Application of Semi-probabilistic Methods to Verification of Series System

Miroslav SÝKORA, Jana MARKOVÁ, Vitali NADOLSKI

Department of Structural Reliability, Klokner Institute, Czech Technical University in Prague, Šolínova 7, Prague, Czech Republic

miroslav.sykora@cvut.cz; jana.markova@cvut.cz; vitali.nadolski@cvut.cz

DOI: 10.35181/tces-2021-0018

Abstract. *Non-linear finite element analysis (NLFEA) has become a widely used tool in the reliability verification of reinforced concrete (RC) structures. Reliability assessment of RC structures is a challenging task and simplified approaches to reliability verifications are needed to allow for routine applications. Simplified semi-probabilistic methods such as the partial factor method (PFM) or the Method of Estimation of Coefficient of Variation (ECoV) may yield over- or under-conservative approximations. This contribution investigates such errors related to the applications of these two methods to series systems; the design resistances obtained by the probabilistic approach are considered as a reference level. It appears that PFM provides good approximations as the method is focused on critical members in the cases under consideration. ECoV may overestimate up to 20% in situations where NLFEA based on mean and/ or characteristic values fails to identify a dominating failure mode. The maximum observed error is attributed mainly to the failure in identifying the type of distribution of the system resistance. The presented limited analysis indicates several directions for further research (analysis of other types of structural systems, cases with non-lognormal resistances, effects of a number of system components, and their mutual correlations, and performance of advanced methods for reliability verification). Recommendations on how to identify situations when a significant error can be expected should be provided for practical applications of simplified semi-probabilistic methods.*

Keywords

NLFEA, ECoV, partial factor method, reinforced concrete, reliability assessment, series systems.

1. Introduction

Non-linear finite element analysis (NLFEA) has become a widely used tool in reliability verification of reinforced concrete (RC) structures as the advancing engineering knowledge makes it possible to design and build more complex structural systems. However, the reliability assessment of RC structures is a challenging task due to the growing complexity of numerical structural models, the large number of random variables, and small failure probabilities related to Ultimate Limit State verification.

The use of partial factor method can be a pragmatic and sufficient approach for many structural systems. However, the global safety concept or the fully probabilistic approach seem to be more appropriate in the cases with strong nonlinear behaviour, the dominating role of tensile and fracture mechanical properties, or with multiple failure modes of similar importance [1], [2], [3]. The fully probabilistic approach makes it possible to realistically consider the randomness of input parameters such as material, geometrical, and load characteristics. However, such an approach is generally time-consuming and semi-probabilistic approaches are commonly applied in engineering practice.

To facilitate the routine applications of semi-probabilistic methods (but also of a fully probabilistic approach), operational rules are being incorporated into the present codes of practice; for instance:

- Draft prEN 1990:2021 for the basis of design should include general provisions for nonlinear analysis and a specific section on the design assisted by numerical simulations (such as outcomes of NLFEA).
- Besides the general concrete-specific rules for NLFEA, prEN 1992:2021 for the design and assessment of concrete structures should include the informative Annex F "Nonlinear analyses procedures". This annex provides operational guidance for applications of NLFEA along with the partial factor method (*PFM*), the global factor method (e.g., using the Method of Estimation of Coefficient of Variation – *ECoV*) or the probabilistic approach. Provisions on how to consider NLFEA-related model uncertainties will

be provided.

- draft *fib* Model Code 2020, providing a background for the concrete structures-related provisions in Eurocodes, is aimed to give detailed guidance regarding the validation, quantification of modelling uncertainty, and application of various reliability verification methods including the *PFM*, *ECoV*, and probabilistic approach.

Simplified semi-probabilistic methods such as *PFM* or *ECoV* are commonly devised to yield adequate estimates in most practically relevant applications (e.g. of design resistance of a structural system in this study). Inevitably, over- or under-conservative approximations might be obtained in some cases where the methods are too simplified.

A detailed study of such discrepancies results in a wide range of design situations of practical relevance is the subject of the research project supported by the Czech Science Foundation under Grant 20-01781S; selected results are presented in this contribution.

First insights regarding the performance of *PFM* and *ECoV* have been obtained by:

- Cervenka J. et al. [4] who investigated shear resistance of a tested beam where the minimum of the stirrups and concrete contributions (thus a series system of two components) were considered. The study indicated that possibly significant errors might be expected for both methods in particular situations. However, all observations were rather fuzzy due to the dominating effect of model uncertainty.
- Sykora et al. [5] took the basis from [4] but adopted a more realistic model for shear resistance where the stirrups and concrete contributions (~parallel system) are summed up. They observed insignificant errors, with both methods providing slightly conservative estimates.

It seems that the verification of series systems presents a challenge concerning applications of *PFM* and *ECoV*. A series system is thus analysed here. In contrast to [4], the following modifications are adopted here:

- Model uncertainty—commonly treated separately, beyond applications of *ECoV* and possibly also of *PFM*—is not considered here to obtain clearer insights into the performance of the two semi-probabilistic methods.
- The probabilistic models for variables are adopted from the background documents to EN 1992-1-1:2004 and prEN 1992-1-1:2021 so as they well correspond to the partial factors for materials. Consequently, for situations with a single failure mode dominating, the *PFM* and *ECoV* design resistances well match those based on the probabilistic approach, and the deficiencies of the semi-probabilistic methods in the situations with truly system behaviour can be well investigated.

The contribution concludes with a discussion about the limitations of the presented analysis and the need for further investigations.

2. Semi-probabilistic Methods Under Consideration

In NLFEA, the reliability condition is commonly formulated as:

$$E_d \leq R_d, \quad (1)$$

where E_d represents the design value of load effect and R_d is the design value of resistance. This contribution is focused on estimating R_d by *PFM* and *ECoV*.

According to *PFM*, design resistance may be calculated by NLFEA using the design values of material parameters. In this study, the following relationship is applied:

$$R_{d,PFM} = \mathrm{R}_{mod}(f_{ck} / \gamma_C; f_{yk} / \gamma_S), \quad (2)$$

where R_{mod} = model resistance; f_{ck} = characteristic value of concrete compressive strength; γ_C = partial factor for concrete (further information is in Section 3); f_{yk} = characteristic value of yield strength of steel reinforcement; and γ_S = 1.15 – partial factor for steel.

Based on the global factor method, Červenka V. [3,4] proposed an approach suitable for routine NLFEA applications – *ECoV*. The method assumes that the distribution of resistance, R, can be described by a two-parameter lognormal distribution [6], which is described by mean R_m and coefficient of variation V_R. The underlying assumption of lognormal resistance is reasonable for resistances of many structural sections, members, and perhaps also for systems. According to *ECoV*, the two parameters of a lognormal distribution can be estimated as:

$$R_m \approx \mathrm{R}_{mod}(f_{cm}; f_{ym}), \quad (3)$$

$$V_{R,\mathrm{ECoV}} = \ln(R_m / R_k) / 1.65, \quad (4)$$

where f_{cm} and f_{ym} = mean values of concrete compressive strength and yield strength of steel reinforcement, respectively; and $R_k \approx \mathrm{R}_{mod}(f_{ck}; f_{yk})$ is the estimate of a characteristic value of resistance (estimate of a 5% fractile of R). The design resistance is then estimated as:

$$R_{d,\mathrm{ECoV}} = R_m \exp(-\alpha_R \beta V_{R,\mathrm{ECoV}}), \quad (5)$$

where α_R = 0.8 is the sensitivity factor for a dominating resistance parameter; and β = 3.8 is the target reliability index according to EN 1990:2002 for Ultimate Limit States (both for a reference period of 50 years). In Eq. (4) and Eq. (5), the effect of model uncertainty is intentionally neglected as discussed above; see also [4]. Note that the adopted values of α_R and β imply that the design value of system resistance is determined as a 1.18‰ fractile of its distribution.

ECoV separates aleatory and epistemic uncertainties and reflects the distinct nature of different failure modes (yielding of reinforcement, failure of concrete in compression or tension, geometrical instability) with only two NLFEA runs to estimate R_m and R_k. As a widely accepted—simple and mostly sufficiently accurate—method, *ECoV* has been introduced in *fib* MC 2010, draft *fib* MC 2020, and prEN 1992-1-1:2021.

3. Cases under Consideration

Two fundamental cases of two-component series systems are considered in the following analysis:

1. *Case 1* with two failure modes acting as a series system with lognormally distributed component resistances,
2. *Case 2* is based on *Case 1*, but assuming normally distributed component resistances.

Design values obtained by *PFM* and *ECoV* are normalised to those obtained by the probabilistic approach, $R_{d,prob}$. For instance, when $R_{d,semi\text{-}prob} / R_{d,probab} > 1$, a semi-probabilistic method overestimates design resistance, thus being on the unsafe side.

As discussed above, model uncertainty is not considered in the numerical analysis as it is typically treated separately, beyond the application of ECoV. Note that the justification of the values of γ_C and γ_S according to [7] indicates that the model uncertainty factors related to the recommended values in EN 1992-1-1:2004 are very close to unity and thus the values of the partial factors are adopted without any adjustment with respect to model uncertainty (intentionally ignored in the following analysis).

4. Series System with Lognormal Component Resistances (*Case 1*)

It is often argued that simplified semi-probabilistic methods may fail in cases with several local extrema that are typically caused by multiple failure modes. To verify this, *Case 1* is focused on a simple series system that can well represent for instance shear resistance of a concrete member that is determined as the minimum of the concrete and stirrups contributions, V_c and V_s respectively:

$$R = \min(V_c, \rho\, V_s), \tag{6}$$

where ρ is a deterministic study parameter arbitrarily varied disregarding practical constraints such as bounds on reinforcement ratios or detailing rules. In engineering applications, it can represent a reinforcement ratio, $\rho = A_s / A_c$ (with denoting an area of steel reinforcement or of section). Relationship (6) can then be re-written as:

$$R / A_c = \min(f_c', \rho\, f_s'). \tag{7}$$

Hereafter, system resistance based on (7) is referred to as R without indicating the normalisation with respect to A_c to simplify the notation.

Uncertainties in the two contributions are described by coefficients of variation of f_c'and f_s' that account for variability of the respective strengths and geometrical variables. Mutually statistically independent contributions are described by lognormal distributions with the following characteristics:

- f_{cm}' = 29.1 1MPa, $V_{fc'}$ = 21.3%, and $f_{c0.05}$' = f_{ck}' = 20.5 MPa,
- f_{ym}' = 489 MPa, $V_{fy'}$ = 10.1%, and $f_{y0.05}$' = f_{yk}' = 414 MPa.

The coefficients of variation are determined in such a way that the design value of the concrete and reinforcement strengths correspond to the value determined by *PFM*:

$$f_{cd}' = f_{cm}' \exp(-\alpha_R\, \beta\, V_{fc'}) = f_{ck}' / 1.35, \tag{8}$$

$$f_{yd}' = f_{ym}' \exp(-\alpha_R\, \beta\, V_{fy'}) = f_{yk}' / 1.15. \tag{9}$$

The value of γ_C is reduced as the common value of 1.5 covers additional factor of 1.15 to account for uncertainty arising from concrete being tested on purpose-made specimens in a lab, rather than in the finished structure [7].

It must be emphasised that the adopted values of CoVs are characteristic rather for resistances than for strengths; this is why the symbol " ' " is used in relationships from (7) to (9).

Fig. 1 displays variability of $R_{d,PFM} / R_{d,probab}$ and $R_{d,ECoV} / R_{d,probab}$ with reinforcement ratio. For low ρ-values, the system resistance is governed by the reinforcement contribution while the concrete contribution becomes more important with increasing ρ.

It appears that *PFM* provides a good approximation in the case of a series system. The method is focused on critical members and thus is well suited to the analysis of series systems. Note that this conclusion is valid only for well-calibrated values of the partial factors for resistances (failure modes) under consideration.

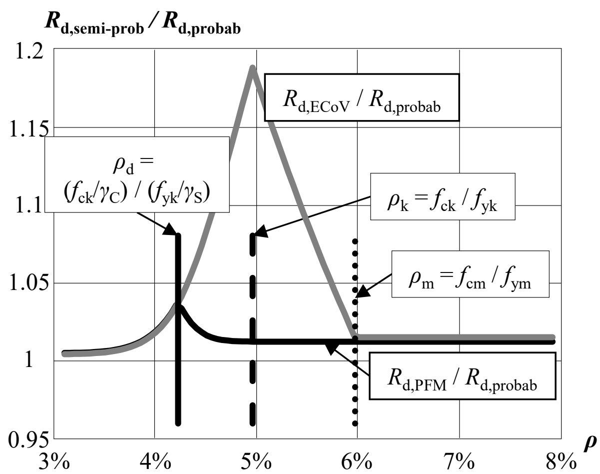

Fig. 1: Variability of $R_{d,PFM} / R_{d,probab}$ and $R_{d,ECoV} / R_{d,probab}$ with ρ (*Case 1*).

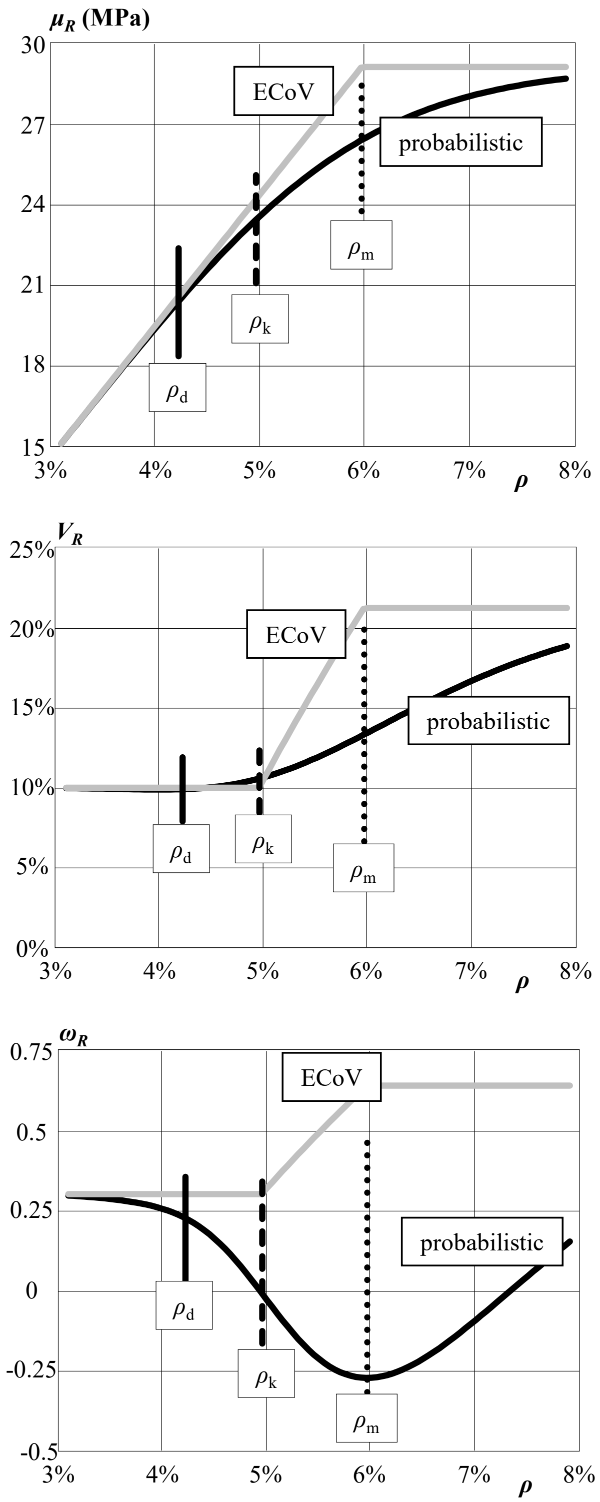

Fig. 2: Variability of statistical characteristics estimated by *ECoV* and probabilistic approach with ρ (from top to bottom – mean values, coefficients of variation, and coefficients of skewness).

In contrast, *ECoV* may overestimate up to 18% for ρ-values ranging from ρ_d to ρ_m. Approximately for $\rho > \rho_d$, the concrete contribution becomes decisive for system resistance while *ECoV* identifies a dominating steel contribution by:

- both analyses based on the mean and characteristic values for $\rho_d < \rho < \rho_k$,
- by analysis based on the mean values for $\rho_k < \rho < \rho_m$.

To further investigate these deficiencies, Fig. 2 shows variability of mean values, coefficients of variation, and coefficients of skewness of system resistance estimated by *ECoV* and by the probabilistic approach.

The unsafe error of 18% observed for $R_{d,ECoV}(\rho_k)$ is attributed to the following:

1. Key is failure in identifying a type of distribution of system resistance – ignoring the bimodal character of the distribution and without regard to an actual coefficient of skewness (Fig. 2 – positive ω_{ECoV} leads to a higher design resistance in comparison to negative ω_{probab}). Detailed analysis indicates that this aspect leads to an error of about 13%.

2. Other two less important effects contributing to the unsafe error are a small overestimation of the mean value (3%) and underestimation of coefficient of variation (2%).

Interesting to note is that the error nearly vanishes for $R_{d,ECoV}(\rho_m)$. Failure to identify a type of distribution and overestimated mean would lead to a significant overestimation, 16% + 10%, but they are outweighed by a markedly overestimated coefficient of variation ($V_{R,ECoV}(\rho_m)$ = 21.3% while $V_{R,probab}(\rho_m)$ = 13.4%). With increasing reinforcement ratio, the errors decrease and vanish as no system behaviour occurs with the concrete contribution entirely dominating.

5. Normal Component Resistances (*Case 2*)

Case 2 investigates the same series system as in *Case 1*, but the concrete and yield contributions are now described by normal distribution with the following adjusted characteristics:

- $V_{fc'}$ = 14.4%, and $f_{c0.05}'$ = 22.2 MPa,
- $V_{fy'}$ = 8.1%, and $f_{y0.05}'$ = 394 MPa.

while the mean values, f_{cm}' and f_{ym}' remain unchanged. Similarly as in Equations (8) and (9), the coefficients of variation are determined to correspond to the partial factors:

$$f_{cd}' = f_{cm}' \, (1-\alpha_R \, \beta \, V_{fc'}) = f_{ck}' \, / \, 1.35, \quad (10)$$

$$f_{yd}' = f_{ym}' \, (1-\alpha_R \, \beta \, V_{fy'}) = f_{yk}' \, / \, 1.15. \quad (11)$$

Fig. 3 displays variability of $R_{d,PFM} / R_{d,probab}$ and $R_{d,ECoV} / R_{d,probab}$ with reinforcement ratio. Variation of the ratios is similar to that observed in *Case 1*. The *ECoV* error reaches up to 20% for $R_{d,ECoV}(\rho_k)$ and converging to 8% even for high reinforcement ratios. Despite the latter is the case with one dominating component (concrete), *ECoV* assumes a lognormal distribution which is inadequate to normally distributed concrete strength. For low ratios, this error is also present, but it is of small magnitude due to a low coefficient of variation of resistance related to steel yielding. The unsafe approximation by *ECoV* is now primarily attributed to failure in identifying a type of distribution of system resistance. Novak L. and Novak D. [8] adjusted *ECoV* for the situation when a normal distribution is assumed for resistances. Fig. 3 shows that this adjustment corrects the *ECoV* estimates for low and high ρ-values where resistances of individual components are dominating. However as in Section 4, the unsafe error of about 20% is found for ratios close to ρ_k.

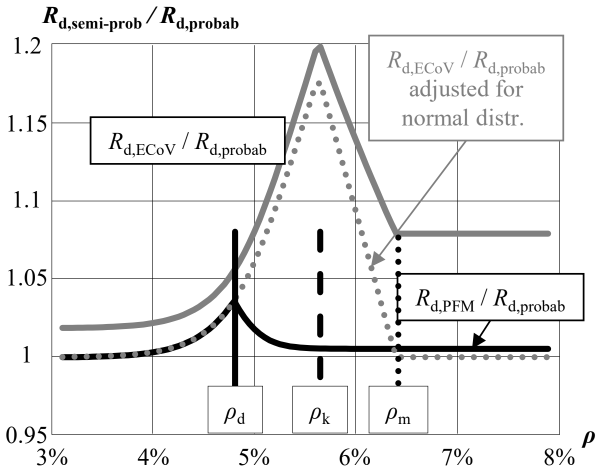

Fig. 3: Variability of $R_{d,PFM} / R_{d,probab}$ and $R_{d,ECoV} / R_{d,probab}$ with ρ (*Case 2*).

6. Discussion

The presented limited analysis of the series systems with two failure modes indicates some directions for further research:

1. Most concrete structural systems are deemed to have properties closer to parallel systems as they are often indeterminate, providing for multiple load paths. Preliminary results for parallel systems, partly presented in [5], indicate that *ECoV* performs very well while *PFM* tends to be rather conservative for parallel systems when $\alpha_R = 0.8$ is considered for both concrete and steel.

2. Positive correlations between failure modes are expected to reduce the *ECoV* error for both types of systems. In contrast, the errors may amplify with an increasing number of failure modes of similar importance. These counteracting effects should be numerically investigated in the future.

3. The error in cases with non-lognormal system resistance may be significant. In this context, situations with important variability of geometrical properties (particularly for small-size members) or with non-lognormal material properties (possibly for UHPC) should be analyzed.

4. Latin Hypercube Sampling (LHS) with a low number of simulations—also included in the draft *fib* MC 2020—is often applied to verify the performance of simplified semi-probabilistic methods. Statistical uncertainty in LHS estimates for cases with multiple failure modes remains to be investigated and quantified.

5. Fig. 2 clearly shows that *ECoV* might considerably overestimate coefficient of variation of system resistance. The advanced variance reduction approaches such as Taylor series expansion or Eigen ECoV [8] may provide significant improvements, but require more complex information about stochastic models and failure regions. Benefits and costs related to the applications of these advanced methods should be further explored.

7. Concluding Remarks

Previous pilot investigations revealed that the Method of Estimation of Coefficient of Variation (*ECoV*) might overestimate the design resistance of some series systems. A detailed analysis of two study cases using *ECoV* and the partial factor method (*PFM*) presented here demonstrates that for the series systems under consideration:

- *PFM* provides good approximations. The method is focused on critical members and thus is well suited to the analysis of series systems. However, this conclusion is valid only for well-calibrated values of the partial factors for resistances (failure modes) under consideration.
- *ECoV* may overestimate up to about 20% in situations where NLFEA based on mean and/ or characteristic values fail to identify a dominating failure mode.
- The maximum observed error is attributed mainly to failure in identifying a type of distribution of system resistance; the other two, less important are a small overestimation of the mean value and underestimation of coefficient of variation.

The presented limited analysis indicates a number of directions for further research, including the analysis of:

- parallel and mixed series-parallel systems,
- cases with strongly non-lognormal resistance,
- effects of many components of the system and correlations between component resistances,
- performance of advanced methods for reliability verification of RC structures.

In general, recommendations on how to identify situations when a significant error can be expected should be provided for practical applications of simplified semi-probabilistic methods.

Acknowledgements

This study has been supported by the Czech Science Foundation under Grant 20-01781S.

References

[1] ALLAIX, D.L., V.I. CARBONE and G. MANCINI. Global safety format for non-linear analysis of reinforced concrete structures. *Struct. Concr.* 2013, Vol. 14, Nr. 1, pp. 29-42. ISSN 1751-7648. DOI: 10.1002/suco.201200017.

[2] CERVENKA, V. Global Safety Format for Nonlinear Calculation of Reinforced Concrete. *Beton- und Stahlbetonbau.* 2008, Vol. 103, Nr. 2008, pp. 37-42. DOI: 10.1002/best.200810117.

[3] CERVENKA, V. Reliability-based non-linear analysis according to fib Model Code 2010. *Struct. Concr.* 2013, Vol. 14, Nr. 1, pp. 19-28. ISSN 1751-7648. DOI: 10.1002/suco.201200022.

[4] CERVENKA, J., V. CERVENKA, M. SYKORA and J. MLCOCH. Evaluation of Safety Formats for Structural Assessment Based on Nonlinear Analysis. In *Proc. EURO-C 2018.* London: CRC Press, 2018, pp. 669-678. ISBN 9781351726764.

[5] SYKORA, M., J. CERVENKA, V. CERVENKA, J. MLCOCH, D. NOVAK and L. NOVAK. Pilot Comparison of Safety Formats for Reliability Assessment of RC Structures. In *Proc. fib Symp. 2019.* Lausanne: International Federation for Structural Concrete, 2019, ISBN 9782940643004.

[6] HOLICKY, M. *Introduction to Probability and Statistics for Engineers.* Berlin: Springer-Verlag, 2013. 181 pp. ISBN 978-3-642-38299-4. DOI: 10.1007/978-3-642-38300-7.

[7] EUROPEAN CONCRETE PLATFORM. *Eurocode 2 Commentary.* 2008.

[8] NOVAK, L. and D. NOVAK. Estimation of coefficient of variation for structural analysis: The correlation interval approach. *Struct.Saf.* 2021, Vol. 92, pp. 102101. ISSN 0167-4730. DOI: https://doi.org/10.1016/j.strusafe.2021.102101.

About Authors

Miroslav SÝKORA was born in České Budějovice, Czech Republic. He was appointed Associate Professor at Faculty of Civil Engineering, CTU in Prague in 2015. His research interests include the basis of structural design and of assessment of existing structures, structural reliability, probabilistic optimisation, load modelling, risk assessment of technical systems, and applications of probabilistic methods in structural design.

Jana MARKOVÁ was born in Prague, Czech Republic. She was appointed Associate Professor at Faculty of Civil Engineering, CTU in Prague in 2007. Her research interests include basis of structural design and assessment of existing structures, structural reliability, load modelling with special focus on climatic action effects, investigations of climate change effects on structural reliability, and applications of probabilistic methods in structural design.

Vitali NADOLSKI was born in Minsk, Belarus. He received his Ph.D. degree from the Brest State Technical University in 2015. His research interests include reliability analyses, FEA-based design, and resistance models of steel structures.