The Prediction of Fatigue Life of Notched Specimens

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DOI: 10.35181/tces-2021-0010

Abstract. *The contribution presents a method for lifetime predictions of components with notches. The method was developed using experimental data gained on smooth and notched samples and numerical analysis of notched specimens. The method employs approaches of generalized fracture mechanics and uses a critical length parameter. Emphasis is placed on the area of gigacycle fatigue.*

Keywords

Wöhler curve, fatigue life, notches, generalized fracture mechanics, stress distribution, gigacycle fatigue.

1. Introduction

The aim of the article is to present a process, which can be used for determination of lifetimes of notched components. Current computational methods bring the possibility of precise analysis of stress concentrations in notches. Fatigue life curves are usually determined for smooth specimens and rarely for notched samples. Two sets of specimens (notched and smooth) were tested in the area of high and very high cycle fatigue [6]. These results were used to estimate fatigue life of specimens with different notch radii. The suggested method forms relation between the life curves of smooth specimens and specimens with model notch and uses knowledge of distribution of the axial stress in the studied specimens. The method was applied on two materials (aluminum alloy and structural steel). The principle is based on the approaches of generalized fracture mechanics.

1.1. Generalized fracture mechanics

Linear elastic fracture mechanics (LEFM) deals with the behavior of objects with cracks in cases where there is a small plastic zone at the crack tip. This occurs under static loading of brittle materials or during cyclic fatigue loading with low stress amplitude. The approaches of LEFM were generalized from cracks to description of sharp or blunt notches. These approaches use local analysis and descriptions of stresses near the notch tip. These local approaches are represented by the generalized fracture mechanics [3], critical quantity criterion [2], the theory of critical distances (TCD) [8, 7, 9], Finite fracture mechanics (FFM) [1, 4] or method of highly stressed volume (HSV) [13].

An approach similar to the critical distance theory has been used to formulate a lifetime prediction procedure. TCD uses a length parameter, the critical distance l. The best-known approaches of TCD are the Point method and the Line method. The Point method states that failure will occur when the elastic stress at a certain distance from the notch reaches some critical value. The Line method is very similar but uses the average stress (Fig. 1).

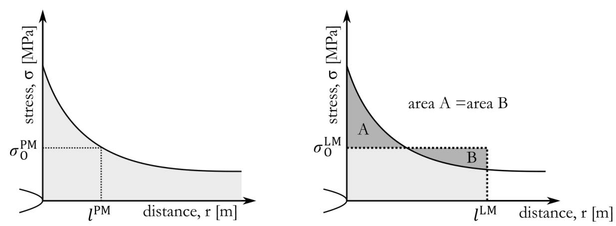


Fig. 1: TCD: Point method, Line method

1.2. Gigacycle fatigue

The fatigue limit can be defined as the maximum stress that does not lead to fracture even if it exceeds the number of cycles 10^7. This definition gave rise to the presumption, if the component is loaded with stress less than the fatigue limit it has an unlimited lifetime. At the end of the twentieth century, it turns out that failure can occur even at stresses below the fatigue limit. This region, called the gigacycle fatigue region, is less studied than the low-cycle and high-cycle fatigue regions. This is due to the cost and time-consuming testing. However, there is an ultrasonic fatigue testing machine that can perform these tests faster than conventional machines. Tests with the number of cycles 10^9, which would last more than a hundred days on classic machines, last less than 14 hours. Claude Bathias is considered to be the pioneer of gigacycle fatigue, together with Paul Paris showed that most of fatigue life is spent on crack initiation, not propagation [5, 11]. Together, they described the mechanisms in the formation of a crack in the area of very high cycle fatigue and justified the formation of the so-called „fish eye" below the surface [12] (Fig. 2).

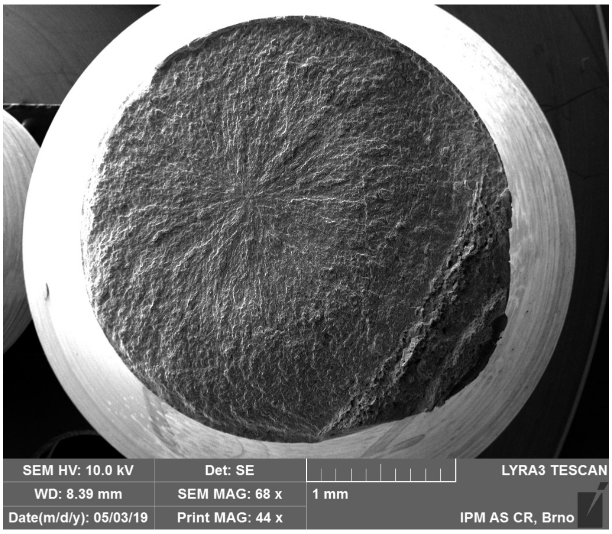


Fig. 2: Fish eye - internal crack initiation in a structural steel

2. Methodology for lifetime predictions

The procedure below leads to the determination of the lifetime curves of notched specimens with various notch radii.

2.1. Design of specimens

The whole process begins with the design of two sets of samples - notched and smooth. The ultrasonic device works on the resonant principle in the range of 19.5 to 20.5 kHz. If the tests are performed using ultrasonic devices, the natural frequencies of the longitudinal vibrations of the designed samples must be close to 20 kHz. This is achieved through parameterizable geometry and an iterative solution. Two materials are analysed in this paper (aluminum alloy and structural steel).

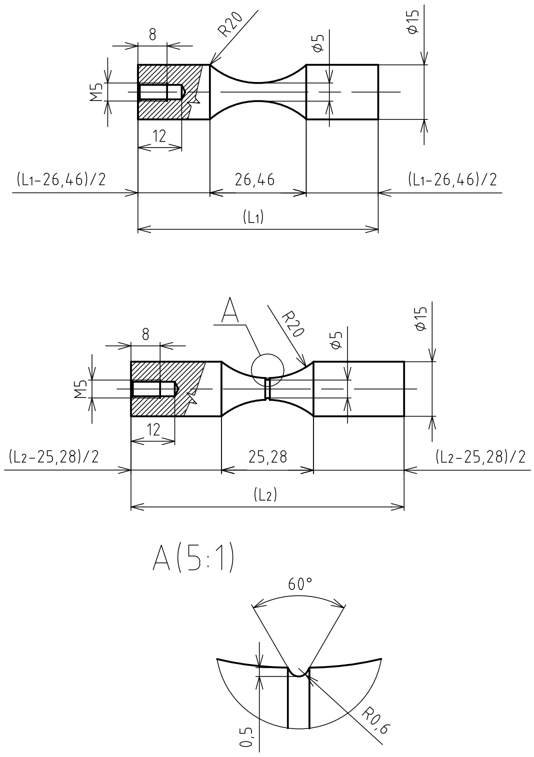


Fig. 3: Geometry of aluminum alloy specimens

The radii of model notches were $R = 0.6$ mm for aluminum specimens and $R = 0.1$ mm for steel specimens. Figure 3 shows the geometry of the aluminum alloy AW7075 samples. The lengths L_n of specimens were determined so that the natural frequencies f_n of the longitudinal oscillations were equal to 20 kHz and are given in the Table 1 below. The subscript $n = 1, 2$ indicates smooth and notched specimens, respectively. The inputs to the calculations were dynamic modulus $E_\mathrm{d} = 72.15$ GPa and mass density $\rho = 2807.95$ kg/m^3.

Tab. 1: Parameters of specimens

		aluminum alloy		steel	
n	specimen	L_n [mm]	f_n [Hz]	L_n [mm]	f_n [Hz]
1	smooth	66.0	20012	78.1	20006
2	notched	75.3	20013	75.0	19999

The finite element models were created using PLANE 183 elements and axisymetry. The planar models of the smooth and notched pattern are shown in Figure 4. The highest density of elements is at the notch.

Similarly, the geometry of a set of steel samples was created for the dynamic modulus $E_\mathrm{d} = 210$ GPa and mass density $\rho = 7850$ kg/m^3. The geometry differ in

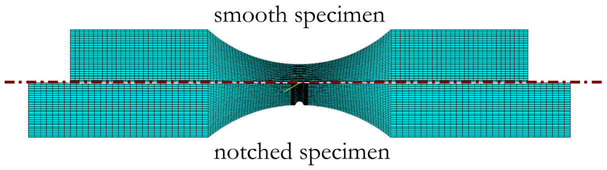


Fig. 4: Finite element models

the radius of the notch, which was 1 mm, the diameter was 10 mm and the narrowest diameter of the samples was 2.4 mm. The parameters are shown in [6]. The lengths L_n were adjusted to gain the intrinsic frequencies f_n close to 20 kHz and are shown in Table 1.

2.2. Harmonic analysis and stress factor

Then, harmonic analysis of the modeled samples were performed to calculate the stress factor and obtain the σ_y distribution across the narrowest cross section of the specimens. The end surfaces of the specimens were loaded with 10 μm and the natural frequency was calculated by modal analysis. Using a defined path - from the surface to the center of the sample, the axial stresses σ_y were plotted and are shown in Fig. 5, 6.

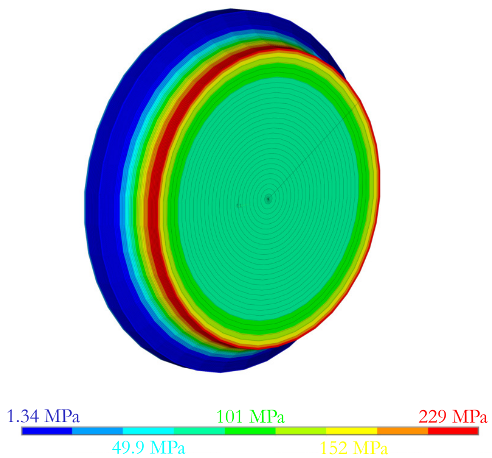


Fig. 5: Harmonic analysis, notched specimen with $R = 0.6$ mm, axial stress, aluminum alloy

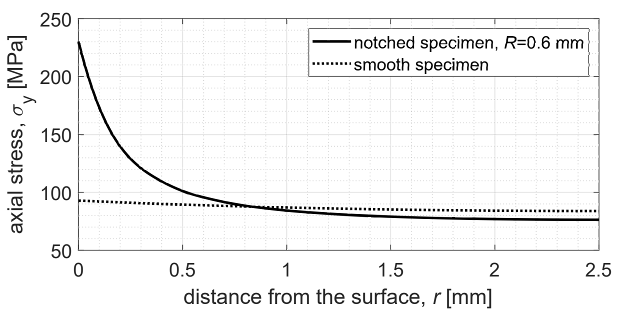


Fig. 6: Axial stress distribution, aluminum alloy

The stress factor is used in gigacycle fatigue tests, in resonant displacement loading. It indicates what axial stress is induced by oscillation amplitude of $A = 1$ μm. There is a nominal and maximum stress factor. The nominal stress factor $S_{\mathrm{f(nom)}}$ is determined as the ratio of the average cross-sectional stress and the magnitude of the oscillations $A = 10$ μm, eq. (1). The maximum stress factor $S_{\mathrm{f(max)}}$ is the ratio of the maximum stress on the sample surface and the magnitude of the oscillations, eq. (2).

$$S_{\mathrm{f(nom)}} = \frac{\sigma_{\mathrm{nom}}}{A} \tag{1}$$

$$S_{\mathrm{f(max)}} = \frac{\sigma_{\mathrm{max}}}{A} \tag{2}$$

The values of these factors for aluminum and steel samples are given in Table 2. The model notch has the radius $R = 0.6$ mm in the case of aluminum alloy and $R = 0.1$ mm in the case of steel specimens.

Tab. 2: Stress factors [MPa/μm]

	aluminum alloy		steel	
	$S_{\mathrm{f(nom)}}$	$S_{\mathrm{f(max)}}$	$S_{\mathrm{f(nom)}}$	$S_{\mathrm{f(max)}}$
smooth specimen	8.67	9.29	40.97	45.91
model notch	9.38	23.02	55.11	210.09

2.3. Fatigue tests

Subsequently, results of fatigue tests of smooth and notched samples of aluminum alloy and structural steel were employed. The S-N data of the aluminum alloy are taken from [10], while S-N data for steel samples are taken from [6]. The experimental data were interpolated by Wöhler curves using the least-squares method, see Fig. 7. The data were fitted by curves A, B, C, D described by equations (3).

$$\begin{array}{ll} \text{A} & \sigma[\mathrm{MPa}] = 343.1 \cdot N_f^{-0.03512} \\ \text{B} & \sigma[\mathrm{MPa}] = 265.8 \cdot N_f^{-0.04827} \\ \text{C} & \sigma[\mathrm{MPa}] = 1116 \cdot N_f^{-0.02953} \\ \text{D} & \sigma[\mathrm{MPa}] = 502.9 \cdot N_f^{-0.03661} \end{array} \tag{3}$$

2.4. Determination of the length parameter

To determine the critical distance, experimentally gained S-N curves of two sets of samples are required: smooth specimens and specimens with a model notch, and numerically determined axial stress distribution of the model notch. Then the critical distance l follows from the formula (4), where $\sigma_{\mathrm{a,nom}}^{\mathrm{notched}}(N_{\mathrm{f}})$, and $\sigma_{\mathrm{a,nom}}^{\mathrm{smooth}}(N_{\mathrm{f}})$ are the values on the S-N curves for an

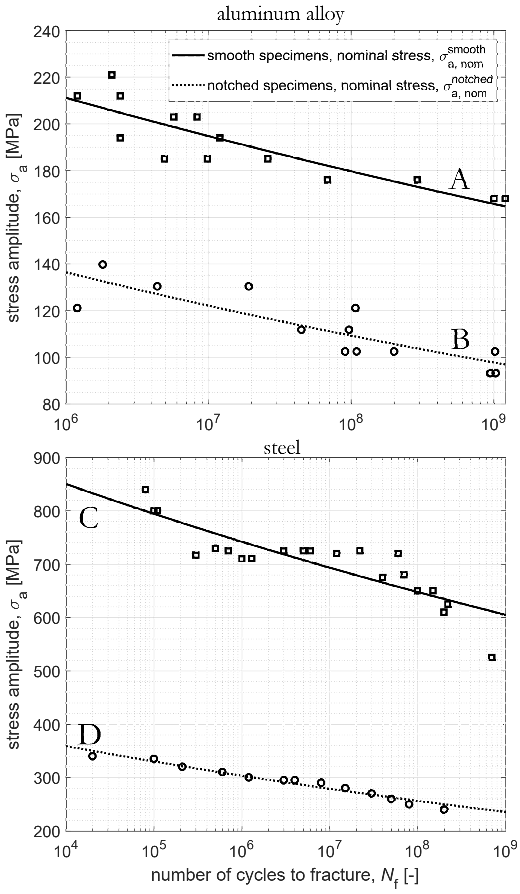


Fig. 7: Lifetime curves, adopted from [10, 6]

arbitrary, but the same number of cycles N_{f}, $\sigma_{y,\mathrm{LM}}(l)$ is the average stress over the distance l, and $\sigma_{y,\mathrm{LM,nom}}$ is the average stress over the entire cross section of the notched specimen with the model radius $R = 0.6$ mm (aluminum alloy) or $R = 0.1$ mm in the case of structural steel.

$$\sigma_{y,\mathrm{LM}}(l) = \frac{\sigma_{\mathrm{a,nom}}^{\mathrm{smooth}}(N_{\mathrm{f}})}{\sigma_{\mathrm{a,nom}}^{\mathrm{notched}}(N_{\mathrm{f}})} \cdot \sigma_{y,\mathrm{LM,\ nom}} \tag{4}$$

The result of equation (4) is an average axial stress over a critical distance l. This average axial stress over the critical distance corresponds to the critical distance l, which can be varied for each number of cycles to failure.

The critical distance l was evaluated for each number of cycles to fracture N_{f}. The dependence of the critical distance l on the number of cycles to fracture is shown in Figure 8. An overview of critical distances is given in Table 3 below.

Tab. 3: Critical distances

	min	max	avg
aluminum alloy	0.3189 mm	0.4540 mm	0.3834 mm
steel	0.0702 mm	0.0894 mm	0.0796 mm

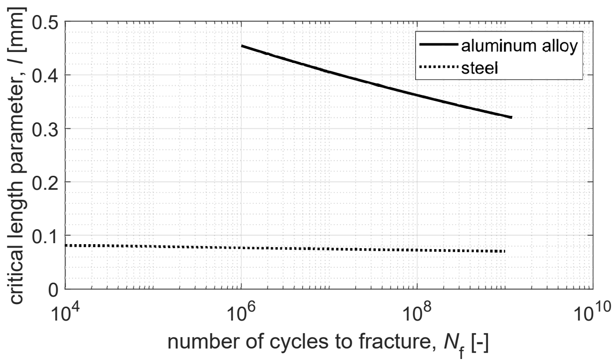


Fig. 8: Critical distance

The dependence of the critical distance parameter l on number of cycles to fracture can be described by equation (5) for aluminum alloy and (6) for steel.

$$l\ [\mathrm{mm}] = 0.8995 \cdot N_{\mathrm{f}}^{-0.04942} \tag{5}$$

$$l\ [\mathrm{mm}] = 0.1085 \cdot N_{f}^{-0.02085} \tag{6}$$

2.5. Prediction of lifetime curves

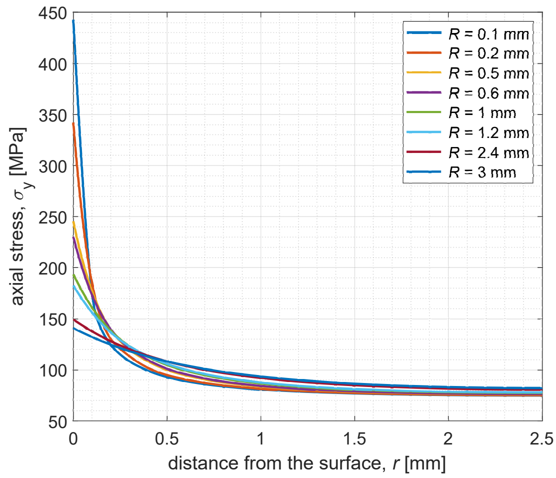
Using the critical length parameter l, it is possible to predict lifetime of notched specimens or components with different notch radii. We assume that this critical distance is a material property and can be used for lifetime predictions. It is an inverse approach of eq. (4).

To estimate the lifetime curves of different notch radii, it is necessary to know the S-N curve of smooth specimens and the critical length parameter for a given material. Then $\sigma_{y,\mathrm{LM}}(l)$ and $\sigma_{y,\mathrm{nom}}$ are determined from numerical calculations of the axial stress distribution for the required notched specimen. The stress concentration factor $K_{\mathrm{t,LM}}(l)$ is determined using the eq. (7). Finally, each point of lifetime curve of notched specimens can be predicted using the eq. (8).

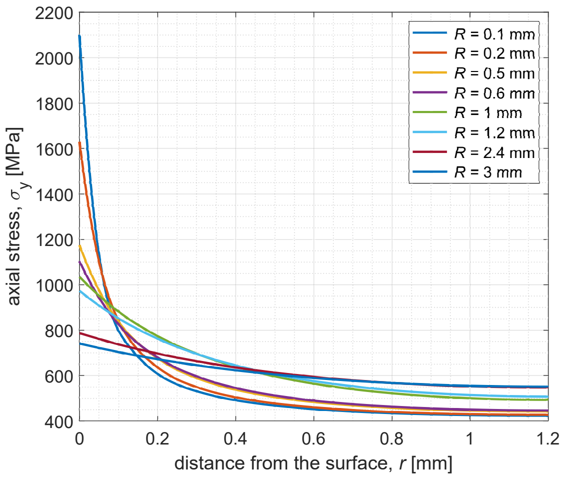
$$K_{\mathrm{t,LM}}(l) = \frac{\sigma_{y,\mathrm{LM}}(l)}{\sigma_{y,\mathrm{nom}}} \tag{7}$$

$$\sigma_{\mathrm{a,nom}}^{\mathrm{notched}}(N_{\mathrm{f}}) = \frac{\sigma_{\mathrm{a,nom}}^{\mathrm{smooth}}(N_{\mathrm{f}})}{K_{\mathrm{t,LM}}(l)} \tag{8}$$

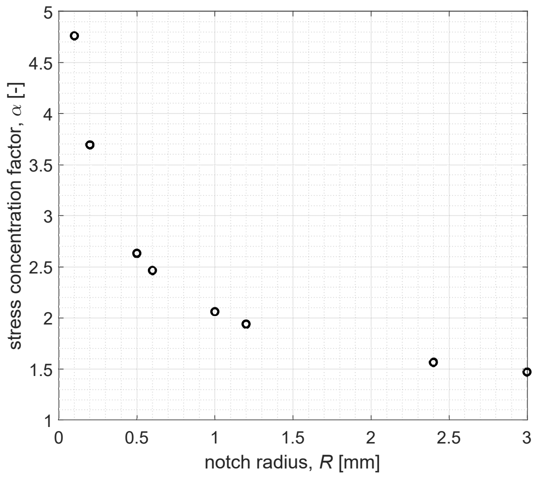
The approach for lifetime predictions was applied for various notch radii, as it is shown in Fig. 9. First of all, axial stresses of each notched specimens with various notch radii from $R = 0.1$ mm to $R = 3$ mm were calculated, see Figures 9(a), 9(b) valid for aluminum alloy and structural steel, respectively. The stress concentration factors α, from eq. (9), of particular notches are plotted in Figures 9(c), 9(d). Finally, in Figures 9(e), 9(f), there are predictions for various notch radii and experimentally determined S-N curves. The S-N curves following from experiments are visualized in black, while the predictions are shown in colors.



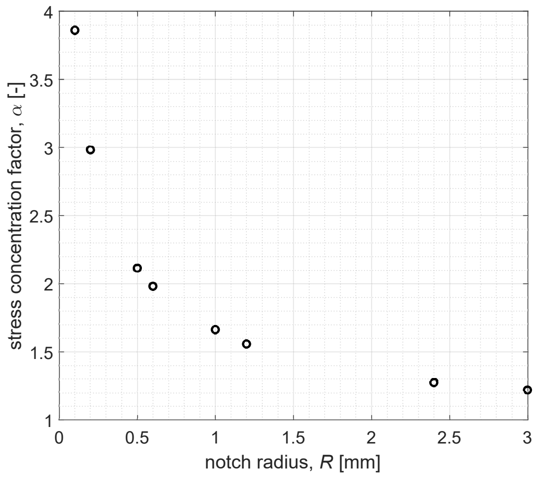
(a) Axial stress distribution, aluminum alloy



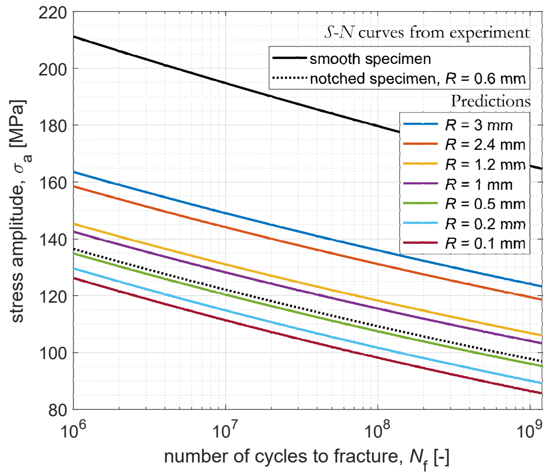
(b) Axial stress distribution, structural steel



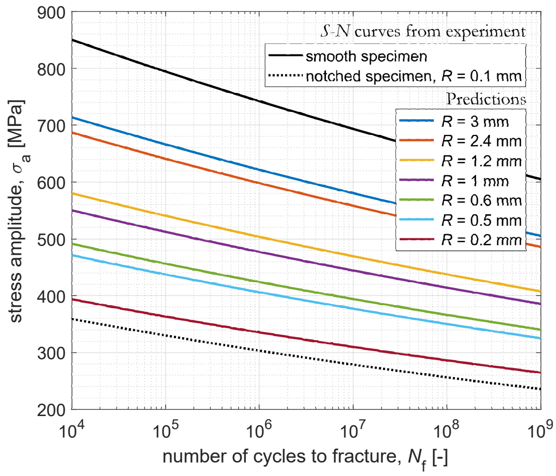
(c) Stress concentration factors, aluminum alloy



(d) Stress concentration factors, structural steel



(e) Lifetime predictions, aluminum alloy



(f) Lifetime predictions, structural steel

Fig. 9: a, b) Axial stress distribution, c, d) Stress concentration factors, e, f) Lifetime predictions

The curves of predictions follow from eq. (8). Note that within calculation of the predictions, critical distances dependent on number of cycles to failure were applied.

$$\alpha = \frac{\sigma_{y,max}}{\sigma_{y,nom}} \quad (9)$$

3. Conclusion

The article summarizes the procedure for prediction of the lifetimes of notched specimens. Using two S-N curves of two sets of samples - smooth and with a model notch, it is possible to predict the lifetime curves of specimens with different notch radii. The study was applied in the field of high-cycle and gigacycle fatigue, where small scale yielding conditions are valid. The main result of the method is the critical distance of two studied materials. The distance is essential for further predictions. It was calculated, that the critical distance slightly depends on the number of cycles to fracture, where the distance decreases with higher number of cycles to fracture. This corresponds to the fact that with increasing N_f the loading level decreases and at the same time crack initiation can be caused by structural defects, voids and inhomogeneities of smaller size. The average value of critical distance was 0.38 mm for aluminum alloy and 0.08 mm for structural steel.

Acknowledgment

The research was supported by the Czech Science Foundation, project No. 21-14886S.

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