# Determination of Critical Length Parameter for Fatigue Lifetime Predictions of Notched Specimens

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DOI: 10.35181/tces-2022-0009

***Abstract.*** *The paper reviews the research results to date on critical length parameters of notched aluminum specimens with various notch radii. The Theory of Critical Distances (TCD) was applied in form of the Line Method for the determination of the critical length parameter. This parameter can be used for lifetime predictions of notched components. The output is a comparison of critical length parameters of specimens with various notch radii. Numerical calculations as well as experimentally obtained data were employed in the analysis. Fatigue experiments were performed at fatigue loading device working at ultrasonic frequency of* 20 kHz*. Thus we were able to reach very high number of cycles to failure.*

## Keywords

***Critical length parameter, Wöhler curve, notches, Theory of Critical Distances, Line Method, lifetime predictions, fatigue, very high cycle fatigue.***

## 1. Introduction

The method for prediction of fatigue lifetime of notched specimens is based on knowledge of experimental fatigue data and numerically determined stress distribution in the notch vicinity. In the study, fatigue loading was realized at an ultrasonic frequency, thus very high cycle fatigue (gigacycle fatigue) region can be reached. The predictions of the lifetime are done using numerical-analytical approaches based on average axial stresses calculated over a critical distance from the notch tip.

### 1.1. Gigacycle fatigue

In recent years, the interest in studying very high cycle fatigue has been developed. Various materials do not exhibit conventional endurance fatigue limits. The experimental data show that fatigue failure can occur at $10^7$ cycles, or $10^9$ cycles and beyond. This fatigue area is called very high cycle or gigacycle fatigue [1]. Among the materials in which this gigacycle fatigue phenomenon is observed are for example high strength steels [2], titanium alloys [6], aluminum alloys [5, 10]. In the field of low-cycle or high-cycle fatigue, surface fracture usually occurs, whereas in the very high cycle fatigue, internal defects (inclusions, voids) play a stronger role, and thus internal failure initiation („fish-eye“) can be observed, Fig. 1.

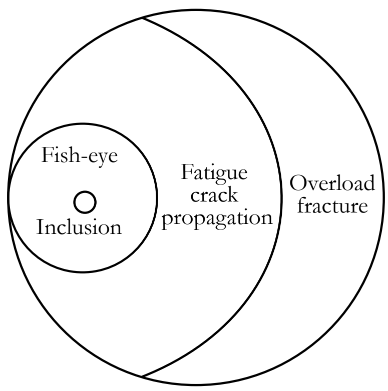



**Fig. 1:** Fracture surface with „fish-eye“ - scheme [4]

Accelerated fatigue tests are realized using an ultrasonic fatigue testing machine [9], which works at the frequency of 20 kHz. The specimens were modeled using axisymmetry and PLANE183 elements. Subsequently, modal analyses were performed and the lengths of the specimens were tuned, so their intrin-

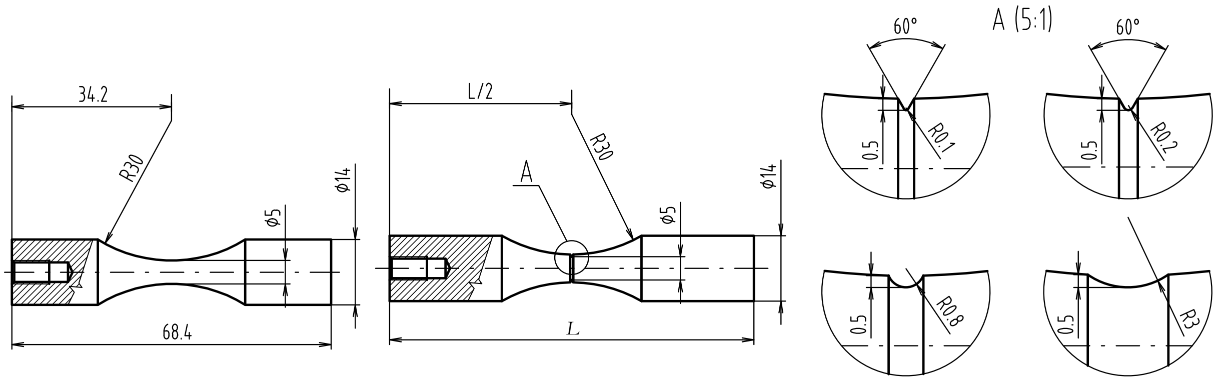



**Fig. 2:** Parameters of smooth and notched specimens

sic frequencies of longitudinal oscillations were close to 20 kHz. In the area of a notch, the density of finite elements was increased. The same models were used to determine the axial stress distributions. The geometries of smooth and notched specimens are shown in Fig. 2. The length $L$ of the notched specimens varies depending on the radius $r$ of the notch. The lengths $L$ of the notched specimens are given in Tab. 1.

| $r$ [mm] | 0.1 | 0.2 | 0.8 | 3.0 |
|---|---|---|---|---|
| $L$ [mm] | 78.1 | 78.0 | 77.6 | 76.7 |

**Tab. 1:** Lengths of notched specimens

## 1.2. A method for prediction of $S$-$N$ curves of notched specimens

In this review, the principles of the Theory of Critical Distances (TCD) are used for the determination of the critical length parameter of notched specimens. For an assessment of notches in the high-cycle fatigue region, the formalism of the TCD was first proposed by Neuber (1958) and Peterson (1959). 50 years later, Susmel and Taylor (2007) reformulated the TCD to estimate fatigue damage [7].

The TCD is a group of theories for predictions of failure conditions of notches and other stress concentrators. All approaches are based on LEFM or can be thought of as an extension of LEFM. The most popular approaches are the Point Method and the Line Method. In the Point Method, the stress is considered and evaluated at a single point. In the Line Method, the stress to be considered is the average stress along the line (critical distance) from the notch root. Failure is predicted to occur if this stress is greater than some critical value [8].

In our research, the TCD was applied in the form of the Line Method, which consists in evaluating the average stress over the critical distance from the tips of notches. There are many formulas how to determine critical length parameters, some of them come from material properties [3]. In our papers, the critical length parameter $l$ was determined from a comparison of experimental fatigue data measured on smooth and notched specimens.

The following method formulates the relationship between the $S$-$N$ curves of experimental data and axial stress in the notched specimen. Experimental data represent failure stresses and numbers of cycles of notched and smooth specimens. Axial stresses in notched specimens were determined by a finite element method software ANSYS Mechanical APDL 2021 R2. Axial stress distribution was calculated using harmonical analysis, where the displacement load applied at the end of the specimens was 10 $\mu$m. The example of stress concentration around the notch with a radius $r = 0.1$ mm is shown in Fig. 4. The principle of the method is shown in Fig. 3 and is based on the equality of stress ratios, eq. 1.

$$\frac{\sigma_{\mathrm{a}}^{\mathrm{n}}}{\sigma_{\mathrm{a}}^{\mathrm{s}}} = \frac{\sigma_{y,\mathrm{nom}}}{\overline{\sigma_y(l)}} \tag{1}$$

Stresses $\sigma_{\mathrm{a}}^{\mathrm{n}}$ and $\sigma_{\mathrm{a}}^{\mathrm{s}}$ represent the fracture stresses of notched and smooth specimens, and they are determined for some specific but equal number of cycles to fracture. The principle of the prediction method is to find the distance $l$ from the notch for which the ratio of the stresses $\sigma_{\mathrm{a}}^{\mathrm{n}}$ and $\sigma_{\mathrm{a}}^{\mathrm{s}}$ is equal to the ratio of stresses $\sigma_{y,\mathrm{nom}}$, and $\overline{\sigma_y(l)}$. These stresses can be found in the stress distribution of the notched specimen, see Fig. 3, and eq. 2, 3.

$$\sigma_{y,\mathrm{nom}} = \frac{1}{R}\int_0^R \sigma_y \, dx \tag{2}$$

$$\overline{\sigma_y(l)} = \frac{1}{l}\int_0^l \sigma_y \, dx \tag{3}$$

Stress $\sigma_{y,\mathrm{nom}}$ represents the average stress over the cross-section of the notched specimen. Stress $\overline{\sigma_y(l)}$ represents average stress over a critical distance $l$ and its

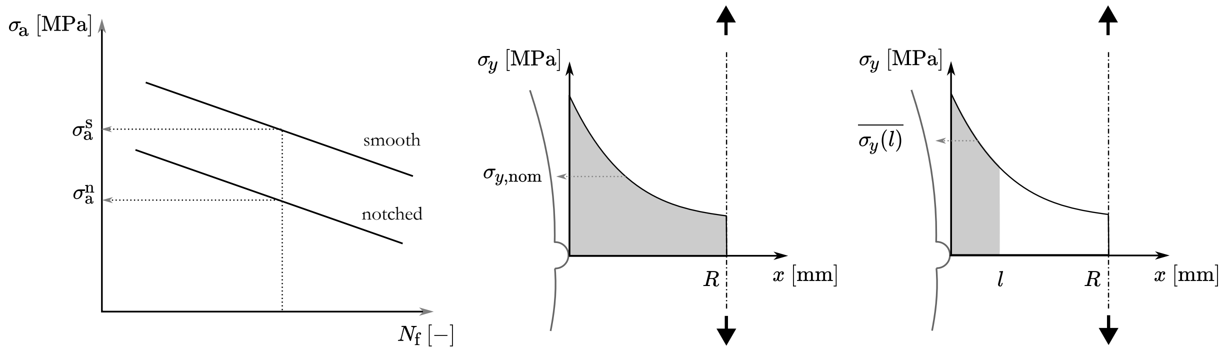



**Fig. 3:** Prediction method: Wöhler curves and axial stress distribution

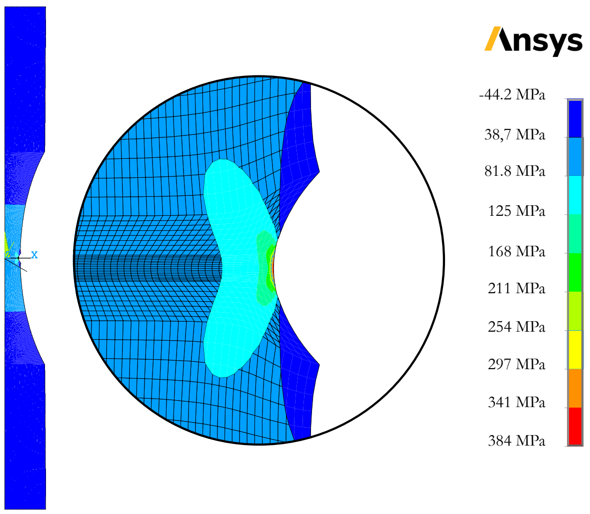



**Fig. 4:** Stress concentration around the notch with radius $r = 0.1$ mm

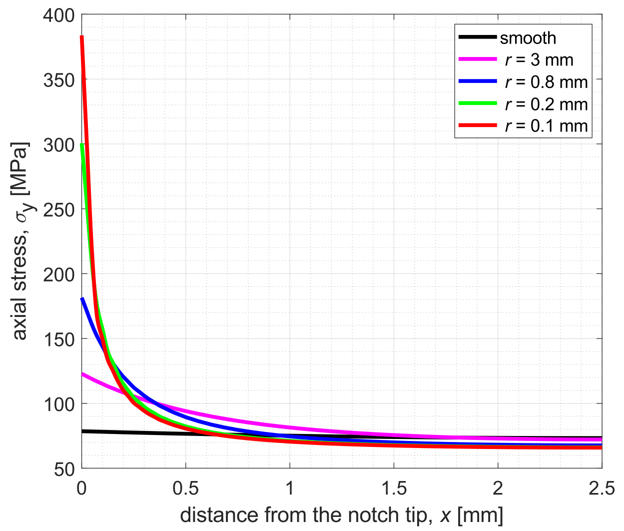



**Fig. 5:** Comparison of axial stresses $\sigma_y$ - smooth and notched specimens

values is first obtained from the formula 1. Subsequently, the critical length $l$ is determined from the distribution of the axial stress and from the known value of the stress $\overline{\sigma_y}(l)$ so that equation 3 is satisfied. The specimens with various notch radii $r$ have different stress distributions, as it is shown in Fig. 5.

## 2. Method application

In the following chapters, experimental data and observed results are described.

### 2.1. Experimental data and Wöhler curves

Experimental results were generated under a fully reversed cycle (load ratio equal to -1). *S-N* curves (Wöhler curves) were described by least-squares approximation of experimental data. Wöhler curves can be described in the form 4.

$$\sigma_a = A \cdot N_f^B \tag{4}$$

The approximations are shown in figure 6(a). The determined constants of the Wöhler curves and the sum of squared errors (SSE) are shown in Tables 2, 3.

| $r$ | 0.1 | 0.2 | 0.8 | 3 | smooth |
|---|---|---|---|---|---|
| $A$ | 310.4 | 297.8 | 419.8 | 297 | 318 |
| $B$ | -796e-5 | -727e-5 | -833e-5 | -471e-5 | -304e-5 |

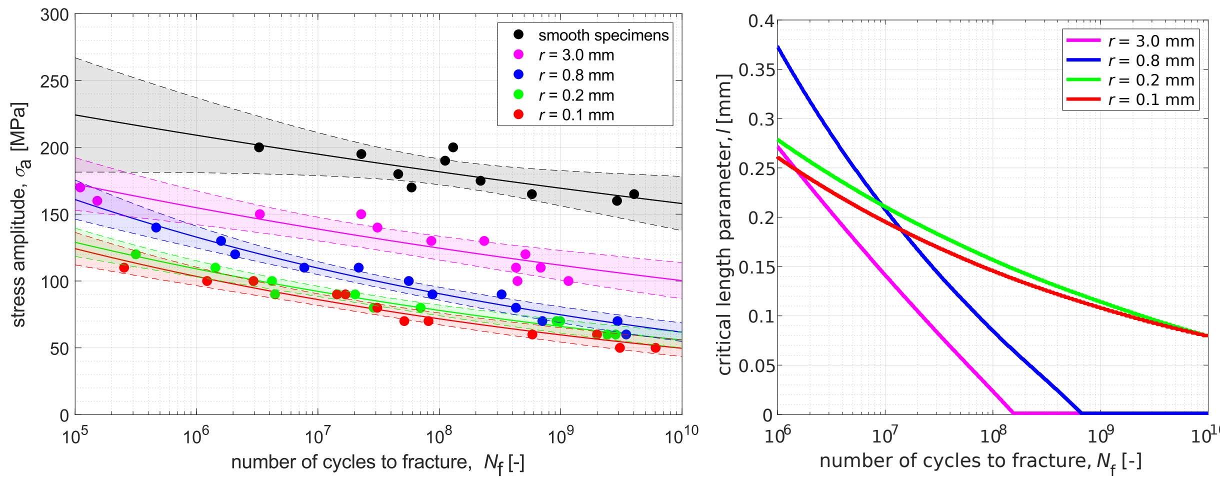
**Tab. 2:** Constants of the Wöhler curves for various $r$ [mm]

| $r$ [mm] | 0.1 | 0.2 | 0.8 | 3 | smooth |
|---|---|---|---|---|---|
| SSE [-] | 235.3 | 149.6 | 284.1 | 1025 | 828.3 |

**Tab. 3:** Goodness of fit: SSE

### 2.2. Fracture surface analysis

After fatigue failure, selected fracture surfaces were investigated by using scanning electron microscopy




(a) (b)

**Fig. 6:** (a) Lifetime curves and 95% prediction bounds, (b) Critical length parameters dependent on number of cycles to fracture

(SEM). SEM images are shown in Figures 7, 8. At fracture surfaces of some smooth specimens, the internal crack initiation and formation of the fish-eye were observed, while in notched specimens, multiple surface crack initiation occurred.

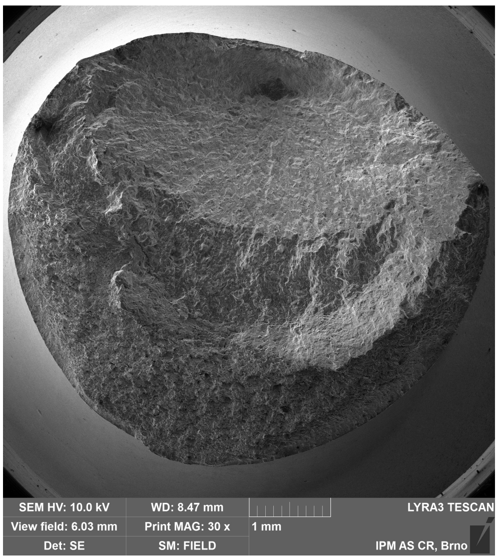



**Fig. 7:** Fracture surface, smooth specimen, $N_\mathrm{f} = 1.1 \cdot 10^8$

## 2.3. Determination of critical length parameters

The critical distances are determined from the $S$-$N$ curves for smooth specimens and each notch radius. For reliable prediction of lifetime of notched specimens, the critical distances gained from various notch radii should be similar. The critical distance depends on the number of cycles to failure $N_\mathrm{f}$. The critical length parameters of all notched specimens were calculated according to the procedure described in Chapter 1.2. Calculations were performed from a set of curves (smooth and specific notch) and the corresponding stress distribution of the notched specimen. The critical length parameter was determined for $N_\mathrm{f} > 10^6$, in this region, there were enough data from experiments. The minimum, maximum, and average critical length parameters for each notch radius are shown in Table 4.

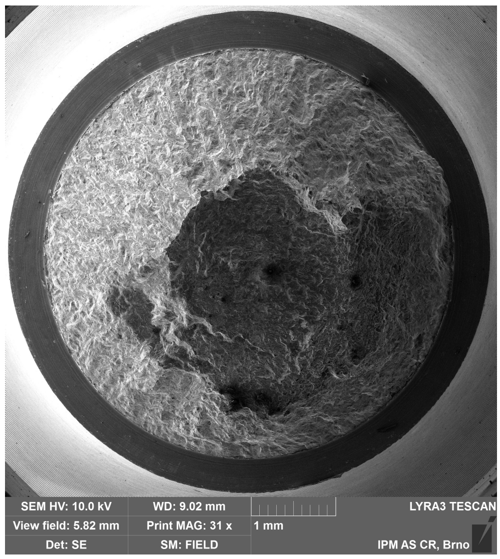



**Fig. 8:** Fracture surface, $r = 0.1$ mm, $N_\mathrm{f} = 1.2 \cdot 10^6$

| $r$ [mm] | 0.1 | 0.2 | 0.8 | 3 |
|---|---|---|---|---|
| $l_{min}$ | 0.08 | 0.08 | 0 | 0 |
| $l_{max}$ | 0.26 | 0.28 | 0.37 | 0.27 |
| $\bar{l}$ | 0.15 | 0.16 | 0.12 | 0.07 |

**Tab. 4:** Critical length parameters $l$ [mm]

## 2.4. Discussion

The obtained critical distances $l$ are to be used to predict the fatigue life of notched parts. Using data obtained from one notch type, it should be possible to predict the service life for a different notch radius. For this prediction to work well, the critical distances for different notches must be the same or similar. It can be seen from the obtained results that the critical distances are not constant, but depend on the number of cycles to fracture. This property is expected and is related, among other things, to the fact that with a higher number of cycles to fracture and at the same time lower amplitude of fatigue load, a smaller area (smaller surface or internal defects or other places suitable for crack initiation) decides on the initiation of failure. For this reason, internal crack initiation occurs in these cases.

However, in terms of predicting the service life of the notched samples, we see that the curves of critical distances are similar for notch radii $r = 0.1$ mm and $r = 0.2$ mm. Therefore, predicting the service life of these types of notches from each other will be reliable. Similarly, it can be seen that the curves of critical distances are close or intersect in the area of the number of cycles between $10^6$ and $10^7$. In this area, therefore, the predictions of the notch life can also be made with a reasonable level of inaccuracy. On the other hand, in the region of the number of cycles to fracture between $10^8$ and $10^9$, the mutual differences of the critical distances obtained from the different radii of the notches $r$ are the largest. Fatigue life predictions need to be handled with care in this area.

Equation 1 also shows that if the life prediction is made from a critical distance value greater than the real one, then the predicted life is also greater than the actual and vice versa. Thus, for example, if we predict the life of a notch with a radius $r = 0.8$ mm based on the critical distance determined for $r = 0.2$ mm, we get a conservative prediction in the area around $10^6$ cycles, a relatively accurate prediction in the area around $10^7$ cycles, but over $10^8$ cycles, we get overestimated results, see Fig. 6(b). Such results lead to dangerous predictions.

The last important fact observed from Figure 6(b) is the size of the critical distance $l = 0$ mm for a very high number of cycles for notches with radii $r = 0.8$ mm and $r = 3$ mm. The stress $\overline{\sigma_y(l)}$ corresponding to the zero value of the critical distance is equal to the maximum stress at the tip of the notch. However, in cases of a very high number of cycles, where $l = 0 = \text{const.}$, it is clear that even the maximum theoretical stress in the notches is not so great that it should lead to crack initiation at a given number of cycles to the fracture $N_\mathrm{f}$. Nevertheless, the failure occurs. This difference between the theoretical results and reality can again be explained by the fact that in the case of gigacycle fatigue, defects of smaller dimensions than in the case of larger load amplitudes play a significant role in failure initiation. Apparently, in gigacycle fatigue, even traces of machining play an important role in initiating failure. Note that smooth test specimens were polished with sandpaper followed by abrasive paste with a grain size of up to 1 $\mu$m, while the notched specimens were tested as produced by turning.

# 3. Conclusion

- The critical length parameter depends on the size of the notch radius.
- The critical length parameter depends on the number of cycles to fracture, decreasing with the increasing number of cycles to fracture.
- If the decrease of the critical distance is ignored, and the greater value of $l$ is used, the lifetime prediction of the notched specimens is overestimated (unsafe side).
- In some notches, the critical length parameter reaches zero value, in these regions all fatigue predictions will be on the dangerous side.
- In the case of notches with notch radii 0.8 mm and 3 mm, the greatest dispersion of the length parameter is observed. In the case of a notch with a radius of 3 mm, the length parameter ranges from 0–0.27 mm. In the case of a notch with a radius of 0.8 mm, the length parameter ranges from 0–0.37 mm. On the other hand, notches with smaller radii are characterized by a small dispersion of length parameters.

## Nomenclature

| | |
|---|---|
| $\overline{\sigma_y(l)}$ | average stress over the critical distance |
| $\bar{l}$ | average critical length parameter |
| $\sigma_\mathrm{a}$ | the amplitude of failure loading |
| $\sigma_\mathrm{a}^\mathrm{n}$ | the amplitude of failure loading of the notched specimen |
| $\sigma_\mathrm{a}^\mathrm{s}$ | the amplitude of failure loading of the smooth specimen |
| $\sigma_{y,\mathrm{nom}}$ | nominal (average) stress over the cross-section of the notched specimen |

| | |
|---|---|
| $\sigma_y$ | axial stress |
| $A$ | the constant of Wöhler curve description |
| $B$ | the constant of Wöhler curve description |
| $L$ | the length of specimen |
| $l$ | critical distance |
| $l_{max}$ | maximum critical length parameter |
| $l_{min}$ | minimum critical length parameter |
| $N_{\mathrm{f}}$ | number of cycles to fracture |
| $R$ | radius of the specimen in the narrowest diameter |
| $r$ | notch radius |
| $x$ | distance from the notch tip |
| SSE | Sum of Squared Errors |
| TCD | Theory of Critical Distances |

## Acknowledgment

The research was supported by the Czech Science Foundation, project No. 21-14886S.

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