post-Buckling Behaviour of Imperfect Slender Web. *Engineering Mechanics*, Vol. 14, 2007, No. 6, pp. 423-429. ISSN 1802-1484.

[4] PSOTNÝ, M. – RAVINGER, J. Stable and Unstable Paths in the Post-Buckling Behaviour. *International Conference VSU´2005*, Sofia, 2005, pp. 42-47.

[5] SAIGAL, S. – YANG, I. Nonlinear Dynamic Analysis with 48 DOF Curved Thin Shell Element. *Int. J. Numer. Methods in Engng*. 1985, 22, pp. 1115-1128. ISSN 0029-5981.

[6] WASHIZU, K. *Variational Methods in Elasticity and Plasticity*. Pergamonn Press, NY, 1982, 630 p. ISBN 0-08-026723-8.

Reviewers:

Prof. Ing. Pavel Kuklík, CSc., Department of Mechanics, Faculty of Civil Engineering, Czech Technical University in Prague.

Ing. Miroslav Rosmanit, Ph.D., Department of Building Structures, Faculty of Civil Engineering, VŠB-Technical University of Ostrava.