CONCRETE SLAB RESPONSE ON MOVING LOAD

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Abstract. *The subject of the article is a numerical simulation of the dynamic response of a concrete slab on an elastic subsoil on the effect of moving vehicle. The vehicle model and the slab model on Winkler and Pasternak subsoil are described. The method of calculating the subsoil constants for both models is presented. The numerical solution is carried out in MATLAB using the ode45 procedure. Considerations are made about the suitability of the used models and possible corrections of results for the Pasternak's model are presented. The results of the solution are presented in numerical and graphical form.*

Keywords

Numerical analysis, moving load, concrete slab, Winkler model, Pasternak model.

1. Introduction

Modeling the movement of vehicles along the transport structures is a current engineering problem that is receiving worldwide attention. The works of Frýba, L. are well known in our conditions [1]. Frýba focused his attention mainly on the analysis of railway bridges [2]. Melcer, J. dealt with dynamic analysis of road bridges [3]. Melcer and his team have recently focused on the dynamic response of pavements to the effects of moving vehicles [4, 5]. In addition to asphalt, also concrete pavements are currently used. Concrete pavements are concreted directly on the construction site. Prefabricated slabs are more often used in the construction of temporary roads. The dynamic response of a concrete pavement to the effects of moving vehicles can be analyzed experimentally or numerically. For numerical analysis, a subsoil model must be selected. In principle, it is possible to choose a spatial or surface model of the subsoil. When choosing a surface model, the Winkler's or Pasternak's model is most often used. Such models are also analyzed in this article.

2. Vehicle Computational Model

For the purpose of this paper the so called quarter model of vehicle [5] was adopted, Fig. 1.

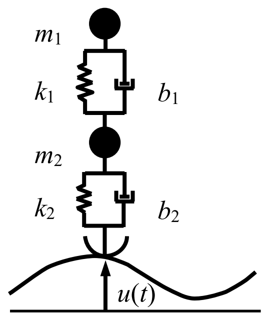

Fig. 1: Quarter model of vehicle.

Equations of motion for calculation of unknown functions $r_i(t)$, describing the vertical displacements of lumped masses m_i, can be written as

$$\ddot{r}_1(t) = \{-k_1 d_1(t) - b_1 \dot{d}_1(t)\}/m_1,$$

$$\ddot{r}_2(t) = \{+k_1 d_1(t) - k_2 d_2(t) + b_1 \dot{d}_1(t) - b_2 \dot{d}_2(t)\}/m_2. \quad (1)$$

The term for calculation of contact force is

$$F(t) = -G_2 + k_2 d_2(t) + b_2 \dot{d}_2(t). \quad (2)$$

The values $d_i(t)$, (i = 1, 2) represent the deformation of connected members of the model in time t. The derivations with respect to time are denoted by the dot above the symbol of dependently variable. G_2 represents the gravity force acting in the contact point. Function $u(t)$ represents the component of kinematical excitation in the contact point of the wheel with the slab.

3. Computational Model of a Concrete Slab

In the spirit of classical mechanics the slab computational model, Fig. 2, is created in the sense of Kirchhoff's theory of thin slabs on elastic foundation of Winkler's and Pasternak's type [1].

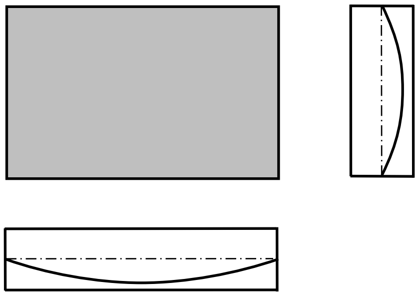

Fig. 2: Slab model, assumption of deflections.

The equation of motion describing the slab vibration on Winkler foundation has the form

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+Cw+\mu\frac{\partial^2 w}{\partial t^2}+2\mu\omega_b\frac{\partial w}{\partial t}= p(x,y,t). \quad (3)$$

In case of Pasternak foundation the equation of motion is

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+C_1 w-C_{2X}\frac{\partial^2 w}{\partial x^2}-C_{2Y}\frac{\partial^2 w}{\partial y^2}+ \mu\frac{\partial^2 w}{\partial t^2}+2\mu\omega_b\frac{\partial w}{\partial t}=p(x,y,t). \quad (4)$$

The assumption about the shape of the bending area of the slab $w_a(x,y)$ due to the load is adopted and the wanted function $w(x,y,t)$ describing the shape of the bending area of the slab in time t is expressed as

$$w(x,y,t)=q(t)w_a(x,y)=q(t)\sin\frac{\pi x}{l_x}\sin\frac{\pi y}{l_y}. \quad (5)$$

The unknown function $q(t)$ has the meaning of generalized Lagrange coordinate. The meaning of other symbols is as follows: D represents the slab stiffness [Nm^2/m], C is the modulus of compressibility of elastic foundation [N/m^3], μ is the mass intensity of the slab [kg/m^2], ω_b is damping angular frequency [rad/s]. The expression $p(x,y,t)$ in equation (3) and (4) is the intensity of continuous dynamic load. In case of discrete moving load the contact force $F_j(t)$ must be transformed on continuous load $p(x,y,t)$ by [1] using the Dirac δ function

$$p(x,y,t)=\sum_j F_j(t)\delta(x-x_j)\delta(y-y_j). \quad (6)$$

4. Pavement Composition

The subject of the analysis is the pavement, the composition of which is shown in Fig. 3. The upper layer is a concrete slab 240 mm thick. For the subsoil it is necessary to determine the constants C, C_1, C_{2X}, C_{2Y}, which characterize its properties in the spirit of Winkler and Pasternak model. We assume an isotropic subsoil $C_{2X}=C_{2Y}=C_2$. The required constants were obtained by numerically simulating the load test in the Laymed program [6]. A circular plate with a radius $a = 0.4$ m is pushed into the subsoil with a pressure $p = 0.07$ MPa ($P = 35.185838$ kN). The calculated subsidences of the subsoil in the distance $a = 0.4$ m and $r = 0.6$ m are $w_0 = 3.29859\text{e-}04$ m and $w_p = 2.37392\text{e-}04$ m, Fig. 4.

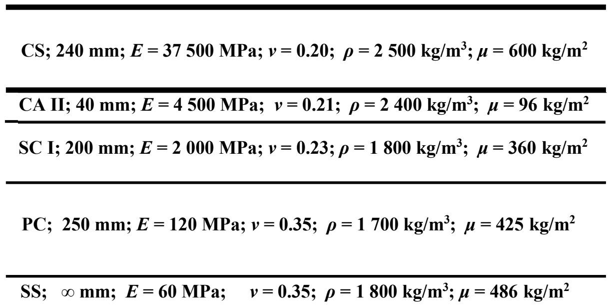

Fig. 3: Pavement composition.

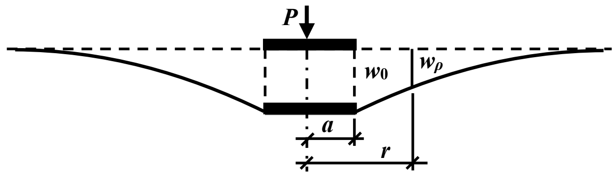

Fig. 4: Loading test.

The Winkler constant C is determined from the ratio

$$C = p/w_0 \quad [\text{N/m}^3]. \quad (7)$$

At Pasternak model we proceed as follows [7]. Dimensionless variables are introduced

$$\rho_0 = a/s\,,\ \ p = r/s\,, \quad (8)$$

where s [m] is the characteristic length of the subsoil model. The ratio w_r is calculated

$$w_r=\frac{w_p}{w_0}=\frac{K_0(\rho)}{K_0(\rho_0)}=\frac{K_0(\frac{r}{s})}{K_0(\frac{a}{s})}\,. \quad (9)$$

From the known value of w_r, the characteristic length of the subsoil model s is calculated using the transcendental equation (9), which must be done in an iterative manner. Then C_1 is calculated using equation (10)

$$C_1=\frac{P}{\pi\cdot w_0\cdot s^2\cdot\rho_0[\rho_0+2K_1(\rho_0)/K_0(\rho_0)]} \quad [\text{N/m}^3]. \quad (10)$$

K_0 and K_1 are Bessel functions of an imaginary argument with the parameter $n = 0$ and 1. Finally, C_2 is calculated using equation (11)

$$C_2 = s^2\cdot C_1 \ [\text{N/m}]. \quad (11)$$

The relationship also applies

$$C_1 = C\cdot k, \quad (12)$$

where

$$k = 1/(1+\frac{2K_1(\rho_0)}{\rho_0 K_0(\rho_0)})\,. \quad (13)$$

The numerical values are as follows: $w_r = 0.719677195$, $s = 1.161901$ m, $C = 212\ 211\ 884.471850$ N/m^3; $C_1 = 16\ 138\ 619.467013$ N/m^3, $C_2 = 21787342.661334$ N/m, $k = 0.076049$. It can be seen from the numerical study that the constant C in the Winkler model and the constant C_1 in the Pasternak model are not numerically identical, $C_1 < C$. From the loading force P, only the $k\cdot P$ is transferred by the slab into the subsoil and the remainder of the

$P(1\text{-}k)$ is transferred by shear distribution to the surrounding environment outside the slab, that is, 7.6 % is transferred by slab and 92.4 % is transferred outside the slab.

5. Numerical Solution

The movement of a quarter model of the vehicle along a concrete slab on an elastic foundation was numerically simulated. The dimensions of the slab are 6.00 x 3.75 x 0.24 m. The vehicle moves in the axis of symmetry of the slab at a speed of $V = 75$ km/h. The vehicle arrives on the slab already vibrated, the slab is at the rest. The initial conditions on the vehicle are as follows: $r_1(0) = -0.02$ m; $r_2(0) = -0.002$ m; $\dot{r}_1(0) = 0.0$ m/s; $\dot{r}_2(0) = 0.0$ m/s (the positive Z axis points upwards). Numerical solution of equations of motion was performed in MATLAB and the ode45 procedure was used [8]. In the first step, the Winkler's subsoil model is considered. The time course of the deflections in the middle of the slab is shown in Fig. 5. The course of the contact forces between the vehicle wheel and the slab is shown in Fig. 6.

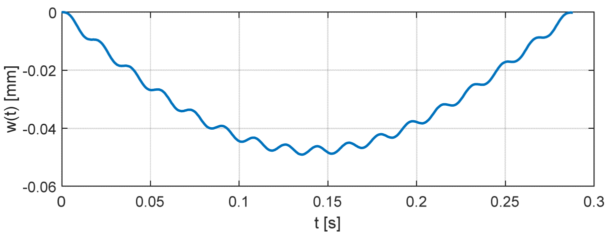

Fig. 5: Deflections in the middle of the slab, Winkler model.

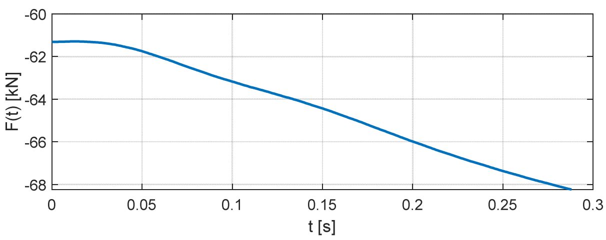

Fig. 6: Contact forces, Winkler model.

The maximum deflection in the middle of the slab has the value $w_{W,\max} = -0.049$ mm and occurs at time $t = 0.135$ s. The maximum and minimum contact forces are as follows $F_{W,\max} = -68.235$ kN, $t_{\max} = 0.288$ s, $F_{W,\min} = -61.293$ kN; $t_{\min} = 0.012$ s.

It is not possible to make a correct solution in this way on Pasternak model, because the model does not take into account the values of shear forces outside the slab on the deflections of the slab. The deflections are unrealistically large, $w_{P,\max} = -0.159$ mm, $t_{\max} = 0.147$ s. However, it is possible to make a correction of the results due to the fact that the slab transmits to the subgrade only the value $k{\cdot}F$ of the contact force, $w_{Pk,\max} = -0.012$ mm, $t_{\max} = 0.147$ s. The corrected course of the deflections in the middle of the slab on Pasternak model is shown in Fig. 7. This correction does not apply to contact forces, $F_{P,\max} = -68.251$ kN; $t_{\max} = 0.288$ s, $F_{P,\min} = -61.231$ kN; $t_{\min} = 0.021$ s, Fig. 8.

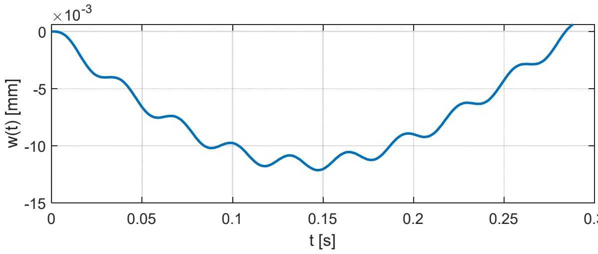

Fig. 7: Corrected deflections in the middle of the slab, Pasternak model.

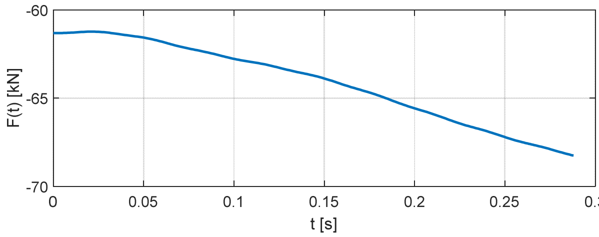

Fig. 8: Contact forces, Pasternak model.

6. Conclusion

The article presents a computational model for the solution of the dynamic response of a concrete slab on an elastic subsoil. The model is derived under certain simplifying assumptions. These assumptions are well suited for the Winkler's subsoil model. In the case of Pasternak subsoil model, the established assumptions are not correct. The model does not take into account the influence of shear forces outside the slab on slab deflections. The deflections of the slab are thus unrealistic. However, corrections can be made to the results with respect to the fact that we know what part of the loading forces will cause deformations under the slab and which part will cause deformations outside the slab. After these corrections, the model provides acceptable results. In the case of contact forces between the vehicle wheel and the slab, both models give practically identical results. A more precise model can be created only in the sense of FEM.

Acknowledgements

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About Author

Jozef Melcer was born in the village Banska Belá, Slovak Republic. He received his PhD. from Applied mechanics in 1987 and DrSc. form Theory of engineering structures in 2001. His research is oriented on numerical and experimental analysis of dynamic problems of transport structure.