

**Jakub VAŠEK<sup>1</sup>, Martin KREJSA<sup>2</sup>****PROBABILISTIC RELIABILITY ASSESSMENT OF TRUSS CONSTRUCTION  
IN MATLAB SOFTWARE PLATFORM****Abstract**

This paper deals with the use of probabilistic methods in assessing the reliability of the planar truss support structure. Classical Monte Carlo simulation technique was chosen for calculation of failure probability in structural elements and the entire support system under assessment. Numerical calculation was applied in MATLAB software system using the random number generator and parallelization using multi-core processors. The aim of the study was to analyze the usability of MATLAB for probability calculations and probabilistic reliability assessments of load-bearing structures.

**Keywords**

MATLAB, reliability assessment, probabilistic methods, Monte Carlo, probability of failure, reliability function, generator of pseudorandom numbers, parallelization.

**1 INTRODUCTION**

The support system of each building structure should fulfill a number of conditions which in the assessment procedures appear in the form of reliability criteria. Determining the likelihood of the required properties of structures deal with the scientific field of structural reliability theory [16]. Application of reliability theory leads to the use of probabilistic algorithms, based on probability theory and mathematical statistics, the development of which has recently experienced a significant increase [1, 9, 21].

**1.1 Probabilistic Calculations**

The main feature of probabilistic methods is the ability to express variability, respectively randomness of input and output variables in probabilistic form such e.g. as histograms. Unlike currently existing standardized processes, based on a deterministic approach to input values [13, 14, 15], the probability procedures lead to higher quality of reliability assessment and ensure the safety of users of structures [11, 20, 22, 24, 25].

This work aims to map out the possibilities of probabilistic calculations in MATLAB with a focus on assessing the reliability of the selected planar truss construction.

**1.2 Monte Carlo Simulation Method**

Calculating the probability of failure for assessment of load-bearing structures allows a number of computational techniques and methods. The largest and most widely used group of

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probabilistic methods represent a simulation methods based on numerical integration of Monte Carlo, i.e. on calculation of repeated quantification (implementation, simulation) the reliability function.

Classical Monte Carlo simulation is easy to apply and universally understood [2, 19]. When solving computationally demanding task, however, is already very efficient, as a sufficiently accurate solution requires a large number of simulations.

For this reason, it is apparent development of other methods based on simulations - called a stratified and advanced simulation methods (e.g. Latin Hypercube Sampling - LHS [18, 23], Response Surface Method - RSM [8]), which can improve the efficiency of quantification the resulting probability of failure by reducing the variance of the individual simulations and their concentration in failure area, which allows a significant reduction in computation time.

### 1.3 Probabilistic Reliability Assessment

In the design process is carried out a number of computational operations related to the reliability assessment of individual components or structure as a whole. Must be satisfied various reliability criteria, defined by the relevant standards, in which figure two key variables - structural resistance  $R$  and load effect  $E$ .

Probabilistic reliability assessment may be based on an analysis of the reliability function, which can be defined as:

$$RF_{(X)} = R - E \quad , \quad (1)$$

where  $X$  is the vector of random input variables – e.g. mechanical properties, structure geometry, load effects or environmental influences on the structure.

Reliability condition can be expressed in the form:

$$E \leq R \rightarrow R - E \geq 0 \rightarrow RF_{(X)} \geq 0 \quad . \quad (2)$$

Failure to comply with condition (2) in terms of reliability is unfavorable state, i.e. fault condition, when the load effect  $E$  exceeds the resistance of the structure  $R$ .

An analysis of the reliability function (1) can obtain the probability of failure  $P_f$ :

$$P_f = P(RF_{(X)} < 0) = P(R - E < 0) \quad , \quad (3)$$

which can be compared with the limit design failure probability  $P_d$ , defined collectively with the target level of reliability in EN 1993-1-1 or in ISO 2394 more detailed. Design of structure is reliable if the following condition of reliability is valid:

$$P_f \leq P_d \quad . \quad (4)$$

Probabilistic assessment can be done also at the level of the reliability index  $\beta$  [3, 10]:

$$\beta \geq \beta_d \quad . \quad (5)$$

## 2 PROBABILISTIC RELIABILITY ASSESSMENT IN A PROGRAM SYSTEM MATLAB

Software MATLAB is a programming environment with a wide degree of application. This software is primarily designed for matrix computations thanks to an extensive library of functions, but it can also be used for example for statistical analysis and to solve probability problems.

### 2.1 MATLAB Functions Suitable for Probabilistic Calculations

The basic operations performed in the probabilistic calculations include work with large volumes of data. In statistical analysis of values, representing e.g. loading and material characteristics, can be used the command "hist". This function creates the histogram from the specified statistical set (vector of numbers), representing the frequency of individual values. The output is a graphical histogram display, absolute frequency vector and the vector containing the mean

of classes, for which the histogram was intended. The number of classes is an input parameter of the function "hist", the default value is 10.

Another option to enter the random input variables to the calculation can be parameterized probability distributions. MATLAB software component library also features command "pdf" (probability density functions), which is used to create a wide range of parametric probability distributions, such as uniform, normal or lognormal. The argument of this function consists of the name of the parametric probability distribution, vector representing the domain specified distribution and relevant parameters. On the "pdf" function follows indirectly command "CDF" (cumulative distribution function). The input values are the same, but the outcome is the distribution function. To facilitate the application of parametric divisions exist within the basic MATLAB software interface also features commands "randtool" and "disttool", which calls up a dialog with option to display all the implemented parametric distributions.

Great importance for probability calculations using simulation techniques has the generator of pseudorandom numbers. For probabilistic reliability assessments of structures are used generator with uniform distribution. The quality of the pseudorandom numbers generator can be expressed as a period of repetition or group of numbers that are repeated during generation. To solve this case, the function "rand" can be used [6], which generates pseudorandom numbers in range 0 to 1. Function "rand" in the earliest versions of MATLAB software used Lehmer recurrent relation:

$$x_{k+1} = (a \cdot x_k + c) \bmod m, \quad (6)$$

where each variable can assume values:  $a = 7^5 = 16807$ ,  $c = 0$  and  $m = 2^{31} - 1 = 2147483647$ .

For these values of the constants is the repeat period over 2 billion numbers. During the improvement of computer technology and the system MATLAB itself has also changed gradually computational algorithm generator. Since the 5<sup>th</sup> version of this software is the repeat period  $2^{1492}$  numbers. This value period is for simulation methods sufficient.

Frequency histogram shown in Figure 1 was created using pseudo-random number generator with uniform probability distribution in MATLAB. Histogram was calculated for the  $1 \cdot 10^6$  pseudo-random numbers generated by command "rand".

## 2.2 Optimization of the Simulation Calculation

To achieve a sufficiently accurate result of simulation methods it is important to use large number of simulation steps. This fact is linked to time-consuming calculation. The measure, which can eliminate the computational time, is parallelization on multi-core processors (like in [7]). This approach can also be applied in the MATLAB programming environment. Commands that shall mobilize core processor is called "matlabpool" [17]. The argument of this command is the number of cores to be subsequently used for the calculation. Due to the fact, that the simulation techniques represent a cycle with known number of repetition, for calculation can be exploited adapted cycle "parfor" [17].

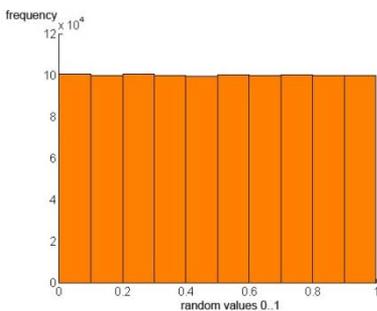


Fig. 1: Histogram based on generated pseudorandom numbers

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87
88 - matlabpool('open',2)
89
90 - parfor i=1:n ...
117
118 - matlabpool close
119

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Fig. 2: Example demonstrated using of the commands „matlabpool“ and „parfor“

An example of the using commands in the MATLAB software is shown in Figure 2. In this case, the computational operations in the simulation cycle is divided into two parts, which are dealt with separately respective core processor. These computing tasks take place dynamically, when the beginning of the next cycle step assigns the appropriate computer operations to free the processor core, which leads to optimal utilization of the processor.

In case of using the cycle "parfor" it is necessary to apply this command with certain restrictions. The first prerequisite for the application of this optimization is the independence of one simulation cycle to another. For the Monte Carlo simulation techniques in the example below, this condition is fulfilled, because in each simulation step for each random variable generating new values are statistically independent. Another condition is the use of only one cycle "parfor". If for some reason it is necessary using the inner loop it can be apply only in a standard cycle "for".

Restrictions, which is reflected in the algorithmic simulation techniques in MATLAB software, related to storing data. If the simulation results are continuously recorded to the matrix, it must be reset first. If the data stored into a matrix during the parallel loop by indexing, it is desirable to know in advance the size of the matrix results. During the parallel calculation is not possible to delete an array, or modify its size. For this reason, all the simulation process was divided into smaller units, after which there was a partial evaluation of a zero matrix results.

### 3 PROBABILITY RELIABILITY ASSESSMENT OF THE TRUSS CONSTRUCTION

The MATLAB software was using the functions above to programming numerical model of the probabilistic reliability assessment of the steel truss, based on Monte Carlo simulation technique. The probability of failure was determined as for the individual structural elements – bars, and for the entire truss structure as a system with a reference period of 50 years. The general deformation method was used to calculate the internal forces in each bar.

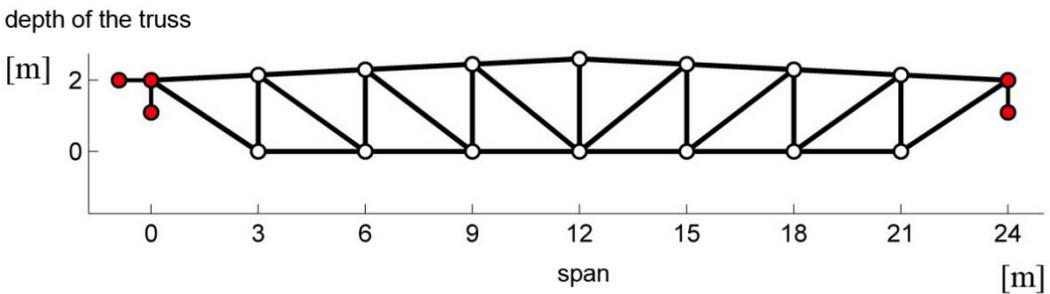


Fig. 3: Static scheme of the truss under reliability assessment

#### 3.1 Description of the Structure under Solution

Assessed truss structure consists of 29 steel bars, strength class S 235 (static scheme see Figure 3). Truss structure consists on bars made from the equal angles profiles. The vertical members and diagonals are formed by a pair of these profiles.

#### 3.2 Input Values

The solved structure was first proposed and evaluated in accordance with existing standardized procedures. The resulting dimension profiles of individual bars were used also for defining the computational model of the probability calculation.

For solving probabilistic task are the input and output random variables expressed in probabilistic form of bounded histograms [4, 12]. The random variables affecting the resistance of

steel structure include the yield stress, the section properties (cross-sectional area and moment of inertia). Variability of cross-sectional properties can be expressed as in [4], the calculation consists in expressing statistically dependent sectional characteristics using the one-parameter histogram  $\varepsilon$  (it is possible to use also the method from [5]).

Input random variables affecting resistance of solved structure are shown in Table 1. Young's modulus of elasticity was expressed deterministically  $E = 210$  GPa.

Tab. 1: Random input variables entered to the calculation of the structural resistance

<i>Variable</i>	<i>Title</i>	<i>Units</i>	<i>Histogram</i>	<i>Range of values</i>	
				<i>Minimal</i>	<i>Maximal</i>
Cross-section variability	$\varepsilon$	[-]	Epsilon	0.0268...	0.0402...
Yield strength of steel	$f_y$	[MPa]	Bars-Fy235-01	207	421

Tab. 2: Random input variables entered to the calculation of the load effect

<i>Variable</i>	<i>Title</i>	<i>Units</i>	<i>Histogram</i>	<i>Range of values</i>	
				<i>Minimal</i>	<i>Maximal</i>
Dead load	DL	1,91	DEAD1	0.818	1
Snow load	SL	10,88	SNOW2	0	1
Wind load	WL	5,01	WIND1	-1	1

For variables representing the load effects were taken with the influence of dead load, snow, wind, and its own weight. In Table 2 is shown the values of the input random variables of all types of loads. Wind load, snow load and dead load work in the joints of the upper chord. The forces representing the self-weight are calculated according to the variable cross-sectional area and the density of steel in each node of the structure. The resulting values of internal forces in bars symmetrical to the structural axis of symmetry are differently due to the variable cross-sectional area.

### 3.3 Calculation Model

The calculation algorithm can be generally divided into three parts. The first is the load data for the corresponding random variables. Subsequently are created histograms and the corresponding distribution functions. Also, it is necessary to retrieve the data, define the geometry of the support system – coordinates of nodes, description of bars, supports and load vectors. To reliability assessment is also necessary to specify the value of buckling lengths.

In the second part of the algorithm proceeds calculation of pseudo-random variables using Monte Carlo simulation technique. It is necessary to determine higher number of simulations for a more precise solution. In this case, the calculation was performed with the number of  $30 \cdot 10^6$  simulation steps. The coefficient of variation can be defined for resulting probability of failure using the formula:

$$v_{P_f} = \frac{1}{\sqrt{N \cdot P_f}}, \quad (7)$$

where  $N$  is number of simulation steps and  $P_f$  is order of calculated failure probability. For a given task can be expected, that outcome dependent on the error, measured by the coefficient of variation,  $\pm 5\%$ . From this perspective, the number of simulation steps can be considered sufficient.

During the simulation are identified the load first. Load vectors represent forces that structure is loaded. The probability is included to the calculation while multiplying each load vector with appropriate randomly generated value of the distribution function. The sum of all load vectors is

possible to get the load vector of the structure. Subsequently, the calculation can be performed using general deformation method, which can be obtained random quantity of the internal forces in the structure.

Finally, the third part of the algorithm processes the data of calculation and compares the values of internal forces of each simulation step with the limit values. To reduce the memory requirements in the actual assessment of the bars the algorithm allows save a value of 1 or 0 only for each of the simulation steps (1 represents the state of failure, 0 reliable condition). This can be used, if necessary, only for obtain the numeric result of the failure probability. If, however, required a graphical presentation of results - histograms of the output variables (e.g. reliability function) algorithm allows to save results of individual simulation steps to complete outputs (e.g. values of internal forces).

### 3.4 Reliability Assessment of the Supporting Element

Assessed truss structure is only subjected to the effects of axial load, the support members may be under tension, or under simple or buckling pressure.

When reliability assessment is done for the bars under tension or simple pressure the resistance of the element is expressed by:

$$R = N_{Rd} = |f_y \cdot A|, \tag{8}$$

where  $f_y$  represents yield strength of the steel [MPa] and  $A$  is cross-section area [m<sup>2</sup>].

When the resistance of the bar under buckling pressure is calculated, definition is based on the Euler critical force:

$$R = N_{Rd} = F_{cr} = \pi^2 \frac{EI_y}{L_{cr}^2}, \tag{9}$$

where  $E$  is Young's modulus of steel [MPa],  $I_y$  moment of inertia [m<sup>4</sup>] and  $L_{cr}$  buckling length [m].

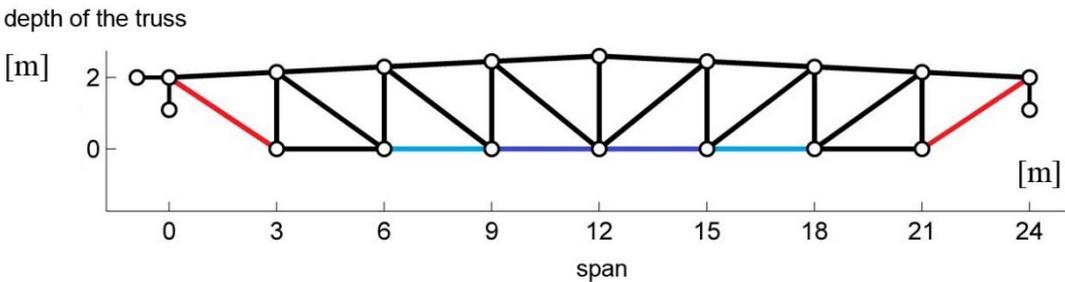


Fig. 4: Schematic representation of the result of the probabilistic calculation

Figure 4 shows the structure under assessment with the results of the probabilistic calculation. Marked bars in black represent a bearing elements for which there is not a failure during simulations. The reason for this result may be that in the design according to EN 1993-1-1, of which the task was based, does not play a role only limit strength, but also limit value of the slenderness. Another factor affecting the zero probability of failure for some bars is that the design of each group of bars is dimensioned from one type of cross-section (diagonals are designed in one profile). In the blue colored marked bar were exceeded tension resistance. When gravity loads the truss, the bars in the lower chord are under tension, which also corresponds to the probabilistic outcome. The last group of bars marked by red color are diagonals at the edge. These bars exceeded the carrying capacity of buckling pressure. These bars under gravity loading are in tension, however in diagonals may also arise compressive normal force due to the effect of wind suction.

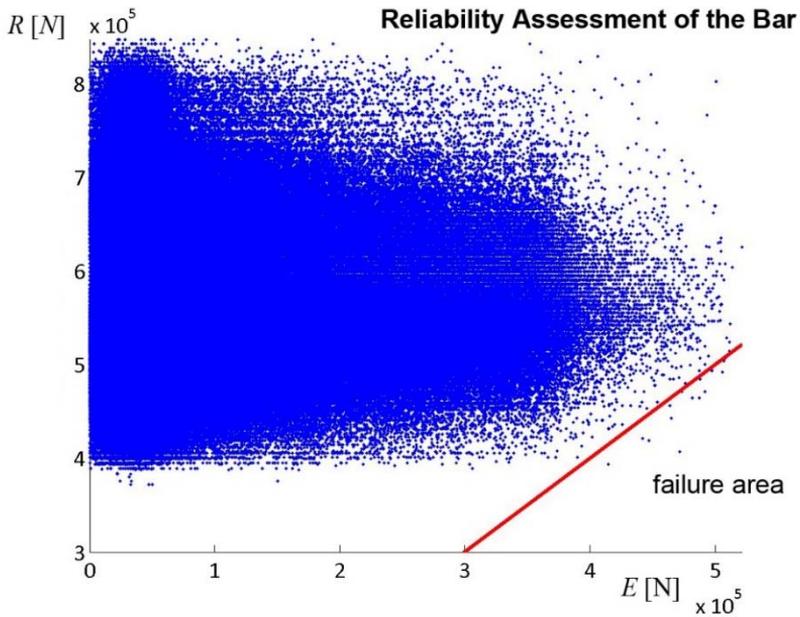


Fig. 5: Output of MATLAB software: Reliability assessment of the bar under tension in the bottom chord of the truss and graphical interpretation of the results - each dot indicates interaction between resultant load effect  $E$  (horizontal axis) and the resistance of the structure  $R$  (vertical axis) for each simulation step, the red line marks the failure boundary that separates the failure area (bottom right) from the area where the structure is reliable

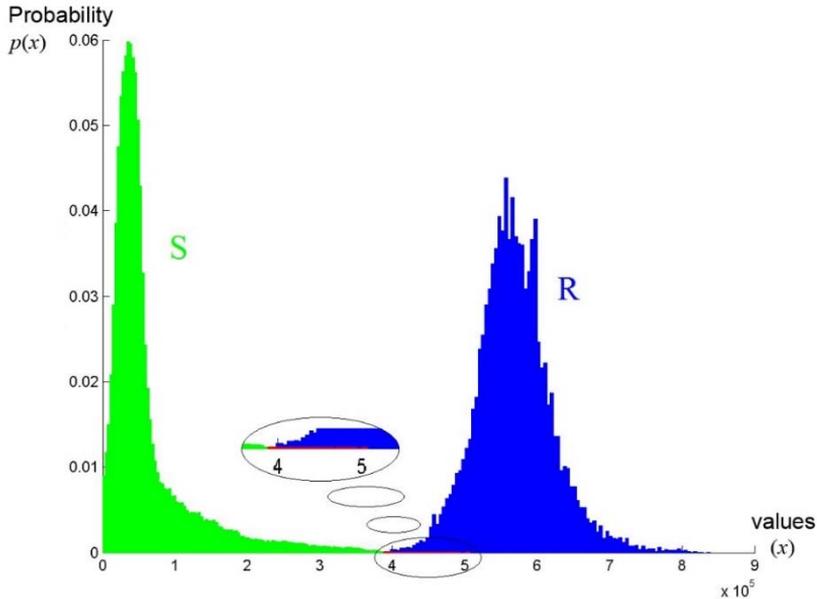


Fig. 6: Output of MATLAB software: Reliability assessment of the bar under tension - left histogram represents random load effect (green), right histogram represents random resistance of the structure (blue). Figure shows also the detail of the failure area according to (3).

Another output from probability calculation in MATLAB is shown in Figure 5. The scatter chart was generated from the calculated values of the bar under tension in the lower chord of the

truss. Each blue point on the graph represents result of one simulation step. The horizontal axis represents the load effects. The vertical axis shows the values of structural resistance. Points located below the marked red line do not meet the reliability criterion (2), thus represent simulations in which there is element under failure state with the resistance of the structure  $R$  smaller than the load effect  $E$ . Failure area of the assessed structural element under tension is shown in detail in Figure 6.

### 3.5 Reliability Assessment of the Structure as the Supporting System

Probabilistic assessment may take place at the level of individual elements, but also at the level of the entire structure as a system. The structure is considered under fault condition when achieved a fault occurs in at least one of its bars. This assumption can be concluded that the probability of structural failure as a whole will be greater than (or equal) than the probability of failure of individual elements, because the calculation of the probability of failure of the entire system outages occur more frequently (just failure in one of the bars) than in assessing the dependability individual members (fault condition occurs only on the element is not affected by the fault conditions of the other bars). This fact correspond to the resulting probability of failure given in Tab. 3. It is necessary to point out that this statement relates only to the specific analysis of the static determinate truss structures and the assumption of elastic material behavior. In the case of statically indeterminate trusses and complex material models, the relationship of the failure probability of the structural elements and the entire system can be complicated. Team of authors the examination of these tasks wanted to focus on further research.

Tab. 3: The results of the probabilistic reliability assessment of the truss structure

<i>Bar under assessment</i>	<i>Decisive stress</i>	<i>Number of failure states</i>	<i>Probability of failure <math>P_f</math></i>	<i>Reliability index <math>\beta</math></i>
Diagonal (left)	Pressure	22	$7.33 \cdot 10^{-7}$	4.82
Diagonal (right)	Pressure	20	$6.67 \cdot 10^{-7}$	4.84
Lower chord (1 <sup>st</sup> on the left)	Tension	3	$1.00 \cdot 10^{-7}$	5.21
Lower chord (2 <sup>nd</sup> on the left)	Tension	186	$6.20 \cdot 10^{-6}$	4.38
Lower chord (3 <sup>rd</sup> on the left)	Tension	184	$6.13 \cdot 10^{-6}$	4.38
Lower chord (4 <sup>th</sup> on the left)	Tension	2	$6.67 \cdot 10^{-8}$	5.29
The entire structure	-	411	$1.37 \cdot 10^{-5}$	4.20

## 4 CONCLUSION

The paper pointed out the possibility of using MATLAB software system for probabilistic calculations based on a probabilistic assessment of structural elements and the steel truss. It has been described probabilistic solution in which was used the programming environment MATLAB with built-in functions, which described a statistical analysis of input data and calculated probabilistic reliability assessment of the truss structure using the Monte Carlo simulation technique and parallelization.

It turned out that the program system allows advantageously solved similarly formulated probabilistic tasks. Due to the possibility of a relatively easy programming MATLAB can be used for probabilistic tasks with more complex computational model (defined with using e.g. the general deformation method or finite element methods).

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