

**Ondřej SLOWIK<sup>1</sup>, Drahomír NOVÁK<sup>2</sup>****ALGORITHMIZATION OF RELIABILITY-BASED OPTIMIZATION****Abstract**

The paper presents newly developed university software FNPO designed for reliability-based optimization. The program works with a newly proposed optimization method called Aimed Multilevel Sampling (AMS) in the optimization cycle of reliability-based optimization. For simulation at different levels of the algorithm AMS and reliability calculations program uses cyclic calls of program FReET – so called double-loop approach. The developed software enables to optimize model of general complexity with consideration of deterministic and/or reliability constraints.

**Keywords**

Optimization, Reliability assessment, Aimed Multilevel Sampling, Monte Carlo, Latin Hypercube Sampling, Probability of failure, Reliability-based design optimization, Small sample analysis.

**1 INTRODUCTION**

Reliability-based optimization is a demanding discipline in which it is necessary to combine the optimization approaches and reliability assessment of structures [1]. Methods for reliability calculation utilize similar simulation techniques and stochastic methods such as optimization approaches - it is also usually a repeated solving of problem. Some particular parts of the reliability calculations can be even formulated as an optimization problem (e.g. calculation of reliability index according Hasofer and Lind [2] or imposing statistical correlation between random variables). Therefore a connection of model optimization with its reliability assessment in the form of optimization constraint is a challenging issue. Thanks to the development of computer technology and stochastic, simulation and approximation methods themselves is such connection of optimization process with reliability assessment possible nowadays [3].

The aim of the paper is to present a newly developed university software designed for reliability-based optimization FReET Nested Probabilistic Optimizer (hereinafter FNPO). This program utilizes a newly suggested optimization algorithm called "Aimed Multilevel Sampling" (hereinafter AMS) for the purpose of stochastic optimization. Software FNPO uses existing program FReET [4], [5] for the reliability calculations and simulation within the AMS algorithm.

**2 GENERAL FORMULATION OF RELIABILITY-BASED OPTIMIZATION PROBLEM**

The basic prerequisite for reliability-based optimization is to model a load and structural response using random variables. Depending on the required robustness and accuracy of the mathematical model it is therefore necessary to randomize some of its input parameters. If any of the

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functional parameters are considered to be random, then analysed function itself is consequently also a random function. The general stochastic formulation of the reliability-based optimization problem can be expressed like this:

$$f(\mathbf{x}, \mathbf{Y}, \mathbf{r}(\mathbf{x}, \mathbf{Y}, \mathbf{y}')) \rightarrow \min \quad (1)$$

under constraints:

$$h_j(\mathbf{x}, \mathbf{Y}, \mathbf{r}(\mathbf{x}, \mathbf{Y}, \mathbf{y}')) = 0 \quad j = 1 \text{ to } p \quad (2)$$

$$g_i(\mathbf{x}, \mathbf{Y}, \mathbf{r}(\mathbf{x}, \mathbf{Y}, \mathbf{y}')) \leq 0 \quad i = 1 \text{ to } m \quad (3)$$

where:

$\mathbf{x}$  – is a vector of deterministic design variables,

$\mathbf{Y}$  – is a vector of random variables,

$\mathbf{r}$  – is a vector of considered probability functions and

$\mathbf{y}'$  – are statistical parameters of random variables.

Numbers  $p$  and  $m$  indicate a numbers of constraints functions.

In the context of the simultaneous application of reliability assessment and stochastic optimization within one procedure, it has to be noted, that the vector  $\mathbf{y}'$  may include two sets of statistical parameters of random variables. The first set of statistical parameters represents the randomization of variables that reflects the natural behaviour of statistical quantities evaluated on the basis of the experiment. This set of statistical parameters is then used for reliability calculations. The second set of statistical parameters of random variables is then used for optimization purposes. For optimization those parameters are randomized, for which optimal input combination is searched. Statistical parameters are then selected with regard to the choice of optimization method so that the design space should be covered as evenly as possible.

Generally structural design is dependent on variables quantifying the response of the investigated structures to the load (e.g. strains and stresses). Therefore we can define the response of the structure as:

$$Y_i = Y_i(\mathbf{A}(\omega), \mathbf{x}) \quad i = 1 \text{ to } m \quad (4)$$

where:

$\mathbf{x}$  – is the vector of deterministic design variables and

$\mathbf{A}(\omega)$  – is a vector of random parameters of the investigated structure (e.g. load or strength).

Design requirements can be formulated as:

$$y_{il} \leq Y_i(\mathbf{A}(\omega), \mathbf{x}) \leq y_{iu} \quad i = 1 \text{ to } m \quad (5)$$

with given boundaries  $y_{il}$  and  $y_{iu}$ . Constraints for deterministic design variables can be determined as:

$$x_{il} \leq x_i \leq x_{iu} \quad i = 1 \text{ to } n \quad (6)$$

Reliability constraints can be expressed by a probability function:

$$P_i(\mathbf{x}) = P(y_{il} \leq Y_i(\mathbf{A}(\omega), \mathbf{x}) \leq y_{iu}) \quad i = 1 \text{ to } m \quad (7)$$

Let us introduce now the function of overall cost of structure  $c=c(\mathbf{z})$ , which will serve as the main criterion of optimality. Optimal design vector of input values  $\mathbf{z}^*$  composed of a vector of deterministic design variables  $\mathbf{x}$  and vector of random variables  $\mathbf{A}(\omega)$  is determined using a stochastic optimization (e.g. [6]). Then the optimization problem can be understood as maximization of reliability, with consideration of constraints, defined as the maximum acceptable cost of structure.

$$P(\mathbf{x}) = P((y_{il} \leq Y_i(\mathbf{A}(\omega), \mathbf{x}) \leq y_{iu}, i = 1 \text{ to } m) \rightarrow \max \quad (8)$$

Constrained by:

$$c(\mathbf{z}) \leq c_{\max} \quad (9)$$

$$x_{il} \leq x_i \leq x_{iu} \quad i = 1 \text{ to } n \quad (10)$$

where the design space for the calculation of the probability is defined as:

$$(\Omega, \Sigma, P), \omega \in \Omega \quad (11)$$

with a given probability distribution, where  $\Omega$  is the sample space for the probability calculations and  $\Sigma$  is a complete design space of variables.

Computational demands of reliability-based optimization are obvious from the formulation above. For the purposes of stochastic optimization it is necessary to repeatedly generate random realizations within the design space  $\Sigma$ . It is also necessary for each of these realizations to calculate the probability of failure in the general case by computationally demanding (mostly numerical) integration of the equation:

$$p_f = \int_{D_f} f(X_1, X_2, \dots, X_n) dX_1, dX_2, \dots, dX_n \quad (12)$$

where:

$D_f$  – represents the failure area (that is the area where value of function indicating a failure is  $< 0$ ) and  $f(X_1, X_2, \dots, X_n)$  – is the joint probability density function of random variables  $\mathbf{X} = X_1, X_2, \dots, X_n$  [7].

The quantification of reliability is associated with the repeated evaluation of structural response. It can bring enormous demands on the computing time. Therefore lot of approximation methods, which aim to reduce the computational complexity of reliability assessment (FORM, SORM, Response surface methods) [8], [9], [10], as well as advanced optimization techniques for the small sample analysis [3], [11] have been developed.

### 3 AIMED MULTILEVEL SAMPLING (AMS)

The simplest heuristic optimization method is to perform Monte Carlo type simulation within a design space and select the best realization of random vector (with regard to optimization criteria). Such a procedure clearly does not converge toward function optimum and the quality of solution depends on the number of the simulations. The exact location of the optimum using only simple simulation is highly improbable. Scatter of the results of such optimization is in the case of small sample analysis very high and strongly dependent on the number of simulations. This approach, however, is very simple requiring no knowledge of features of the objective function and from the engineering point of view is transparent and relatively easy to apply.

Method Aimed Multilevel Sampling was first suggested in [12] (called Nested LHS). Its basic idea is to sort the course of the simulation into several levels. An advanced sampling within a defined space will be performed at each level. Subsequently, the sample with the best properties with respect to the definition of the optimization problem will be selected. Design vector  $X_{i,best}(x_1, x_2, \dots, x_n)$  corresponding to the best in the  $i$ -th level generated sample is determined as a vector of mean values of random variables for simulation within the next level of algorithm AMS. Subsequently, the sampling space is scaled down around the best sample. Another LHS simulation is then performed in this reduced space. This leads to more detailed search in the area around the samples with the best properties with respect to the extreme of the function. The general algorithm of AMS method along with a detailed description of the settings of input parameters and comparison of suggested method with other common optimization techniques is presented in [3].

### 4 FREET NESTED PROBABILISTIC OPTIMIZER (FNPO)

#### 4.1 Description of the program

FNPO is university software developed primarily for the purposes of reliability-based optimization and testing of algorithm AMS. The program works as control software for process of reliability-based optimization using program FReET that provides basic calculations on the various levels of the algorithm of program FNPO.

The program uses so called double loop approach to reliability-based optimization. In this approach algorithm works in two (or three) basic cycles:

- **The outer loop** represents the optimization part of the process. The simulation within the design space is performed in this cycle. For obtained design vectors of  $n$ -dimensional space  $X_i(x_1, x_2, \dots, x_n)$  objective function values are calculated. The best realization is then selected based on these values. Consequently the best realization of random vector  $X_{i,best}$  is compared with optimization constraints. These constraints may be formulated by any deterministic function which functional value we can compare with a defined interval of allowed values. Constraints are also possible to formulate as allowed interval of reliability index  $\beta$  for any limit state function (within design space of given problem). Calculations of reliability index of each generated random vectors  $X_i$  takes place in the inner loop. If the best random vector fit constraints, it is accepted as the next starting point of algorithm AMS or in the case of optimization by simple simulation as a feasible solution.
- **The inner loop/s** are used to calculate reliability index either for the need of checking of generated solutions – if they satisfy constraints, or to calculate the actual value of the objective function, if the target reliability index is set as goal of optimization process.

FNPO is not the stand alone program. The program needs software FReET, in which takes place definitions of all functions and variables, set up of the relevant probability distributions and correlation matrix. Internal cycles to calculate the reliability index are also performed within FReET using approximation method FORM. FNPO then processes .fre files appropriately and manages the concurrence of all levels of reliability-based optimization process. The program is therefore fully dependent on calculations of FReET and cannot be used independently, both for deterministic and reliability-based optimization.

For the optimization process itself, FNPO application offers two methods. The user has the choice to use either a simple simulation using one of simulation methods available in FReET or AMS algorithm. AMS algorithm can be fully controlled and all options of its settings described in [3] are implemented in the program.

Detailed description of possible settings for each section along with the user instructions is available in the user manual [3].

## 4.2 Possible problem definitions in FNPO

Using FNPO, numbers of user-defined reliability-based optimization problems can be solved. But the described version of program, however, is still in the testing stages. Some of the features we consider for the future development have not been fully implemented yet. Program thus suffers from several limitations. Solved problems currently cannot be multi-criterial and there is the option to specify only one function as a constraint (deterministic or reliability-based). These and many other minor problems (related to the GUI environment) should be removed in subsequent versions of the program. Therefore, in the following, we will focus only on the definitions of optimization problems that are solvable using FNPO today.

### Possible definitions of optimization goal

The simplest type of tasks that can be solved using FNPO is unbounded optimization of the objective function either using a simple simulation with one of simulation methods implemented in FReET, or using the AMS algorithm. Optimization problems defined in this way were mainly used for testing of the efficiency of algorithm AMS [3]. Example of unbounded minimization of the objective function may be defined as:

$$f(\mathbf{X}) \rightarrow \min \quad \mathbf{X} \in R_n \quad (13)$$

where:

$\mathbf{X}$  – is a random vector defined within  $n$ -dimensional design space  $R_n$ .

The optimization problem defined in this way can be used to solve equations that cannot be solved analytically in closed form. This type of equations appears for example in mathematical models of the behaviour of the rope.

FNPO also allows searching for such random vectors  $\mathbf{X}$  within the defined design space  $R_n$ , which corresponds to the defined objective function values  $f(\mathbf{X})$ .

$$|f(\mathbf{X}) - k| \rightarrow \min \quad \mathbf{X} \in R_n \quad (14)$$

where:

$k$  – is defined functional value for which the vector of input values is searched.

The task is thus defined as minimization of the absolute value of the difference between the functional value and defined value  $k$ . This type of optimization is often part of the design of cable structures. An architect defines the shape of the proposed cable structure. The engineer then has to find such a combination of load, the cross-sectional area and the prestressing of wires that result in the final deflection of the each wire as close as possible to proposed deflection.

Another option is to define the target reliability index  $\beta_d$  for a given limit state function. For the generated random vectors thus except objective function value also reliability index is calculated, which is the selection criterion of the best realization in a given cycle.

$$|\beta(\mathbf{X}) - \beta_d| \rightarrow \min \quad \mathbf{X} \in R_n \quad (15)$$

In such way algorithm AMS can be directed to the solution with defined value of reliability.

### Possible constraints definitions

For the above-specified objectives of optimization a constraint can be simultaneously defined in the form of allowed interval of functional values of a selected function (also objective function values may be limited). Allowed interval of functional values can be defined as open or closed interval of real numbers. Constraint function can generally be any function defined in  $R_n$ . An example of optimization with constraint:

$$f(\mathbf{X}) \rightarrow \min \quad \mathbf{X} \in R_n \quad (16)$$

constrained by:

$$d < c(\mathbf{X}) < h \quad (17)$$

or:

$$c(\mathbf{X}) < h \quad (18)$$

or:

$$c(\mathbf{X}) > d \quad (19)$$

where:

$d$  – is lower limit of the functional value and

$h$  – is upper limit of the functional value.

Constraints can also be formulated as reliability-based. Similar to the deterministic constraints, they could be defined as an open or closed interval of allowed values of reliability index for a given limit state function.

$$d < \beta_o(\mathbf{X}) < h \quad (20)$$

The definitions of open intervals are for reliability constraints similar to (18) and (19). The user can define more than one limit state function (e.g. for the ultimate limit state and serviceability limit state). In this case, the task can be defined as optimization of the reliability index of one limit state function subject to the limitation of allowed reliability index of second limit state function. This procedure is clearly demonstrated in the example given in section 5. Such a definition would correspond the equation (15) under constraints (20).

## 5 APPLICATION OF FNPO FOR PRACTICAL TASK

Let us define the problem (taken from [13]) for the needs of the optimization task. It is the task of reliability-based optimization of a wooden beam. Analytical relationships needed to define the limit state functions are taken from [14].

Wooden simply supported beam of length  $l$  with rectangular cross-section is loaded by uniform continuous loading along its entire length. The load is the sum of the permanent load  $g$  and variable load  $q$ . Static scheme is illustrated in Fig. 1.

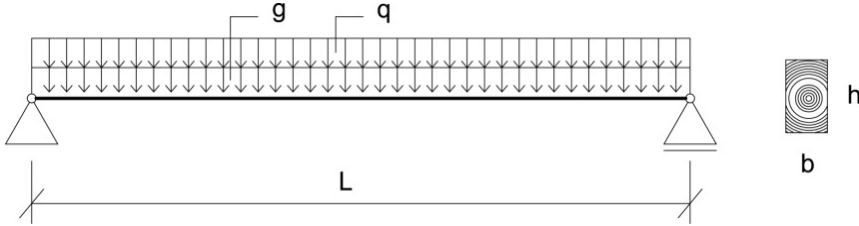


Fig. 1: Static scheme of simply-supported wooden beam with rectangular cross-section

Parameters  $h$  and  $b$  represent the height and width of the cross-section. For the purpose of reliability-based optimization in accordance with the requirements of EUROCODE 5 [14] two basic limit state functions were defined. For the ultimate limit state:

$$g_1 = M_R - M_E \quad (21)$$

and for serviceability limit state:

$$g_2 = u_{lim,fin} - u_{net,fin} \quad (22)$$

where:

$M_R$  – is the critical moment (23).

$$M_R = \theta_R \frac{1}{6} b h^2 k_{mod} f_m \quad (23)$$

$M_E$  – is moment induced by imposed loads:

$$M_E = \theta_E \frac{1}{8} (g + q) l^2 \quad (24)$$

In equations (23) and (24) values  $\theta_R$  and  $\theta_E$  represent model uncertainties of structural response and load effect selected according to [15].  $k_{mod}$  is normative coefficient taking into account the influence of ambient humidity and duration of load. Value  $f_m$  represents the bending strength of used wood. In the case of equation (22) the limit state function is defined by deflections. The limit deflection is defined by equation:

$$u_{lim,fin} = \frac{l}{200} \quad (25)$$

The value of deflection due to the applied load is defined as follows:

$$u_{net,fin} = \theta_E (u_{1,fin} + u_{2,fin}) \quad (26)$$

where:

$$u_{1,fin} = \frac{5}{384} \frac{g l^4}{E \frac{1}{12} b h^3} (1 + k_{1,def}) \quad (27)$$

$$u_{2,fin} = \frac{5}{384} \frac{q l^4}{E \frac{1}{12} b h^3} (1 + k_{2,def}) \quad (28)$$

$u_{1,fin}$  and  $u_{2,fin}$  are the deflections induced by dead and live loads,  $E$  is the modulus of elasticity of used wood,  $k_{1,def}$  is a factor taking into account the effect of creep for dead load,  $k_{2,def}$  is a factor taking into

account the effect of creep for live load. In performed calculations, these values of normative factors were considered:

- $k_{mod} = 0,8$
- $k_{1,def} = 0,8$
- $k_{2,def} = 0,25$

Reliability calculations as well as optimization process requires randomization of individual parameters of vector of input values. For the purposes of calculations of the reliability index (using FORM method within the internal cycle of reliability-based optimization) randomization of individual parameters according to Tab. 1 was performed [13].

Tab.1: Randomization of parameters for reliability calculations

Variable	Distribution	mean	Standard deviation	COV
l [m]	Normal	3.5	0.175	0.05
b [m]	Normal	Optimised	--	0.05
h [m]	Normal	Optimised	--	0.05
E [Gpa]	Lognormal (2 par)	10	1.3	0.13
$f_m$ [Mpa]	Lognormal (2 par)	34	8.5	0.25
g [kN/m]	Gumbel max EV 1	1.686	0.169	0.1
q [kN/m]	Gumbel max EV 2	2.565	0.77	0.3
$\Theta_R$ [-]	Lognormal (2 par)	1	0.1	0.1
$\Theta_E$ [-]	Lognormal (2 par)	1	0.1	0.1

Mean values of height and width of the cross-section area were the subjects of optimization. Presented task is therefore nine-dimensional in terms of reliability calculations and two-dimensional in terms of the optimization process. For the purpose of optimization of cross-sectional area parameters  $b$  and  $h$  were randomized according to Tab. 2.

Tab. 2: Randomization of parameters  $b$  and  $h$  for purpose of optimization

Variable	Distribution	mean	Standard deviation	a	c
b [m]	Rectangular	0.125	0.0144	0.1	0.15
h [m]	Rectangular	0.225	0.0144	0.2	0.25

Values  $a$  and  $c$  in Tab. 2 are parameters of utilized rectangular distribution. The example described was solved by an artificial neural network (ANN) in [13]. The aim of the task was to find such combination of height and width of the cross-section, which corresponds to the value of reliability index for the ultimate limit state function (ULS) given by (21) equal to 3.8 and simultaneously for the serviceability limit state function (SLS) given by (22) equal to 1.5. The result of the solution of described problem by artificial neural network (appear in [13]) is displayed in Tab. 3.

Tab. 3: Results of solution obtained by artificial neural network

mean h	mean b	$\beta_1$	$\beta_2$	$\beta_1$ - target	$\beta_2$ - target
0.132	0.214	3.8001	1.5001	3.8	1.5

If we define optimization problem by equation (15) with reliability constraints given by equation (20), then we can solve the same problem of reliability-based optimization using program FNPO. Since that process is not a multi-criteria optimization solution in the real sense, we cannot expect a similar accuracy of solution as in the case of neural network. Future implementations of multi-criteria optimization together with an extension of options to defining constraints could allow the FNPO solve that kind of problems with greater precision.

During the solution of the problem using program FNPO was utilized the option to determine a target value of reliability index for the selected limit state function. Therefore target reliability index for the limit state function given by (22) was defined as  $\beta = 1.5$ . As a constraint was set interval of allowable values of reliability index for the limit state function given by equation (21)  $3.75 < \beta < 3.85$ .

During solution of the tasks (using AMS algorithm) the total number of 300 simulations was used. Training of ANN utilized in [13] needed 100 simulations. Note that simple two-dimensional optimization task would AMS algorithm probably master successfully with a lower number of simulations. Due to the specification of the task, it was necessary to ensure that the strict definition of the constraints met by at least one realization in the first cycle of optimization algorithm AMS. This problem should probably remove the application of multi-criteria optimization. The result solution of the task using FNPO is displayed in Tab. 4 and in Fig. 2.

Tab. 4: The result solution of the task using FNPO

mean h	mean b	$\beta$ 1	$\beta$ 2	$\beta$ 1 - target	$\beta$ 2 - target
0.131	0.215	3.793	1.50009	3.8	1.5

Final solution therefore corresponds well to the values obtained using neural network [13]. The resulting cross-sectional area has a size 0.028 m<sup>2</sup>.

The graph in Fig. 2 shows the gradual convergence of generated solutions toward the required values of reliability indices.

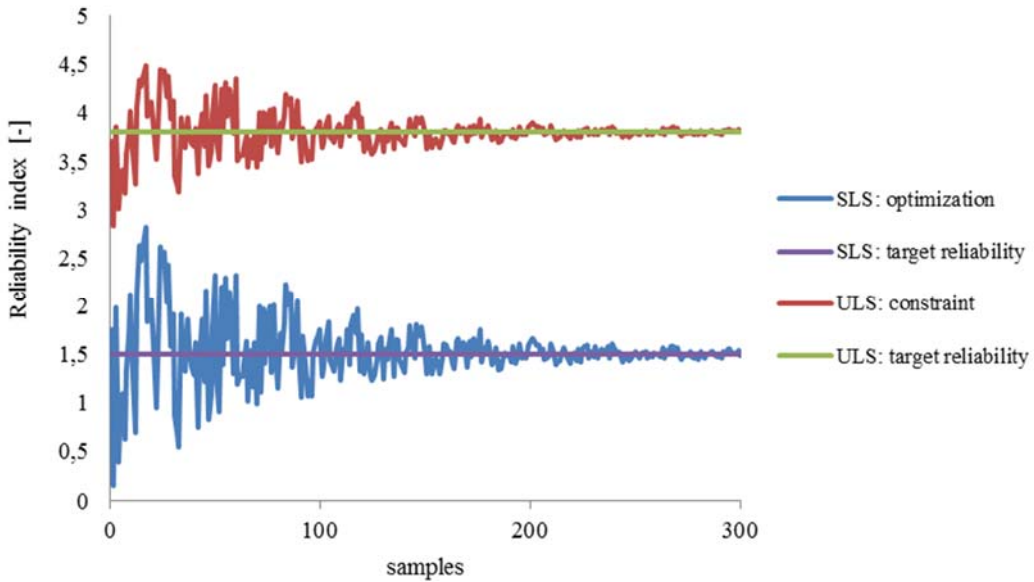


Fig. 2: Evolution of values of reliability indices during optimization



## 6 CONCLUSION

The paper presents newly developed university software FNPO designed for reliability-based optimization. The program uses a newly proposed optimization algorithm AMS, which was developed for small sample analysis and existing program FReET for simulation and reliability calculations. Tests of AMS algorithm and program FNPO performed so far provide promising results. However, it is necessary to make another series of tests, especially for high-dimensional problems to determine more accurately effectiveness of the proposed method. Detailed information about the software FNPO and algorithm AMS are available in [3].

## ACKNOWLEDGMENT

The presented results were obtained with the support of projects GAČR (SPADD), n. 14-10930S, TAČR (SIMSOFT), n. TA01011019 and the project of the specific university research at Brno University of Technology, registered under the number FAST-J-14-2425.

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