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NUMERICAL MODELLING OF TIME-DEPENDENT BEHAVIOUR OF REINFORCED CONCRETE STRUCTURE WITH USE OF B3 MODEL

Abstract

The paper proposes an implementation of creep analysis of reinforced concrete structures which utilizes the B3 model and the direct stiffness method for reinforced concrete frames. The analysis is based on a numerical integration and it is implemented in an algorithmic programming language. There is presented a solution with the mentioned approaches which is compared with solution based on the EN 1992-1-1 technical standard.

Keywords

Numerical modelling, viscoelasticity, reinforced concrete, direct stiffness method, B3 model.

1 INTRODUCTION

Many of building materials change their properties in time. These changes can result in time-dependent deformations during the life-span of the structures. These effects are important for timber-based materials [2, 5] and also for concrete (swelling and relaxation effects) [3, 6, 10, 12]. There are cases when these two materials are combined. For example, some of modern multi-storey living houses are based on a combination of reinforced concrete and timber-based structural elements. For these reasons it is important to study the influence of long-term changes and deformations on structural behavior and on serviceability and usability of these objects. The most important effect which has to be taken into account is a difference between long-term deformations of different materials (in this case they are timber and concrete). These differences can cause disintegration of joints, excessive deformations of structures, damage of insulation elements and structures, for example. These effects can be omitted for small structures but they are very important for large ones (for example for multi-storey buildings or for bridges) [7].

There is a wider research programme at the Faculty of Civil Engineering of the VŠB-Technical University of Ostrava which aims to understand a long-term behaviour of large composite structural systems which are based on a combination of reinforced concrete and timber-based elements. This paper discusses only a part of initial works in this area. These works are going to be utilised for preparation of numerical tools for approximation of long-term behaviour of such structures. There is discussed a use of the B3 model for concrete [1] and its implementation in an algorithmic language. The main use of the written code will be a verification of selected approaches for the expected problems. After the verification of the approach, more robust computational tools will be prepared. The further works will be focused also on larger computational problems.

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2 MODELLING OF TIME-DEPENDENT EFFECTS IN CONCRETE

2.1 Viscoelastic models

Time-dependent changes in the concrete (chemical processes, drying) cause time-dependent changes of mechanical properties (increase of strength and of modulus of elasticity) but they are also directly related to time-dependent deformations (creep and shrinkage).

There are many approaches for description of these deformations. The common way is use of linear viscoelastic material models. These approaches are relatively easy but they are able to provide acceptable results only if the maximum stress in the concrete is much lower than the limit strength [8,9]. The viscoelastic-based approach is often used by current design codes [4]. The time-dependent compliance function of the viscoelastic material or the aging coefficient is often used.

The compliance function can be used in the form of Kelvin chain [8] but a relatively huge amount of input data is unnecessary for its correct description. It is often uneasy to obtain all the necessary parameters for real use. For this reason the approach used in the design codes is usually more suitable. For special cases it is possible to use more refined numerical models, for example the B3 model [1,9].

2.2 Model B3

The numerical examples which are discussed in the following text use the B3 material model which was proposed by Bazant [1]. The simplified version of the model has been used. The B3 is based on an analysis of long term investigations and experiments on reinforced concrete structures. This model requires several input parameters which require some effort to be correctly obtained. In the ideal case the parameters should be obtained from short-term tests of the material. The original author also provides limits for use of the model. The model can be used outside of these limits but its behaviour in these cases is not tested. The limits are: water ratio from 0.35 to 0.85, cylinder strength after 28 days from 17 MPa to 70 MPa and cement weight from 160 kg to 720 kg in cubic meter of the concrete. For the most common concrete mixtures it is possible to find recommended values of input parameters. The recommended parameters were also used for the computations discussed in this article.

The compliance function J can be written in the following (it is valid for the simplified version of the B3 model):

$$J(t,t') = \frac{1}{E_o} + q_s \ln\left[(1+\psi(t'^{-m}+\alpha)(t-t')^n\right], \tag{1}$$

where:

t' – time when loads were applied [days],

t – time when the deformations are computed [days],

E_o – asymptotic modulus of elasticity [Pa].

Other parameters of the function are constants and should be obtained from laboratory tests. For usual concrete mixes the following values are recommended: ψ=0,3, m=0,5, n=0,1, α=0,001 [8]. The Equation (1) does not include shrinkage effects. According to [9] the Pickett effect was included into the computations.

2.3 Computational approaches

The value of the compliance function J for given time t it is possible to use a numerical integration or other numerical approaches. (an exponential method can be used, for example [9]). The 2D strame stuctures can be effectively analysed with use of the direct stiffness method [11] but a use of this metod require also computation of the relaxation function *R(t,t')*. The relation between the compliance function and the relaxation function is shown in the Equation (2).

$$J(s,t_0)\frac{1}{J(t_0,t_0)} + \int_{t_0}^{s} J(s,t)\frac{\partial R(t,t_0)}{\partial t}dt = 1, \tag{2}$$

In many cases in is impractical to use the Equation (2). For concrete it is possible to use and approximation by Bazant et al [8]. The approximation is shown in the Equation (3).

$$R(t,t') = \frac{0{,}992}{J(t,t')} - \frac{0{,}115}{J(t,t-\Delta t)}\left[\frac{J(t_m,t')}{J(t,t_m)} - 1\right], \tag{3}$$

where:

t_m – half of time interval between t and t' [days],

Δt – 1 day [days].

A comparison of numerical solution which is based on the Equation (2) with the approximation (3) is shown in the Fig. 1. The input data were based on recommendation from section 2.2 of this paper. The computations in the following sections of this text use the numerical solution.

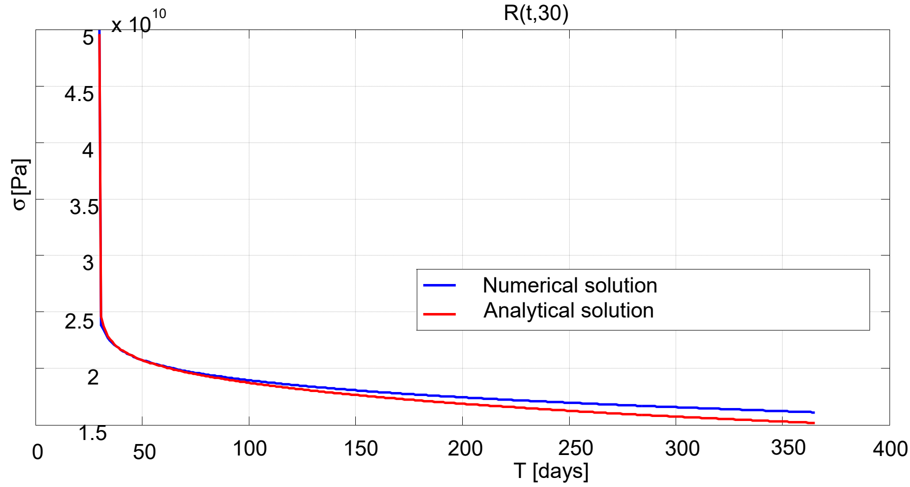

Fig. 1: Comparison of numerical and analytic computation of compliance function

3 COMPARISON OF RESULTS FROM MODEL B3 AND EN 1992-1-1

3.1 Numerical example

To compare the differences between results obtained from computation which uses the B3 model and the analysis which is based on the EN 1992-1-1 design code (Appendix B) a simple example was prepared. The example is a simply supported beam with 3 m span. The load was assumed to be continuous with size 7 kN/m. The load was applied 30 days after the beam was built. The concrete was assumed to be the C30/37 (it was assumed that average concrete strength after 28 days is 38 MPa and the tangent modulus of elasticity is 32 GPa). The main reinforcement bars are 4x10-B420B and they are located 38 mm from bottom surface of the beam. The relative humidity 50% has been assumed. The computations didn't include influence of possible tensile cracks. The detailed scheme of the analysed beam is shown in the Fig. 2.

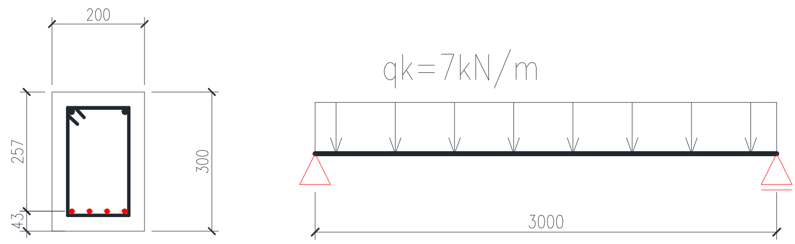

Fig. 2: Geometry and load of example

The computational model of the beam consisted from 10 finite elements. Two alternatives were studied. In the first alternative the equivalent reinforcement bars were included in a form of an addition to an idealised homogenous cross section. The second alternative included additional finite elements to simulate effects of the reinforcement bars. These additional finite elements were eccentrically located to respect their location inside the cross-section of the beam.

3.2 Results

The results are shown in the Fig. 3. The vertical deflection in the center of the beam is compared. It is visible that the effects of the reinforcement are important especially for larger times and thus they have to be included in computations. The B3 model gives larger deformations than the EN model for this particular case.

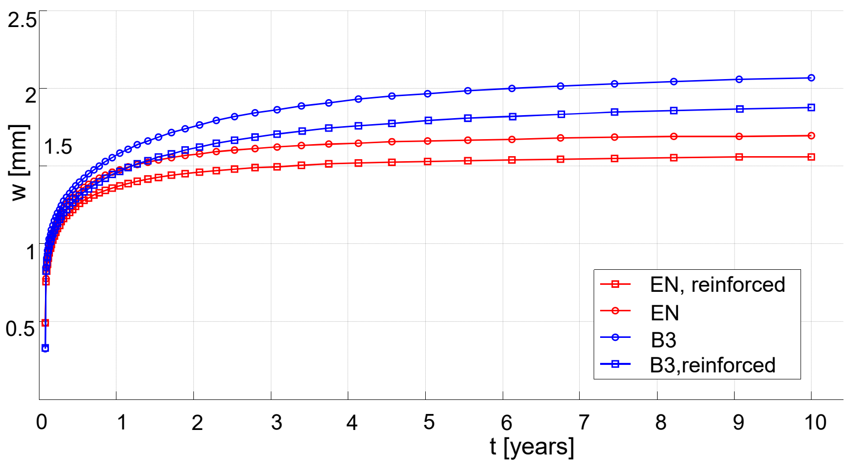

Fig. 3: Time-dependent deformations of beam

If the structure is idealised as a statically determinate problem then time–dependent effects should have no elations to changes of internal forces. In the cases when presence of reinforcement is respected in the model there are time-dependent changes of internal forces. This effect is shown in Fig. 4. This figure shows forces in reinforcement in times 3 days and 10 years which were obtained from computation which was based on the EN code. The Fig. 5 shows forces in reinforcement which were obtained for which used of model B3. Differences in initial stresses are caused by differences of input data types for the individual models (the B3 model uses asymptotic modulus of elasticity).

The stepped shape of the diagrams of internal forces is caused by model simplifications. The interaction between concrete and reinforcement is modelled only in finite element nodes.

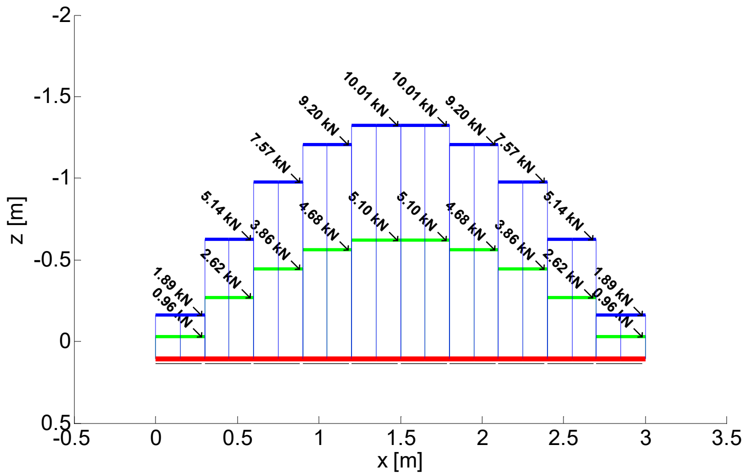

Fig. 4: Axial forces in reinforcement computed with use of EN code

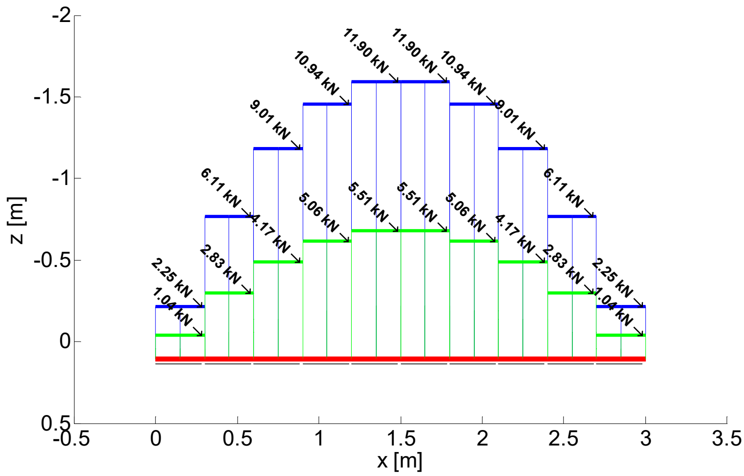

Fig. 5: Axial forces in reinforcement computed with use of model B3

The J and R functions are obviously highly non-linear as well as the obtained results (see Fig. 3). The use of numerical integration in the abovementioned example requires further verification as the step size may affect precision of computations. Several time divisions up to 100 time steps were studied. According to [9] a logarithmic step size division was studied, too (up to 20 time steps). The results are shown in Fig. 6.

It can be concluded that to constant steps size it is necessary to use at least about 50 steps. If a logarithmic division is used then good results can be obtained for as little as 20 steps It is also obvious that convergence is very good for higher numbers of steps.

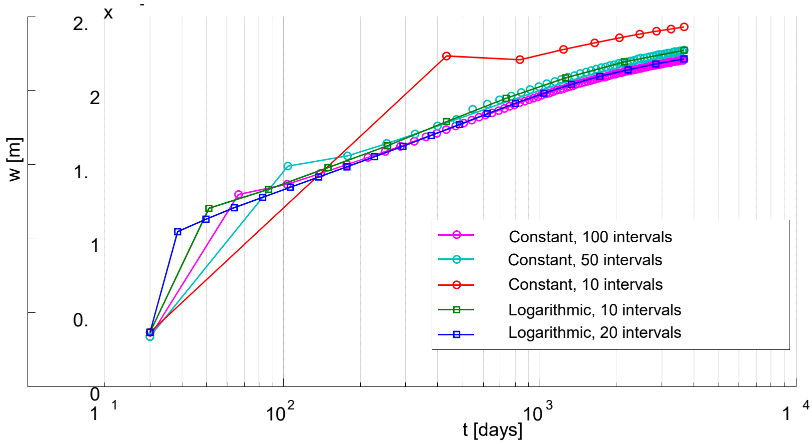

Fig. 6: Comparison of steps sizes

4 ANALYSIS OF SIMPLE FRAME

4.1 Problem description

The geometry and the dimensions of the problem and also the loads are shown in Fig. 7. A quasi-static combination of loads was used. It was assumed that the loads were applied 40 days after structure was built. The load on horizontal members was $q = g_1 + g_2 + \psi_2 s_k = 25{,}8 + 2 + 0{,}2 \cdot 14{,}4 = 30{,}7\text{kN/m}$. The load on columns was $n = g_3 = 1\text{kN/m}$.

The material was assumed to be a C20/25 concrete. The main reinforcement was the B500B. The compliance function was obtained with use of the simplified version of the B3 model with following input data: average compression strength$f_{cm} = 24\text{MPa}$, time of concrete treatment was assumed to be 28 days.

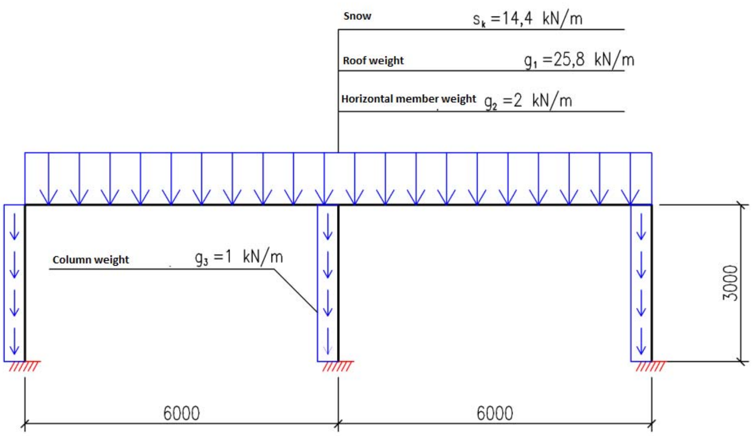

Fig. 7: Computational model

The number and dimensions of main reinforcement in columns were and 4xϕ20. The horizontal members had 4xϕ16 at the bottom and 4xϕ20 at the top. The concrete cover was 30 mm in all cases. The cross-sections are shown in the Fig. 8.

Fig. 8: Cross sections: a column (left) and a horizontal member (right).

4.2 Results

The Fig. 9 shows deformations in the center of the left horizontal member. The Fig. 10 illustrates frame deformations for three selected times.

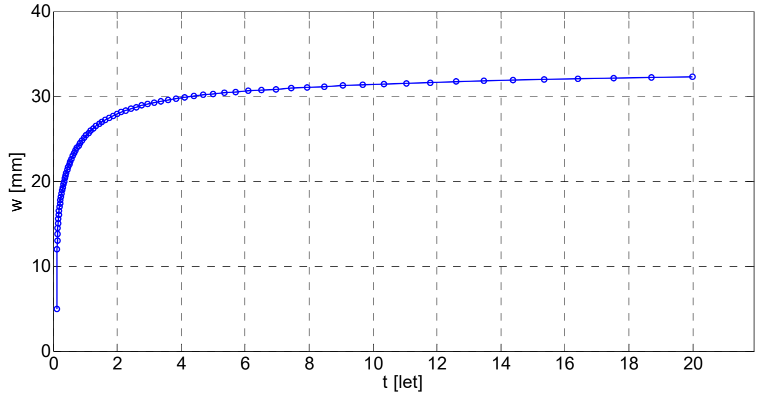

Fig. 9: Deformations of horizontal member in time

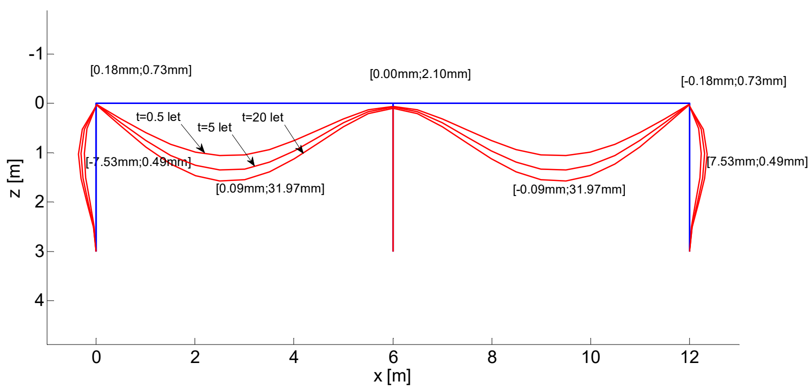

Fig. 10: Frame deformations after 0.5 year, 5 years and 20 years

6 CONCLUSIONS

The article discussed computational analysis of time-dependent deformations of simple concrete frames. The direct stiffness method has been used for analysis of stress state and the B3 model was used for time-dependent constitutive relations. It was shown that the mentioned approaches can be implemented with use of an algorithmic language (for example Octave or Matlab). It is important to remember that use of precise constitutive models is only useful it the input data can be verified with help of short-time laboratory tests. It is not possible to use recommended values from design codes as inputs for more advanced models because these data were derived for different purposes. Their use may lead to results that might not be correct in a particular case (see the Fig. 3).

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