DYNAMIC ANALYSIS OF BLOCK CONCRETE FOUNDATION

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Abstract. *The paper is dedicated to the assessment of the dynamic response of a block concrete foundation to dynamic effect of an external force. The basic dynamic characteristic of the system is analysed. The state of resonance and introduction of the resonance curves are assessed. Description of the possibilities of numerical solution of the foundation response to dynamic loading and analysis of the behaviour of the foundation under different conditions are presented.*

Keywords

Block foundation, harmonic load, dynamic response, numerical modelling.

1. Introduction

Reinforced block concrete foundations are a common type of foundations for various equipment. The foundation is placed directly on the subsoil, or an insulating layer is inserted between the foundation and the subsoil. This layer can be made of a cork, a rubber, or spring insulators. The reason for its establishment can be different, for example, to increase the damping, to influence the natural frequency of the system or to reduce the transmission of vibrations to the surrounding environment. The analysis of the dynamic response of such foundations was analysed by various authors [1], [2], [3] and [4]. Beside the analytical methods, numerical methods are preferred nowadays [4], [5]. The paper assesses the dynamic response of the block foundation to the harmonically varying force generated by the hydraulic cylinder of the pulsator at different excitation frequencies.

2. Parameters of the System

The foundation is a reinforced concrete block placed on a cork slab. The dimensions of the foundation are $a = 2.0$ m, $b = 2.5$ m, $h = 1.6$ m, the ground plane of the foundation $A = a \cdot b = 2.0 \cdot 2.5 = 5.0$ m^2, the volume of the foundation $V = A \cdot h = 5.0 \cdot 1.6 = 8.0$ m^3, bulk density $\rho = 2\,500$ kg/m^3, the mass of foundation $m_z = V \cdot \rho = 8.0 \cdot 2\,500 = 20\,000$ kg. The subsoil compressibility module $K_z = 3.0 \cdot 10^8$ N/m^3. The elastic modulus of the cork slab $E_k = 1.0 \cdot 10^7$ N/m^2, the thickness of the cork slab $h_k = 0.2$ m. The mass of the tested element $m_s = 4\,000$ kg. The damping angular frequency $\omega_b = 12.566\,371$ rad/s. To solve the problem, the single degree of freedom (SDF) calculation model is chosen, Fig. 1.

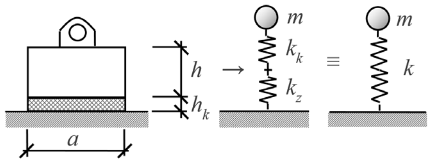

Fig. 1: SDF calculation model.

In the first step, the basic dynamic characteristics, which define the dynamic individuality of the system, are calculated. The mass of the system
$m = m_s + m_z = 4\,000 + 20\,000 = 24\,000$ kg.
The stiffness of the subsoil
$k_z = K_z \cdot A = 3.0 \cdot 10^8 \cdot 5.0 = 1.5 \cdot 10^9$ N/m.
The stiffness of the cork slab
$k_k = E_k \cdot A / h_k = 1.0 \cdot 10^7 \cdot 5.0/0.2 = 2.5 \cdot 10^8$ N/m.
The stiffness of the whole system
$k = k_k \cdot k_z/(k_k + k_z) = 2.5 \cdot 10^8 \cdot 1.5 \cdot 10^9/(2.5 \cdot 10^8 + 1.5 \cdot 10^9) = 214\,285\,714.2857$ N/m.
The natural angular frequency of undamped vibration
$\omega_0 = \sqrt{(k/m)} = \sqrt{(214\,285\,714.2857/24\,000)} = 94.491$ rad/s.
The angular frequency of damping $\omega_b = 12.566$ rad/s.
The natural angular frequency of damped vibration
$\omega_d = \sqrt{(\omega_0^2 - \omega_b^2)} = \sqrt{(94.491^2 - 12.566^2)} = 93.651$ rad/s.
The angular resonance frequency
$\omega_R = \sqrt{(\omega_0^2 - 2 \cdot \omega_b^2)} = \sqrt{(94.491^2 - 2 \cdot 12.566^2)} = 92.804$ rad/s.
The frequency ω_R corresponds to the motor revolutions in the value $n_R = 60 \cdot \omega_R/(2\pi) = 60 \cdot 92.8/(2\pi) = 886.175$ re/min.

In the second step, the response of the foundation block on dynamic effect of the pulsating force is analysed. The hydraulic cylinder of the pulsator acts on the foundation with a harmonically varying force $F(t) = F \cdot \sin(\omega \cdot t)$. The amplitude of the exciting force $F = 10\,000$ N. Static

deflection due to the amplitude of the exciting force
$v_{st} = F/k = 10\,000/214\,285\,714.2857 = 0.0466 \cdot 10^{-3}$ m.
Dynamic factor in resonance
$\delta_R = \omega_0^2/(2 \cdot \omega_b \cdot \omega_d) = 94.491^2/(2 \cdot 12.566 \cdot 93.651) = 3.7934$.
Amplitude of steady-state forced vibration in resonance
$v_{re} = v_{st} \cdot \delta_R = 0.0466 \cdot 10^{-3} \cdot 3.7934 = 0.177 \cdot 10^{-3}$ m.
The phase shift φ_R corresponding to ω_R
$\varphi_R = \mathrm{arctg}(-\sqrt{((\omega_0/\omega_b)^2-2)}) = \mathrm{arctg}(-\sqrt{((94.491/12.566)^2-2)}) = -1.4362$ rad.

The response of the system depends on the frequency ω of the exciting force $F(t)$ [2]. Let us define three dimensionless quantities

$$\eta = \omega/\omega_0, \ \eta_R = \omega_R/\omega_0 \text{ and } \xi = \omega_b/\omega_0. \quad (1)$$

We can now express the dynamic factor δ depending on the two parameters η and ξ as follows

$$\delta = 1/\sqrt{((1-\eta^2)^2 + 4 \cdot \eta^2 \cdot \xi^2)}. \quad (2)$$

It is valid for the phase shift angle φ

$$\varphi = -2 \cdot \xi \cdot \eta/(1-\eta^2). \quad (3)$$

The amplitude and phase resonance curves describe the influence of the frequency ω of the exciting force $F(t)$ on the response of the system. The amplitude resonance curve is a graphic dependence of the dynamic factor δ on the quantity η, plotted for a specific value ξ, Fig. 2. The phase resonance curve is a graphic dependence of the phase shift angle φ on the quantity η, plotted for a specific value ξ, Fig. 3.

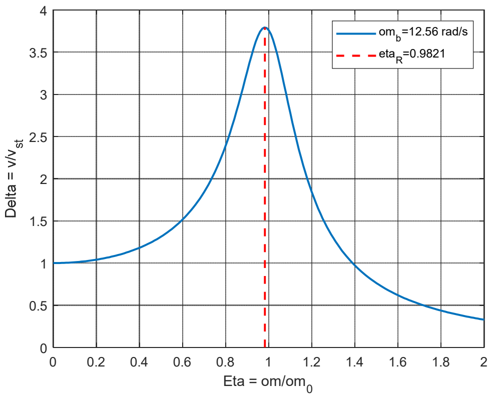

Fig. 2: Amplitude resonance curve at $\xi = 0.13299$, $\eta_R = 0.9821$.

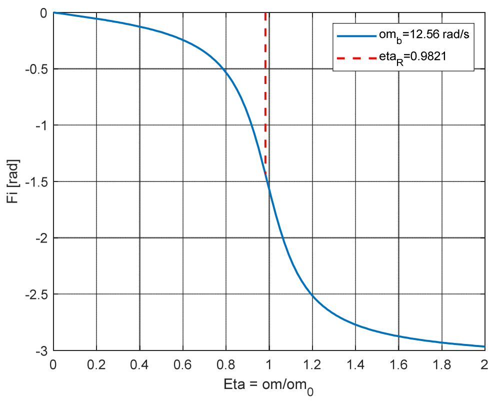

Fig. 3: Phase resonance curve at $\xi = 0.13299$, $\eta_R = 0.9821$.

3. Response in Time Domain

The equation of motion describing the vibration of the computational model from Fig. 1 is obtained on the basis of d'Alember principle as a condition of equilibrium of forces acting on the mass m. If we mark the vertical deflection in time $v(t)$ then we can write

$$\ddot{v}(t) + 2\omega_b \cdot \dot{v}(t) + \omega_0^2 \cdot v(t) = \frac{F}{m}\sin(\omega \cdot t). \quad (4)$$

The general solution of Equation (4) consists of a general solution of the equation without a right-hand side and a particular solution of the equation with a right-hand side

$$v(t) = v_o(t) + v_p(t) =$$
$$= e^{-\omega_b \cdot t} \cdot [A_o \cdot \sin(\omega_d \cdot t) + B_o \cdot \cos(\omega_d \cdot t)] +$$
$$+ A \cdot \sin(\omega \cdot t) + B \cdot \cos(\omega \cdot t). \quad (5)$$

The Equation (6) for the velocity is

$$\dot{v}(t) = e^{-\omega_b \cdot t} \cdot \omega_d \cdot [A_o \cdot \cos(\omega_d \cdot t) - B_o \cdot \sin(\omega_d \cdot t)]$$
$$+ \omega \cdot [A \cdot \cos(\omega \cdot t) - B \cdot \sin(\omega \cdot t)]. \quad (6)$$

The solution can also be written in the form

$$v(t) = e^{-\omega_b \cdot t} \cdot v_0 \cdot \sin(\omega_d \cdot t + \varphi_0) + v \cdot \sin(\omega \cdot t + \varphi). \quad (7)$$

The Equation (8) for the velocity

$$\dot{v}(t) = -\omega_b \cdot e^{-\omega_b \cdot t} \cdot v_0 \cdot \sin(\omega_d \cdot t + \varphi_0) +$$
$$+ \omega_d \cdot e^{-\omega_b \cdot t} \cdot v_0 \cdot \cos(\omega_d \cdot t + \varphi_0) + \omega \cdot v \cdot \cos(\omega \cdot t + \varphi). \quad (8)$$

The constants A, B, v, φ are calculated from the relations

$$A = \frac{F \cdot (\omega_0^2 - \omega^2)}{m \cdot [(\omega_0^2 - \omega^2)^2 + 4 \cdot \omega_b^2 \cdot \omega^2]}, \quad (9)$$

$$B = \frac{-2 \cdot F \cdot \omega_b \cdot \omega}{m \cdot [(\omega_0^2 - \omega^2)^2 + 4 \cdot \omega_b^2 \cdot \omega^2]}, \quad (10)$$

$$v = \sqrt{A^2 + B^2}, \quad (11)$$

$$\varphi = \mathrm{arctg}(B/A). \quad (12)$$

It is assumed that the foundation is initially at rest and then a harmonically variable force is applied. The initial conditions are as follows $t = 0$, $v(t) = 0$, $\dot{v}(t) = 0$. Substituting the initial conditions into Equations (5) and (6) gives the relations for calculating the integration constants A_0, B_0 in the form

$$B_0 = -B, \quad (13)$$

$$A_0 = -\omega \cdot A/\omega_d, \quad (14)$$

$$v_0 = \sqrt{A_0^2 + B_0^2}, \quad (15)$$

$$\varphi_0 = \mathrm{arctg}(B_0/A_0). \quad (16)$$

The response of the system in time depends on the angular frequency ω of the exciting force $F(t)$. In the sub-resonance area ($\omega < \omega_R$, $n < n_R$), dynamic deflections increase with increasing engine revolutions. The reason for this is that the time course of the excitation force $F(t)$ and the component of the steady-state forced vibration $v_p(t)$ change in phase, i.e. they support each other, Fig. 4. In the over-resonance area ($\omega > \omega_R$, $n > n_R$), dynamic deflections decrease with increasing engine revolutions. They can even reach a lower value than the static deflection. The reason for this is that the time course of the excitation force $F(t)$ and the component of the steady-state forced oscillation $v_p(t)$ change in opposite phase, i.e. they disturb each other, Fig. 5.

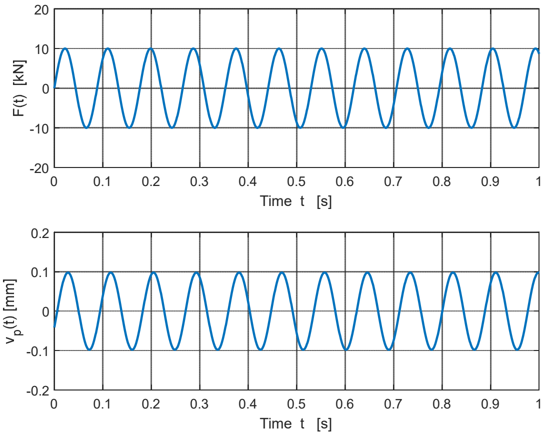

Fig. 4: $F(t)$ and $v_p(t)$ at ω = 71.209 rad/s, sub-resonance area.

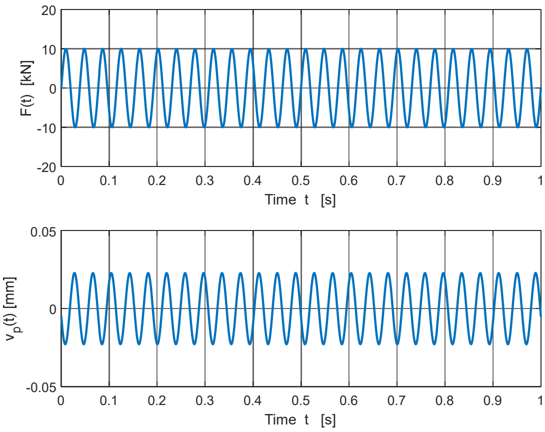

Fig. 5: $F(t)$ and $v_p(t)$ at ω = 163.363 rad/s, over-resonance area.

The resulting response in time $v(t)$ can preferably be calculated numerically in the environment of the Matlab program system [5]. For the need of a numerical solution, it is convenient to rewrite the motion Equation (4) in the form

$$\ddot{v}(t) = \frac{F}{m}\sin(\omega \cdot t) - 2\omega_b \cdot \dot{v}(t) - \omega_0^2 \cdot v(t). \qquad (17)$$

The **ode45** procedure [6] is used for the solution. The substitution $y_1(t) = v(t)$, $y_2(t) = \dot{v}(t)$ is introduced. Then the system of two equations of the 1st order is solved: $\dot{y}_1(t) = y_2(t)$ and $\dot{y}_2(t) = \ddot{v}(t)$.

```
% fder.m
function yder=fder(t, y)
F=10000;m=24000;om=92.804871;
omb=12.566371;omo2=94.491118^2;
yder=[y(2);((F/m)*sin(om*t)-
2*omb*y(2)-omo2*y(1))];
```

The solution itself is done using the **fdere.m** script

```
% fdere.m
[t,y]=ode45('fder',[0,1.0],[0,0]);
subplot(211)
plot(t,y(:,1)*1000,'LineWidth',1.5); grid
set(gca,'GridAlpha',0.7);
xlabel('Timet[s]');ylabel('v(t)[mm]');
```

The time course of the deflection $v(t)$ in resonance is shown in Fig. 6. The maximum deflection reaches the value of $0.177 \cdot 10^{-3}$ m.

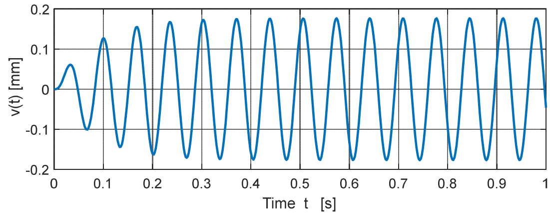

Fig. 6: Deflection $v(t)$ in resonance, v_{max} = 0.177 mm.

Due to damping, the component of natural vibration $v_0(t)$ disappears quickly (practically within 0.3 s) and only the component of steady forced vibration $v_p(t)$ remains, representing the particular solution of the motion Equation (4). The time course of the individual components in the sub-resonance area is shown in Fig. 7 and in the over-resonance area in Fig. 8.

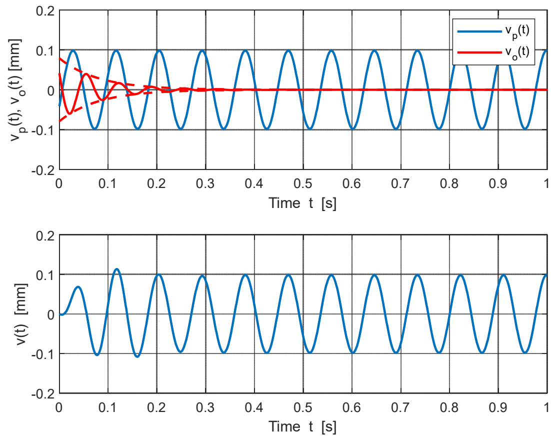

Fig. 7: Component. $v_0(t)$, $v_p(t)$, $v(t)$ at ω = 71.209 rad/s, sub-resonance area.

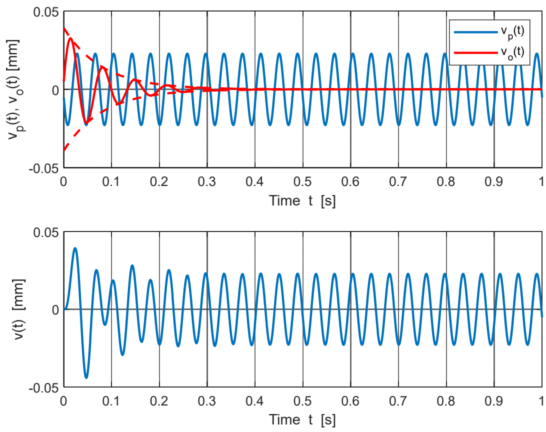

Fig. 8: Component $v_0(t)$, $v_p(t)$, $v(t)$ ω = 163.363 rad/s, over-resonance area.

4. Influence of Damping

The dynamic response of the system is significantly affected by the damping. which is clearly visible in Fig. 9. Amplitude and phase resonance curves are displayed for different values of the angular frequency of damping ω_b in the interval from $\omega_b = 1.0$ rad/s to the $\omega_b = \omega_0/\sqrt{2} = 66.81$ rad/s, corresponding to so-called semi-critical damping.

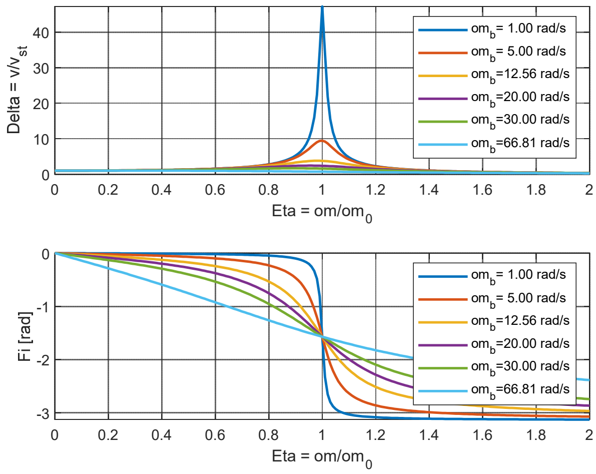

Fig. 9: Amplitude and phase resonance curves at various damping.

5. Conclusion

Reinforced block concrete foundation is a common type of foundation for various equipment. The insulating layer between the foundation and the subsoil is intended to provide the appropriate damping or to change the natural frequency of the system. A layer of cork represents good solution in such a case. The foundation analysed above shows a satisfactory dynamic response in the entire range of working frequencies of the pulsator. Static deflection due to the amplitude of the exciting force $v_{st} = 0.0466 \cdot 10^{-3}$ m. At the resonant frequency, $\omega_R = 92.804$ rad/s the dynamic factor has value $\delta_R = 3.7934$. Amplitude of steady-state forced vibration is $v_{re} = 0.177 \cdot 10^{-3}$ m and phase shift $\varphi_R = -1.4362$ rad. This is a favourable condition that does not have a negative effect on the operation of the equipment or on the surrounding environment. If $\eta > 1.389$ ($\omega > 131.248$ rad/s) the dynamic factor $\delta < 1$ and $v < v_{st}$. A problem could arise if the damping properties of the cork layer are about to change. If ω_b would fall below 0.5, the amplitude of steady-state forced vibration can be greater than 0.5 mm, which could directly affect the reliable operation of the machine.

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